Econ 582 Midterm Exam

Instructions: This is an open book and open notes exam. Answer all question. Time limit is 2 hours. Total points = 100.

1. (15 points) Consider the linear regression model with normal errors conditional on the $\boldsymbol{x}$'s:

$$y_i = \boldsymbol{x}_i'\boldsymbol{\beta} + \varepsilon_i$$
$$\varepsilon_i \sim iid\ N(0, \sigma^2)$$

a) Recall, the maximum likelihood estimator of $\boldsymbol{\beta}$ maximizes

$$-SSR(\boldsymbol{\beta}) = -(\boldsymbol{y} - \boldsymbol{X\beta})'(\boldsymbol{y} - \boldsymbol{X\beta})$$

Show that the Newton-Raphson algorithm for maximizing $-SSR(\boldsymbol{\beta})$ converges in 1 iteration to the least squares estimate $\hat{\boldsymbol{\beta}} = (\boldsymbol{X'X})^{-1}\boldsymbol{X'y}$ regardless of the initial value chosen for the iteration.

2. (25 points) Let $X_1, \ldots, X_n$ be an *iid* sample with $X \sim N(\mu, \sigma^2)$. Consider the maximum likelihood estimator (mle) of σ^2

$$\hat{\sigma}^2_{mle} = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

a) What is the asymptotic distribution of $\hat{\sigma}^2_{mle}$?
b) What is the mle for σ?
c) Using the delta method, derive the asymptotic distribution for $\hat{\sigma}_{mle}$

Hint:

$$I(\theta) = \begin{pmatrix} \dfrac{n}{\sigma^2} & 0 \\ 0 & \dfrac{n}{2\sigma^4} \end{pmatrix}$$

3. (30 points) Consider the consumption function regression

$$\Delta c_t = \beta_0 + \beta_1 \Delta y_t + \beta_2 r_t + \varepsilon_t$$

where $\Delta c_t = \Delta \ln(C_t)$ denotes real consumption growth, $\Delta y_t = \Delta \ln(Y_t)$ represents real income growth and r_t represents the real interest rate. Let I_t denote the information set at time *t*, which contains current and lagged values of all variables, and assume that ε_t is a martingale difference sequence so that $E[\varepsilon_t \mid I_{t-1}] = 0$. The pure form of the permanent income hypothesis (PIH) has $\beta_1 = \beta_2 = 0$. The PIH with a non-constant real interest rate has $\beta_1 > 0$. If some part of the population are current income consumers then $\beta_1 > 0$.

a) Is the method of least squares an appropriate estimation procedure for the consumption function regression? Briefly explain.
b) What are the assumptions that justify using two-stage-least-squares (2SLS) to estimate the parameters of the consumption function?
c) If you were to estimate the consumption function by 2SLS, what instruments would you use? How would you determine if these instruments were valid instruments?
d) Describe how you would estimate the consumption function using the efficient generalized method of moments (GMM) estimator based on a given set ***X*** of $k > 2$ instruments.
e) What are the hypotheses being tested with Hansen's J-statistic for overidentifying restrictions? What do you conclude if reject the null hypothesis?
f) How is the efficient GMM estimator different from the 2SLS estimator based on the instrument set ***X***?
g) Using the efficient GMM estimator, describe how you would test the PIH.

4. (30 points) The binomial probability model is to be based on the following index function model:

$$\begin{aligned} &y^* = \alpha + \beta d + \varepsilon, \\ &y = 1, \quad if \quad y^* > 0, \\ &y = 0, \quad otherwise \end{aligned}$$

The only regressor, d, is a dummy variable. The data consist of 100 observations that have the following:

		y	
		0	1
d	0	24	28
	1	32	16

Using a logit model,

a) Obtain the maximum likelihood estimators of α and β
b) Estimate the asymptotic standard errors of your estimates.
c) Test the hypothesis that β equals zero by using a Wald test

Hints:

- Formulate the log-likelihood function in terms of α and $\delta = \alpha + \beta$
- $\Lambda(z) = \dfrac{e^z}{1+e^z}$
- $\dfrac{\partial \Lambda(z)}{\partial z} = \Lambda(z)(1-\Lambda(z)) = \dfrac{e^z}{(1+e^z)^2}$