Econ 583 Lec 1

Note Title 1/7/2013

Topics

- Course introduction
- Large sample theory
  - introduction to asymptotic analysis
  - Laws of large numbers
  - Consistency of estimators

Motivating Example

$$X_1, \ldots, X_n \quad \text{iid} \quad E[X_i] = \mu, \ \text{var}(X_i) = \sigma^2$$

A natural estimator of μ is

$$\hat{\mu} = \bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

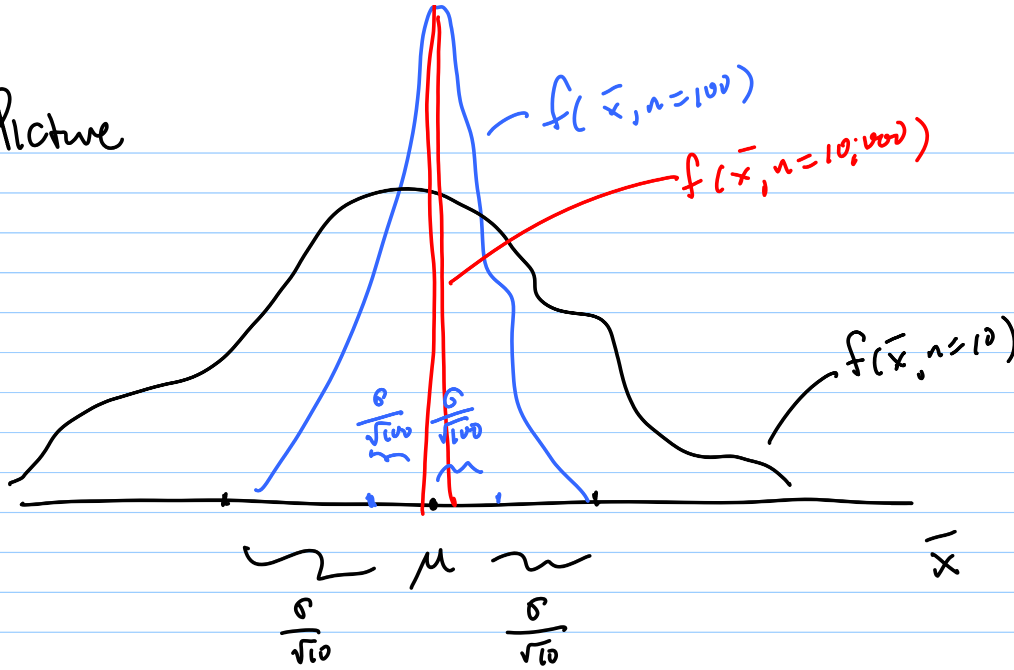
Properties

$$E[\bar{X}] = \frac{1}{n} \sum_{i=1}^{n} E[X_i] = \frac{1}{n} \sum_{i=1}^{n} \mu = \mu$$

$$\text{var}(\bar{X}) = \text{var}\left(\frac{1}{n} \sum_{i}^{n} X_i\right) = \frac{1}{n^2} \sum_{i=1}^{n} \text{var}(X_i)$$

$$= \frac{1}{n^2} \sum_{i=1}^{n} \sigma^2 = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

Picture



As $n \to \infty$ the pdf of $\bar{x}$ collapses at μ

Notation:

$$Var(\bar{x}) = \frac{\sigma^2}{n}, \quad \sqrt{Var(\bar{x})} = \underbrace{SE(\bar{x})}_{\text{Standard error of } \bar{x}} = \frac{\sigma}{\sqrt{n}}$$

Asymptotic Normality

Finite sample distn of $\bar{x} = \frac{1}{n}\sum_{i}^{n} x_i$ is not known b/c the distn of x is not known. $f(x)$ is unknown.

However, we know that

$$E\{\bar{X}\} = \mu$$

$$\mathrm{var}(\bar{X}) = \sigma^2/n$$

Create the standardized r.v.

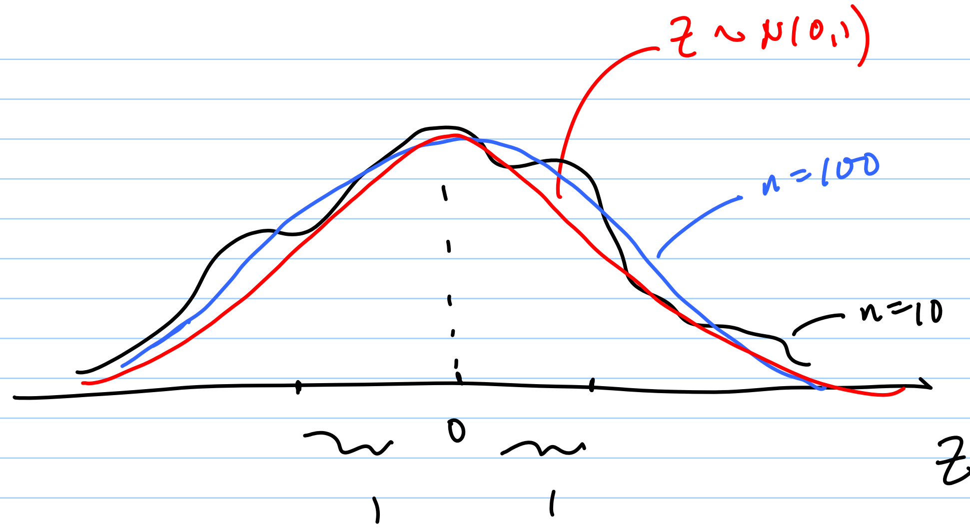
$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \sqrt{n}\left(\frac{\bar{X} - \mu}{\sigma}\right)$$

$$E\{Z\} = 0, \quad \mathrm{var}(Z) = 1$$

So the CLT says for large enough n

$$Z = \sqrt{n}\left(\frac{\bar{X} - \mu}{\sigma}\right) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n}\left(\frac{X_i - \mu}{\sigma}\right)$$

has a distn that is well approximated by a standard normal distribution



Notation

$$Z = \sqrt{n}\left(\frac{\bar{X}-\mu}{\sigma}\right) \overset{A}{\sim} N(0,1)$$

" is asymptotically distributed as "

$N(0,1)$ is the approximating distn for

$$\sqrt{n}\left(\frac{\bar{x}-\mu}{\sigma}\right)$$

What about $\bar{x}$?

$$\sqrt{n}\left(\frac{\bar{x}-\mu}{\sigma}\right) = z \overset{A}{\sim} N(0,1)$$

$$\Rightarrow \quad \bar{x}-\mu = \frac{\sigma}{\sqrt{n}} \cdot z$$

$$\Rightarrow \quad \bar{x} = \mu + \frac{\sigma}{\sqrt{n}} \cdot z$$

$$E\{\bar{x}\} = \mu$$

$$\operatorname{var}(\bar{x}) = \frac{\sigma^2}{n}$$

$$\bar{x} \overset{A}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$$

Notation:

$\operatorname{avar}(\bar{x}) = \frac{\sigma^2}{n}$ = variance of the asymptotic normal distribution.

$$\operatorname{avar}(\sqrt{n}(\bar{x}-\mu)) = \sigma^2$$

ase or ASE

$$ASE(\bar{x}) = \sqrt{avar(\bar{x})}$$
$$= \sigma/\sqrt{n}$$

If σ is unknown, we compute an estimate ASE using a consistent estimate of σ:

$$\widehat{ASE}(\bar{x}) = \hat{\sigma}/\sqrt{n} \quad \text{s.t.}$$

$\hat{\sigma}$ is consistent for σ

A natural estimate of σ^2 is the sample variance

$$\hat{\sigma}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

Inference: 95% confidence for μ

$$\bar{x} \pm 1.96 * ASE(\bar{x})$$
$$\bar{x} \pm 1.96 * \widehat{ASE}(\bar{x})$$

i.e. $\bar{x} \pm 1.96 \cdot \hat{\sigma}/\sqrt{n}$

$H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$

$$\bar{X} \overset{A}{\sim} N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$t_{\mu=\mu_0} = \frac{\bar{X} - \mu_0}{ASE(\bar{X})}$$

$$= \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}} = \sqrt{n}\left(\frac{\bar{X} - \mu_0}{\sigma}\right)$$

Under $H_0: \mu = \mu_0$

$$t_{\mu=\mu_0} \overset{A}{\sim} N(0,1)$$

Reject $H_0: \mu = \mu_0$ vs. $H_1: \mu \neq \mu_0$

if $|t_{\mu=\mu_0}| > 1.96$

asymptotic size of the test is 5% but in finite samples (e.g. $n = 25$) the actual size could be different from 5%.