Econ 583 Lec 5

Note Title 1/23/2013

Topics

Single equation linear GMM

- Linear regression with endogenous variables
- Sources of endogeneity
- endogeneity bias
- Method of moments estimation
- Identification
- Generalized Method of Moments (GMM)
- Efficient GMM
- 2-step GMM
- iterated GMM
- Continuous updating GMM

Endogeneity bias

$i = 1, \ldots, n$

$$y_i = z_i' \delta_0 + \varepsilon_i \qquad E\{z_i \varepsilon_i\} \neq 0$$

$$\hat{\delta} = \left(\sum_1^n z_i z_i'\right)^{-1} \sum_1^n z_i y_i$$

$$= \delta_0 + \left(\sum_1^n z_i z_i'\right)^{-1} \sum_1^n z_i \varepsilon_i$$

$$= \delta_0 + \left(\frac{1}{n}\sum_1^n z_i z_i'\right)^{-1} \frac{1}{n} \sum_1^n z_i \varepsilon_i$$

$$\left(\frac{1}{n}\sum_1^n z_i z_i'\right)^{-1} \xrightarrow{p} E\{z_i z_i'\}^{-1} \qquad \frac{1}{n}\sum_1^n z_i \varepsilon_i \xrightarrow{p} E\{z_i \varepsilon_i\} \neq 0$$

Estimate β_0 using moment conditions: Method of Moments Estimation

Ex: OLS as a method of moments estimator

$$y_i = z_i'\beta_0 + \varepsilon_i \qquad i = 1, \dots, n$$

$$E\{z_i \varepsilon_i\} = 0 \iff E[z_i(y_i - z_i'\beta_0)] = 0$$

Sample moment

$$\frac{1}{n}\sum_1^n \hat{z}_i \hat{\varepsilon}_i = \frac{1}{n}\sum_1^n \hat{z}_i (y_i - z_i'\hat{\beta})$$

$\hat{\beta}$ is some estimator of β_0

Choose $\hat{\beta}$ s.t. sample moment = pop moment

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i.e. $\frac{1}{n}\sum_1^n \hat{z}_i (y_i - z_i'\hat{\beta}) = 0$ = F.O.C for OLS

$$\Rightarrow \frac{1}{n}\sum_1^n \hat{z}_i y_i = \frac{1}{n}\sum_1^n \hat{z}_i z_i' \hat{\beta}$$

$$\Rightarrow \hat{\beta} = \left(\frac{1}{n}\sum_1^n \hat{z}_i z_i'\right)^{-1} \frac{1}{n}\sum_1^n \hat{z}_i y_i = \hat{\beta}_{OLS}$$