Econ 583 Lec 3

Note Title 1/14/2013

Topics

- Central Limit Theorems
- Slutsky's Theorem ; Continuous Mapping Theorem
- Delta method
- Time Series Concepts
- Ergodic Theorem

Ex. Illustrate Slutsky theorem

$X_1, \dots, X_n$ iid $E[X_i] = \mu$, $var(X_i) = \sigma^2$

Suppose σ^2 is unknown.

We showed that $\hat{\sigma}^2 = \frac{1}{n} \sum_i^n (X_i - \bar{X})^2 \xrightarrow{p} \sigma^2$

By Slutsky thm part 4, $\hat{\sigma} \xrightarrow{p} \sigma$. Consider

$$Y_n = \frac{\bar{X} - \mu}{\hat{\sigma}/\sqrt{n}} = \left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}\right)\left(\frac{\sigma}{\hat{\sigma}}\right)$$

$$= X_n \cdot Z_n$$

$X_n \xrightarrow{d} N(0,1)$ by CLT

$Z_n \xrightarrow{p} 1$ by LLN $\therefore$ Slutsky's thm

So by extension to Slutsky's thm

$$Y_n = X_n \cdot Z_n \xrightarrow{d} Z \cdot 1 \sim N(0,1)$$

Hence,
$$\bar{X} \overset{A}{\sim} N\left(\mu, \frac{\hat{\sigma}^2}{n}\right)$$

This justifies replacing σ^2 by $\hat{\sigma}^2$ in the asymptotic distribution

Ex: Illustration of CMT

$$Y_n = \left(\frac{\bar{X}-\mu}{\hat{\sigma}/\sqrt{n}}\right) \xrightarrow{d} N(0,1)$$

Then by CMT

$$(Y_n)^2 = \left(\frac{\bar{X}-\mu}{\hat{\sigma}/\sqrt{n}}\right)^2 \xrightarrow{d} N(0,1)^2 \equiv \chi^2(1)$$

because $f(x) = x^2$ is continuous for all x.

Delta Method

Ex: Asymptotic distn of $\frac{1}{\bar{X}}$

$X_1, \ldots, X_n$ iid $E[X_i] = \mu$, $var(X_i) = \sigma^2$

$$\sqrt{n}(\bar{X} - \mu) \xrightarrow{d} N(0, \sigma^2)$$

$$\eta = g(\mu) = \frac{1}{\mu} = \mu^{-1}, \quad g'(\mu) = -\mu^{-2}$$

$$g'(\mu)^2 = \mu^{-4}$$

Delta method gives

$$\sqrt{n}(\hat{\eta} - \eta) = \sqrt{n}\left(\frac{1}{\bar{X}} - \frac{1}{\mu}\right) \xrightarrow{d} N\left(0, \mu^{-4} \cdot \sigma^2\right)$$

$$\frac{1}{\bar{X}} \overset{A}{\sim} N\left(\frac{1}{\mu}, \frac{\mu^{-4} \cdot \sigma^2}{n}\right)$$

$$\widehat{avar}\left(\frac{1}{\bar{X}}\right) = \frac{\bar{X}^{-4} \cdot \hat{\sigma}^2}{n}, \quad \hat{\sigma}^2 = \frac{1}{n}\Sigma_1^n (X_i - \bar{X})^2$$

Asymptotic 95% C.I. for $\eta = \frac{1}{\mu}$

$$\frac{1}{\bar{X}} \pm 1.96\sqrt{\frac{\bar{X}^{-4}\hat{\sigma}^2}{n}}$$