Economic Models that generate Geometric DL Behavior

I. Adaptive Expectations Model

$$y_t = \alpha + \beta \cdot x_t^e + \epsilon_t$$

x_t^e = unobservable expectations variable e.g.
could have $x_t^e = E_t\{x_{t+1}\} \cdot = E\{x_{t+1}$

I_t = info a
at the

Model for expectations formation:

$$\underset{\text{revision in expectations}}{\Delta x_t^e} = \underbrace{(1-\lambda)}_{\text{speed of adj. to new info.}} \underbrace{(x_t - x_{t-1}^e)}_{\text{deviation of actual value from previous expectation}} \quad , 0 \le \lambda < 1$$

$I_t = \{x_t$,

Alternatively

$$x_t^e = \lambda x_{t-1}^e + (1-\lambda) x_t$$

$\lambda = 1$: ignore current data — no revision in expectation

$\lambda = 0.9$ very slow revision to new info

$\lambda = 0.1$ very fast revision.

Solving the expectations model

$$X_t^e = \lambda L X_t^e + (1-\lambda) X_t$$

$$\Rightarrow \quad (1-\lambda L) X_t^e = (1-\lambda) X_t$$

$$\Rightarrow \quad X_t^e = (1-\lambda L)^{-1}(1-\lambda) X_t \qquad \text{provided } |\lambda|<1$$

$$= (1-\lambda)\sum_{k=0}^{\infty} \lambda^k L^k X_t$$

or

$$X_t^e = (1-\lambda)\sum_{k=0}^{\infty} \lambda^k X_{t-k}$$

$\Rightarrow$ expectation = ~~geometric~~ weighted average of all past observations.

Substituting into the original regression gives

$$y_t = \alpha + \beta(1-\lambda)\sum_{k=0}^{\infty} \lambda^k X_{t-k} + \epsilon_t$$

$$= \alpha + \beta B(L) X_t + \epsilon_t$$

= Geometric distributed lag model.

II. Partial Adjustment Model

$$y_t^* = \alpha + \beta \cdot x_t$$

desired level of y_t

= long-run equilibrium value

variable that determines long-run

Adjustment mechanism to "equilibrium"

$$\Delta y_t = (1-\lambda)\underbrace{(y_t^* - y_{t-1})}_{\text{deviation from equilibrium.}} + \epsilon_t \qquad 0 \le \lambda < 1$$

$\epsilon_t \sim iid\ (0, \sigma^2)$

Note: $E\{\Delta y_t\} = (1-\lambda)(y_t^* - y_{t-1})$

- $y_{t-1} = y_t^* \Rightarrow E\{\Delta y_t\} = 0$
- $y_{t-1} \neq y_t^* \Rightarrow$ adjustment & λ measures speed of adjustment

$\lambda \approx 0 \Rightarrow$ Fast adjustment

$\lambda \approx 1 \Rightarrow$ slow adjustment.

Rearranging gives

$$y_t = (1-\lambda) y_t^* + \lambda y_{t-1} + \epsilon_t$$

Substitute $\quad y_t^* = \alpha + \beta \cdot x_t \quad$ to give

$$y_t = (1-\lambda)(\alpha + \beta \cdot x_t) + \lambda y_{t-1} + \epsilon_t$$

$$= (1-\lambda)\alpha + \beta(1-\lambda) x_t + \lambda y_{t-1} + \epsilon_t$$

Which is a regression with a lagged dep. variable.

To get to the geometric lag model, solve for y_t:

$$(1-\lambda L) y_t = (1-\lambda)\alpha + \beta(1-\lambda) x_t + \epsilon_t$$

$$\Rightarrow \quad y_t = (1-\lambda)(1-\lambda L)^{-1}\alpha + \beta(1-\lambda)(1-\lambda L)^{-1} x_t$$

$$+ (1-\lambda L)^{-1} \epsilon_t$$

$$= \alpha + \beta(1-\lambda) \sum_{k=0}^{\infty} \lambda^k x_{t-k} + \sum_{k=0}^{\infty} \lambda^k \epsilon_{t-k}$$

$$= \alpha + \beta \cdot B(L) + u_t$$

where

$$u_t = \lambda u_{t-1} + \epsilon_t \quad = AR(1) \text{ disturbance.}$$

$$\Delta y_t = (1-\lambda)(y_t^* - y_{t-1}) + \epsilon_t.$$

Interpretation of Coefficients in Partial adjustment model

$$y_t = (1-\lambda)\alpha + \beta(1-\lambda)x_t + \lambda y_{t-1} + \epsilon_t$$

$0 \cancel{\leq} \lambda < 1$, $\epsilon_t \sim iid(0, \sigma^2)$

λ = speed of adjustment parameter = coeff on y_{t-1}

$\lambda \simeq 1 \Rightarrow$ large persistence in y_t & slow adjustment

$\lambda \simeq 0 \Rightarrow$ little persistence in y_t & fast adjustment.

$$\frac{\partial y_t}{\partial x_t} = \beta(1-\lambda) = \text{"short-run" impact of } x_t \text{ on } y$$

Since $\lambda \in (0,1)$, $1-\lambda \in (0,1)$ and

$|\beta(1-\lambda)| < |\beta|$ = long-run impact of x on y.

A quick & dirty way to determine the LR effect of x on y is as follows.

In steady state $x_t = \bar{x}$, $y_t = \bar{y}$, $\epsilon_t = 0$. Then

$$\bar{y} = (1-\lambda)\alpha + \beta(1-\lambda)\bar{x} + \lambda\bar{y}$$

$$\Rightarrow (1-\lambda)\bar{y} = (1-\lambda)\alpha + \beta(1-\lambda)\bar{x}$$

$$\Rightarrow \bar{y} = \alpha + \beta\cdot\bar{x} \quad \text{and} \quad \frac{\partial\bar{y}}{\partial\bar{x}} = \beta.$$

Berndt gives the following interpretation

Due to the partial adjustment mechanism the short-run impact of ~~[illegible]~~ x on y is

$$\frac{\partial y_t}{\partial x_t} = \beta(1-\lambda)$$

In the long-run, $\lambda \to 0$ so that we are adjusted to the long-run equilibrium. Notice that

$$\lim_{\lambda \to 0} \frac{\partial y_t}{\partial x_t} = \lim_{\lambda \to 0} \beta(1-\lambda) = \beta$$

gives the appropriate LR effect.

Estimation of Coefficients in Partial adjustment model

$$y_t = (1-\lambda)\alpha + \beta(1-\lambda)x_t + \lambda y_{t-1} + \epsilon_t$$

$0 < \lambda < 1$, $\epsilon_t \sim iid\,(0,\sigma^2)$, x_t exogenous.

This is a regression with a lagged endogenous variable

$$y_t = \beta_0 + \beta_1 x_t + \beta_2 y_{t-1} + \epsilon_t$$

where

$$\beta_0 = (1-\lambda)\alpha, \quad \beta_1 = \beta(1-\lambda), \quad \beta_2 = \lambda.$$

Provided $|\lambda| < 1$ and x_t is exogenous OLS gives consistent and asymptotically normal estimates. but is biased in finite sample

Consistent estimates of α, λ and β are given by

$$\hat{\lambda} = \hat{\beta}_2$$

$$\hat{\beta} = \frac{\hat{\beta}_1}{1-\hat{\beta}_2}$$

$$\hat{\alpha} = \frac{\hat{\beta}_0}{1-\hat{\beta}_2}$$

Standard errors for these estimates can then be easily computed via the delta method.

motivated by Adaptive Expectations.

Estimation of the Geometric Lag model : Koyck Transformation

$$y_t = \alpha + \beta \cdot B(L) x_t + \epsilon_t$$

$$= \alpha + \beta (1-\lambda)(1-\lambda L)^{-1} x_t + \epsilon_t .$$

Trick: multiply both sides by $(1-\lambda L)$ to give

$$(1-\lambda L) y_t = (1-\lambda L)\alpha + \cancel{(1-\lambda L)} \beta (1-\lambda) \cancel{(1-\lambda L)^{-1}} x_t + (1-\lambda L)\epsilon_t$$

$\Rightarrow$

$$y_t = \lambda y_{t-1} + (1-\lambda)\alpha + \beta(1-\lambda) x_t + (\epsilon_t - \lambda \epsilon_{t-1})$$

$$= \delta_0 + \lambda y_{t-1} + \delta_1 x_t + u_t ,$$

Koyck Transformation turns infinite order lag model to a regression model with
~~ETR Regression~~

(1) lagged dependent variable

(2) MA(1) error term.

Problem: lagged dep. variable is correlated with MA(1) error term

$\Rightarrow$ OLS is biased & inconsistent !

IV estimation

$$y_t = \delta_0 + \lambda y_{t-1} + \delta_1 x_t + u_t \qquad t = 2, \dots, T$$

or

$$y = Z\delta + u$$

where

$$Z = \begin{bmatrix} 1 & x_2 & y_1 \\ \vdots & \vdots & \vdots \\ 1 & x_T & y_{T-1} \end{bmatrix}, \quad \delta = (\delta_0, \delta_1, \lambda)'$$

Need to instrument for y_{t-1}

- Use x_{t-1}

  x_{t-1}, y_{t-1} are correlated via structure of model

  x_{t-1} is uncorrelated with $u_t = \epsilon_t - \lambda\epsilon_t$

  since x_t is assumed exogenous.

Then

$$\hat{\delta}_{IV} = (X'Z)^{-1}X'y, \quad \hat{\delta}_{IV} \xrightarrow{p} \delta$$

$$\widehat{var}(\hat{\delta}_{IV}) = \hat{\sigma}^2_{IV} (X'Z)^{-1}X'X(Z'X)^{-1}$$

$$\hat{\sigma}^2_{IV} = (y - Z\hat{\delta}_{IV})'(y - Z\hat{\delta}_{IV})$$

The estimation of the Adaptive expectations model is complicated by serial correlation in the error term.

Hence, a topic of considerable interest is testing for serial correlation in the error term of a regression model with a lagged dependent variable.

<u>Model</u> $y_t = \beta_0 + \beta_1 x_t + \beta_2 y_{t-1} + \epsilon_t$

H_0: $\epsilon_t \sim iid\,(0, \sigma^2)$ (No serial correlation)

H_1: $\epsilon_t \sim AR(1)$ <u>or</u> $\epsilon_t \sim MA(1)$

<u>Note:</u> The DW statistic is not valid in the presence of a lagged dependent variable and is not designed for MA(1) errors.

Bruesh e Pagan develop a simple LM test in this context and is programmed into Eviews. (Greene pg. 595

Procedure to compute LM test

1. Estimate model by OLS giving

$$y_t = \hat{\beta}_0 + \hat{\beta}_1 x_t + \hat{\beta}_2 y_{t-1} + \hat{\epsilon}_t$$

(which is a valid regression under Ho)

2. Estimate the auxillary regression by OLS

$$\hat{\epsilon}_t = \tilde{\alpha}_0 + \tilde{\alpha}_1 x_t + \tilde{\alpha}_2 y_{t-1} + \tilde{\alpha}_3 \hat{\epsilon}_{t-1} + \tilde{u}_t$$

and compute R^2

$\tilde{\alpha}_3$ will be non zero if $H_1: \epsilon_t \sim AR(1)$ or $\epsilon_t \sim MA(1)$ & R^2 will be non zero.

3. Form the LM statistic : (proof in Econ 583)

$$LM = T \cdot R^2$$

where T = sample size from the auxillary regression & R^2 = auxillary regression R^2.

Result: Under Ho: $\epsilon_t \sim iid\ (0, \sigma^2)$, $LM \overset{A}{\sim} \chi^2(1)$

Reject Ho at 5% level if $LM > \chi^2_{0.95}(1)$.