Econ 583
Winter 2013
HW #2

Eric Zivot

Due: Monday, January 21

1. Hayashi, Questions for Review, page 123, questions 2 and 3.
2. Hayashi, Questions for Review, page 130, question 5.
3. Hayashi, Problem Set for Chapter 2 (pages 168-176)
   (a) Analytic Exercise 4
   (b) Analytic Exercise 5
   (c) Analytic Exercise 9
4. Let $X_1, \ldots, X_n$ be iid random variables representing monthly continuously compounded returns on an asset (i.e., $X_i = \ln(P_i/P_{i-1})$ where P_i denotes the price at the end of of month i) with $X_1 \sim N(\mu, \sigma^2)$. Common consistent estimators of μ and σ^2 are

$$\hat{\mu} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i, \;\; \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \bar{X})^2.$$

Let $\theta = (\mu, \sigma^2)'$ and $\hat{\theta} = (\hat{\mu}, \hat{\sigma}^2)'$. It can be shown (we will show this later in the class)

$$\begin{aligned} \hat{\theta} &\overset{A}{\sim} N(\theta, V), \\ V &= \begin{pmatrix} \sigma^2/n & 0 \\ 0 & 2\sigma^4/n \end{pmatrix} = \text{avar}(\hat{\theta}) \end{aligned}$$

(a) How would you consistently estimate $\text{avar}(\hat{\theta})$?

(b) For $\alpha \in (0,1)$, show that the $\alpha \times 100\%$ quantile of the $N(\mu, \sigma^2)$ distribution can be expressed as

$$q_\alpha^X = \mu + \sigma \times q_\alpha^Z,$$

where q_α^Z is the $\alpha \times 100\%$ quantile of the $N(0,1)$ distribution (i.e, $\Pr(Z \leq q_\alpha^Z) = \alpha$).

(c) An estimate of the $\alpha \times 100\%$ quantile is

$$\hat{q}_\alpha^X = \hat{\mu} + \hat{\sigma} \times q_\alpha^Z$$

Show that $\hat{q}_\alpha^X$ is consistent for q_α^X, and using the asymptotic distribution of $\hat{\theta}$ and the delta method compute an asymptotic standard error for $\hat{q}_\alpha^X$.

(d) In risk management, lower quantiles of return distributions are used to compute so-called value-at-risk (VaR) measures. VaR represents the dollar loss over an investment horizon with a stated probability. For example, with an initial investment of \$$W$ the 5% VaR for a monthly investment with continuously compounded return $X \sim N(\mu, \sigma^2)$ is

$$VaR_{0.05} = \$W \times \left[\exp(q_{0.05}^X) - 1\right].$$

An estimate of $VaR_{0.05}$ is

$$\widehat{VaR}_{0.05} = \$W \times \left[\exp(\hat{q}_{0.05}^X) - 1\right].$$

Show that $\widehat{VaR}_{0.05}$ is consistent for $VaR_{0.05}$, and use the delta method to compute an asymptotic standard error for $\widehat{VaR}_{0.05}$.

(e) A common performance measure is the so-called Sharpe ratio (SR)

$$SR = \frac{\mu - r_f}{\sigma},$$

where r_f is a fixed (non-random) risk-free rate. An estimate of the SR is

$$\widehat{SR} = \frac{\hat{\mu} - r_f}{\hat{\sigma}}.$$

Show that $\widehat{SR}$ is consistent for SR, and use the delta method to compute an asymptotic standard error for $\widehat{SR}$.