Econ 583 Lab 6 Solutions

Note Title Nonlinear GMM 12/18/2009

4 $Y_1, \dots, Y_n$ iid Gamma (α, β):

$$f(y|\theta) = \frac{\alpha^{\beta}}{\Gamma(\beta)} e^{-\alpha y} y^{\beta - 1}, \quad y > 0$$

$\alpha > 0, \ \beta > 0$

Moments: $E[Y] = \beta/\alpha$

$E[Y^2] = \beta(\beta+1)/\alpha^2$

$E[\ln Y] = \psi(\beta) - \ln\alpha$, $\psi(\beta) = \frac{d}{d\beta}\ln\Gamma(\beta)$

$E[1/Y] = \alpha(\beta - 1)$

a. Let $w_t = \{Y_t, Y_t^2, \ln Y_t, 1/Y_t\}$. Then define $\theta = (\alpha, \beta)'$ and

$$\underset{4\times 1}{g(w_t, \theta)} = \begin{bmatrix} Y_t - \beta/\alpha \\ Y_t^2 - \beta(\beta+1)/\alpha^2 \\ \ln Y_t - \psi(\beta) + \ln\alpha \\ 1/Y_t - \alpha/(\beta - 1) \end{bmatrix}$$

Under the true model $g(w_t, \theta_0) = 0$

which can be verified directly given the moment equations

b. Because $Y_1, \ldots, Y_n$ is a random sample $g(w_t, \theta)$ is ergodic-stationary.

c. For efficient GMM using $g(w_t, \theta)$, the GMM objective function is

$$J(\theta, \hat{S}^{-1}) = g_n(\theta)' \hat{S}^{-1} g_n(\theta)$$

Where

$$g_n(\theta) = \frac{1}{n} \Sigma_i^n g(w_t, \theta)$$

$$\hat{S} = \frac{1}{n} \Sigma_1^n g(w_t, \hat{\theta}) g(w_t, \hat{\theta})'$$

$$\hat{\theta} \xrightarrow{p} \theta.$$

For example, $\hat{\theta}$ could be computed from an inefficient GMM estimation with $\hat{W} = I_4$:

$$\hat{\theta} = \underset{\theta}{\operatorname{argmin}}\ g_n(\theta)' I_n g_n(\theta)$$

The F.O.C's for minimizing $J(\theta, \hat{S}^{-1})$ are

$$\underset{2\times 1}{0} = \frac{\partial J}{\partial \theta} = \underset{2\times 4}{\frac{\partial g_n(\hat{\theta})'}{\partial \theta'}} \underset{4\times 4}{\hat{S}^{-1}} \underset{4\times 1}{g_n(\hat{\theta})}$$

Given the moment equations this produces a system of non-linear equations in α, β and no analytic solution exists. The GN iteration scheme is

$$\hat{\theta}_{n+1} = \hat{\theta}_n + (G_n' \hat{S}^{-1} G_n)^{-1} G_n' \hat{S}^{-1} g_n(\hat{\theta}_n)$$

Where $\underset{4\times 2}{G_n} = \dfrac{\partial g_n(\hat{\theta}_n)}{\partial \theta'}$