Econ 583 HW 5

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Winter 2013
Due: Monday, February 11.

1 Reading

1. Hayashi, Chapters 3 and 6.
2. Hall, Chapter 2 and Chapter 3, section 5.
3. Bakaert, G., and R. Hodrick, 1993, "On Biases in the Measurement of Foreign Exchange Risk Premiums," *Journal of International Money and Finance*, 12, 115-138.

2 Derivations of Asymptotic Results

1. Hayashi, Chapter 3, Analytic Exercises, pages 243-250, #7 and 9. Note, these questions are very matrix algebra intensive but informative. Hayashi does a good job walking you through the steps.
2. Consider the simple AR(1) model

$$\begin{aligned} y_t &= \rho y_{t-1} + \varepsilon_t,\ \varepsilon_t \sim WN(0, \sigma^2),\ t = 1, \ldots, T \\ |\rho| &< 1,\ y_0 \text{ is fixed.} \end{aligned}$$

   (a) Is $\{y_t\}$ covariance stationary and ergodic? What are $E[y_t]$ and $\text{var}(y_t)$?

   (b) Consider the sample mean $\bar{y} = T^{-1} \sum_{t=1}^{T} y_t$. Show that $\bar{y}$ is an unbiased and consistent estimator for $E[y_t]$. For the consistency result, be sure to state the appropriate LLN.

   (c) What is the asymptotic distribution of $\sqrt{T}\bar{y}$? Be sure to state the appropriate CLT to justify your result.

   (d) How would you estimate the asymptotic variance of $\sqrt{T}\bar{y}$?

(e) The least squares estimator of ρ is $\hat{\rho} = \left(\sum_{t=1}^{T} y_{t-1}^2\right)^{-1} \sum_{t=2}^{T} y_{t-1} y_t$. Is $\hat{\rho}$ an unbiased estimator of ρ? Briefly explain.

(f) Show that $\hat{\rho}$ is a consistent estimator of ρ. Be sure to state the appropriate LLN to justify this result.

(g) Let $g_t = y_{t-1}\varepsilon_t$ and $I_t = \{y_t, y_{t-1}, \ldots y_0\}$. Show that $\{g_t, I_t\}$ is a MDS.

(h) What is the asymptotic distribution of $\sqrt{T}(\hat{\rho} - \rho)$? Be sure to state the appropriate CLT to justify your result.

(i) How would you estimate the asymptotic variance of $\sqrt{T}(\hat{\rho} - \rho)$?

3 GMM Estimation with Serial Correlation

1. Hayashi, Chapter 6, Analytic Exercises, page 437-438, #9

2. Hayashi, Chapter 6, Empirical Exercises (pages 438 - 440): 1(b) - 1(f). You can use any software that can do OLS estimation with HAC standard errors (e.g. Eviews, Matlab, R, S+FinMetrics, Stata).