Econ 583
Winter 2013
HW #3

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Due: Monday, January 28

Reading

- Hayashi, *Econometrics*, Chapter 1, Section 1.4 (Hypothesis Testing under Normality and Monte Carlo Exercises (pages 81 - 84); Chapter 2, Section 2.4. (Hypothesis Testing).
- My lecture notes on hypothesis testing (see webpage, class slides part 3)
- Kleiber and Zeileis, *Applied Econometrics in R*, chapter 7 (Programming your own analysis).

Part I: Questions from Hayashi.

Chapter 2

1. Questions for Review, page 149, questions 1, 2 and 3.

Chapter 3

1. Hayashi, Questions for Review, page 193, questions 3 and 4.
2. Hayashi, Questions for Review, page 197, question 1.
3. Hayashi, Questions for Review, page 204, questions 6 and 7.

4. Hayashi, Questions for Review, page 208, questions 4.

Part II: Computer Exercises

On the class homework page is the file `econ583Lab3MonteCarlo.r`, which contains R code for performing a simple Monte Carlo analysis to evaluate the asymptotic normal approximation to the finite sample distribution of the sample mean computed from an iid sample of $\chi^2(1)$ random variables. Use this code as a template to do this assignment. Copy and paste relevant results (output, graphs etc.) from running this program in a document to answer the questions.

Exercises

Let $X_1, \ldots, X_n$ be an iid sample with $X_i \sim \chi^2(df)$ (chi-square with df degree of freedom) where df is assumed to be known. It can be shown that $E[X_i] = \mu = df$ and $\text{var}(X_i) = \sigma^2 = 2 \cdot df$. Consider estimating μ and σ^2 using the sample mean and variance

$$\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} X_i, \ \hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\mu})^2.$$

The Central Limit Theorem gives the following asymptotic approximation to the finite sample distribution of $\hat{\mu}$

$$\hat{\mu} \overset{A}{\sim} N\left(\mu, \ \text{avar}(\hat{\mu})\right), \ \text{avar}(\hat{\mu}) = \frac{\sigma^2}{n} = \frac{2df}{n} \tag{1}$$

Notice that in this example, $\text{avar}(\hat{\mu})$ does not involve any unknown parameters because we assumed df is known. Still we could estimate it using $\widehat{\text{avar}}(\hat{\mu}) = \frac{\hat{\sigma}^2}{n}$, if we wanted to. The finite sample distribution (the distribution of $\hat{\mu}$ for a fixed n) of $\hat{\mu}$ is not normal for any fixed n. We will use Monte Carlo methods to simulate the unknown finite sample distribution of $\hat{\mu}$ for given values of df and n, and then compare the asymptotic approximation (1) to the simulated finite sample distribution. We will evaluate the asymptotic approximation by (1) comparing the finite sample pdf/CDF to the asymptotic pdf/CDF; (2) computing the coverage probability of asymptotic 95% confidence intervals; (3) computing the rejection frequencies of nominal 5% t-tests and Wald tests. Asymptotic 95% confidence intervals are computed as

$$\hat{\mu} \pm 1.96 \times \sqrt{\text{avar}(\hat{\mu})} = 1.96 \times \text{ase}(\hat{\mu}).$$

The hypotheses to be tested are

$$H_0 : \mu = 1 \text{ vs. } H_1 : \mu \neq 1$$

and the asymptotic test statistics are

$$t_{\mu=1} = \frac{\hat{\mu} - 1}{\text{ase}(\hat{\mu})}, \text{ Wald}_{\mu=1} = (t_{\mu=1})^2 .$$

Under $H_0 : \mu = 1$, $t_{\mu=1} \sim N(0, 1)$ and $\text{Wald}_{\mu=1} \sim \chi^2(1)$.

1. Run a Monte Carlo simulation with $n = 10$ and $df = 1$ using 10,000 simulations (this is the default parameterization in `econ583Lab3MonteCarlo.r`). In this simulation, use the true value of $\text{avar}(\hat{\mu})$ given in (1) (set the variable `useTrueAVAR` to `TRUE` in the code).

    (a) Using the simulated values of $\hat{\mu}$, compute the finite sample bias of $\hat{\mu}$, compare the histogram of the $\hat{\mu}$ values to the asymptotic normal pdf, and compare the empirical quantiles of the $\hat{\mu}$ to the quantiles of the asymptotic normal pdf. Comment on what you see.

    (b) Compute the empirical coverage probability of the asymptotic 95% confidence interval. How close is it to 95%?

    (c) Compare the histograms of the simulated t-stats and Wald stats to the $N(0, 1)$ and $\chi^2(1)$ pdfs, respectively. For the t-stats, create normal QQ-plot and for the Wald stats create a chi-square QQ-plot. How well do the asymptotic distributions capture the finite sample distributions?

    (d) Compute the empirical rejection frequencies of the nominal 5% t-test and Wald test and use these values to compute the finite sample size distortion of these asymptotic tests. Compare the finite sample 5% critical values to the 5% asymptotic critical values.

    (e) Overall, do you think the asymptotic approximation is good or bad? Briefly justify your answer.

2. Repeat the analysis of question 1 using $n = 10$ and $df = 1$ but use the estimated value of $\text{avar}(\hat{\mu})$ (set the variable `useTrueAVAR` to `FALSE` in the code). That is, use $\widehat{\text{avar}}(\hat{\mu}) = \frac{\hat{\sigma}^2}{n}$.

3. Repeat the analysis of question 1 using $n = 100$ and $df = 1$. In this simulation, use the true value of $\text{avar}(\hat{\mu})$ given in (1) (set the variable `useTrueAVAR` to `TRUE` in the code).

4. Repeat the analysis of question 1 using $n = 100$ and $df = 1$. In this simulation, use the estimated value of $\text{avar}(\hat{\mu})$ given in (1) (set the variable `useTrueAVAR` to `FALSE` in the code).