University of Washington
Department of Economics

Economics 583
Assignment #4
Single Equation Linear GMM
Due: Monday, 2/4/13

Readings:

1. Hayashi, *Econometrics*, Chapter 3
2. Hall, *Generalized Method of Moments*, Chapter 2.
3. Baum, C.F., M.E., Schaffer, and S. Stillman (2003). "Instrumental Variables and GMM: Estimation and Testing, The Stata Journal, 3(1), 1-31.
4. Cliff, M. (2003). "GMM and MINZ Program Libraries for Matlab" (.pdf on class syllabus page)
5. Newey, W. and West, K. (1987). "Hypothesis testing with efficient method of moment estimators," *International Economic Review*, 28, 777-787.
6. *Eviews:* See help listings on IV and GMM estimation; Wald test, coefficient restriction; J-statistic; Hausman test.

Data: (available on class homework page)

1. grilic.xls. Cross sectional data set for wage equation estimation. See the description of the variables on page 250 of Hayashi..

Analytic Exercises

1. Hayashi, Chapter 3, Questions for Review, pages 215-216, # 7; Questions for Review, page 235, #1, 2 and 6.
2. Hayashi, Chapter 3, Analytic Exercises, page 245, #1, 2, 3.
3. In this exercise you will prove that the GMM-LR statistic has an asymptotic chi-square distribution. In the simple linear GMM model

$$y_t = z_t'\delta + \varepsilon_t$$
$$E[g_t] = E[x_t\varepsilon_t] = 0$$

where z_t is a L x 1 vector and x_t is a K x 1 vector with $K > L$, consider testing the hypothesis

$$H_0 : \delta = \delta_0$$

using the GMM-LR statistic

$$LR_{GMM} = J(\delta_0, \hat{S}^{-1}) - J(\hat{\delta}(\hat{S}^{-1}), \hat{S}^{-1})$$
$$J(\delta_0, \hat{S}^{-1}) = n \cdot \overline{g}(\delta_0)' \hat{S}^{-1} \overline{g}(\delta_0)$$
$$J(\hat{\delta}(\hat{S}^{-1}), \hat{S}^{-1}) = n \cdot \overline{g}(\hat{\delta}(\hat{S}^{-1}))' \hat{S}^{-1} \overline{g}(\hat{\delta}(\hat{S}^{-1}))$$
$$\overline{g}(\delta) = S_{xy} - S_{xz}\delta, \; S_{xy} = n^{-1}\sum x_t y_t, \; S_{xz} = n^{-1}\sum x_t z_t{}'$$
$$\hat{\delta}(\hat{S}^{-1}) = (S_{xz}' \hat{S}^{-1} S_{xz})^{-1} S_{xz}' \hat{S}^{-1} S_{xy}$$

Assume that $\hat{S} \xrightarrow{p} S = E[g_t g_t{}']$.

a) Show that

$$J(\delta_0, \hat{S}^{-1}) = \left(\sqrt{n} S_{x\varepsilon}\right)' \hat{S}^{-1} \left(\sqrt{n} S_{x\varepsilon}\right)$$
$$J(\hat{\delta}(\hat{S}^{-1}), \hat{S}^{-1}) = \left(\sqrt{n} S_{x\hat{\varepsilon}}\right)' \hat{S}^{-1} \left(\sqrt{n} S_{x\hat{\varepsilon}}\right)$$

where $\hat{\varepsilon}_t = y_t - z_t{}'\hat{\delta}(\hat{S}^{-1}) = \varepsilon_t - z_t{}'(\hat{\delta}(\hat{S}^{-1}) - \delta_0)$

b) Next, show that

$$J(\hat{\delta}(\hat{S}^{-1}), \hat{S}^{-1}) = J(\delta_0, \hat{S}^{-1}) - 2n(\hat{\delta}(\hat{S}^{-1}) - \delta_0)' S_{xz}{}' \hat{S}^{-1} S_{x\varepsilon}$$
$$+ n(\hat{\delta}(\hat{S}^{-1}) - \delta_0)' S_{xz}{}' \hat{S}^{-1} S_{xz} (\hat{\delta}(\hat{S}^{-1}) - \delta_0)$$

c) Finally, using $\sqrt{n}(\hat{\delta}(\hat{S}^{-1}) - \delta_0) = (S_{xz}' \hat{S}^{-1} S_{xz})^{-1} S_{xz}{}' \hat{S}^{-1} \sqrt{n} S_{x\varepsilon}$ show that

$$J(\delta_0, \hat{S}^{-1}) - J(\hat{\delta}(\hat{S}^{-1}), \hat{S}^{-1}) = \left(S_{xz}{}' \hat{S}^{-1} \sqrt{n} S_{x\varepsilon}\right)' (S_{xz}' \hat{S}^{-1} S_{xz})^{-1} S_{xz}{}' \hat{S}^{-1} \sqrt{n} S_{x\varepsilon}$$
$$\xrightarrow{d} \chi^2(L)$$

4. Consider the continuous updating GMM estimator under the assumption of conditional homoskedastity :

$$\delta(\hat{S}_{CU}^{-1}) = \arg\min_{\delta} n g_n(\delta)' \hat{S}_{CU}^{-1}(\delta) g_n(\delta)$$
$$g_n(\delta)' = s_{xy} - S_{xz}\delta = n^{-1}X'y - n^{-1}X'Z\delta$$
$$\hat{S}_{CU}(\delta) = \hat{\sigma}^2(\delta) \cdot n^{-1}X'X = n^{-1}(y - Z\delta)'(y - Z\delta)n^{-1}X'X$$

Show that

$$ng_n(\delta)'\hat{S}_{CU}^{-1}(\delta)g_n(\delta) = n\frac{(y-Z\delta)'P_X(y-Z\delta)}{(y-Z\delta)'(y-Z\delta)}$$

Empirical Exercises

For the empirical exercises, you may use any statistical software package you like (e.g. EVIEWS, STATA, S-PLUS, MATLAB, R etc). However, for this assignment (and for linear GMM in general) I encourage the use of either Eviews or Stata. I you are using Stata then you should use the ivreg2 library. If you are using EVIEWS, be aware that you cannot fix the weight matrix during GMM estimation in Eviews 6. This is important for computing certain test statistics. Eviews 7 contains many improvements for GMM estimation.

1. Hayashi, Chapter 3, Empirical Exercises, pages 250-254, # (a) – (g)