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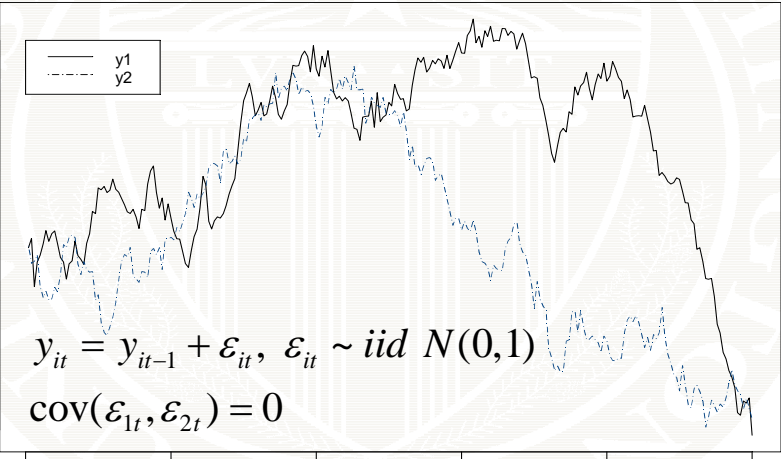
Econ 582 Cointegration

Eric Zivot
May 17, 2012

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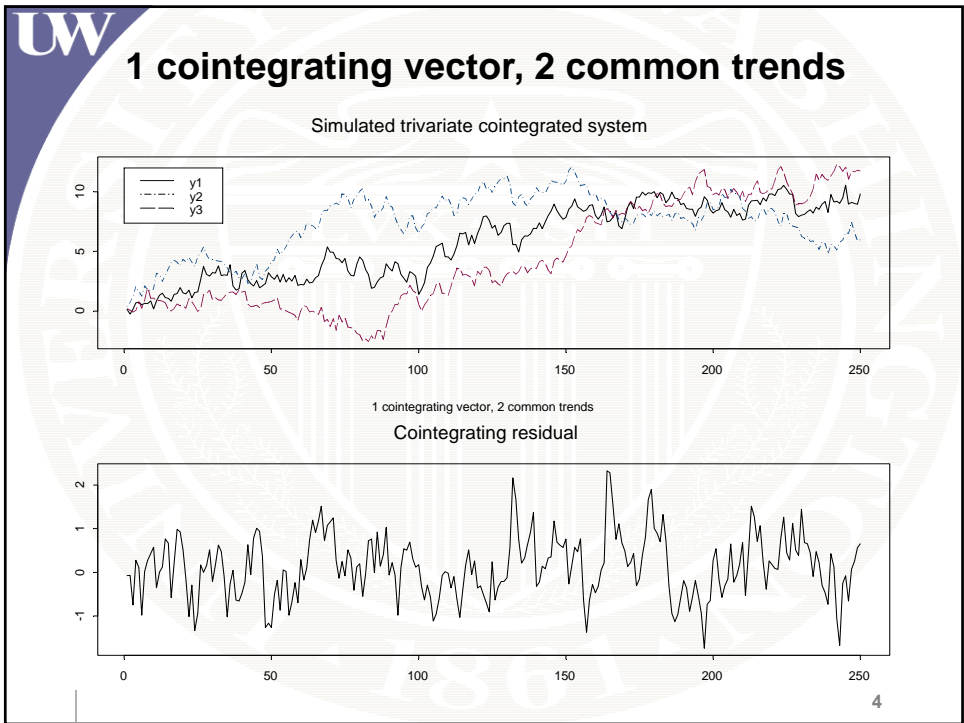
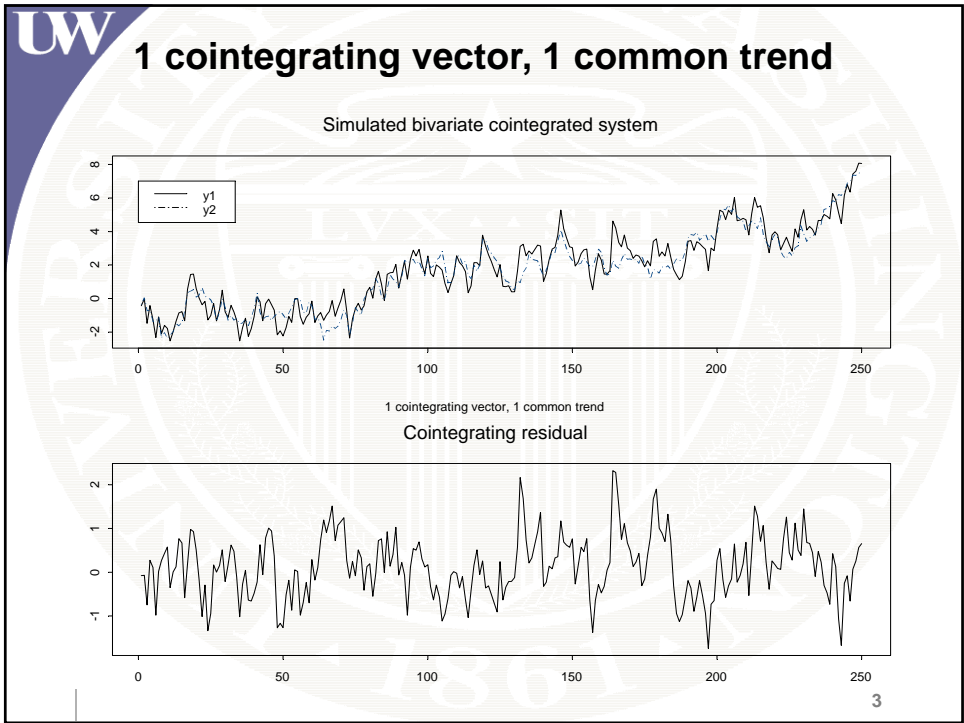
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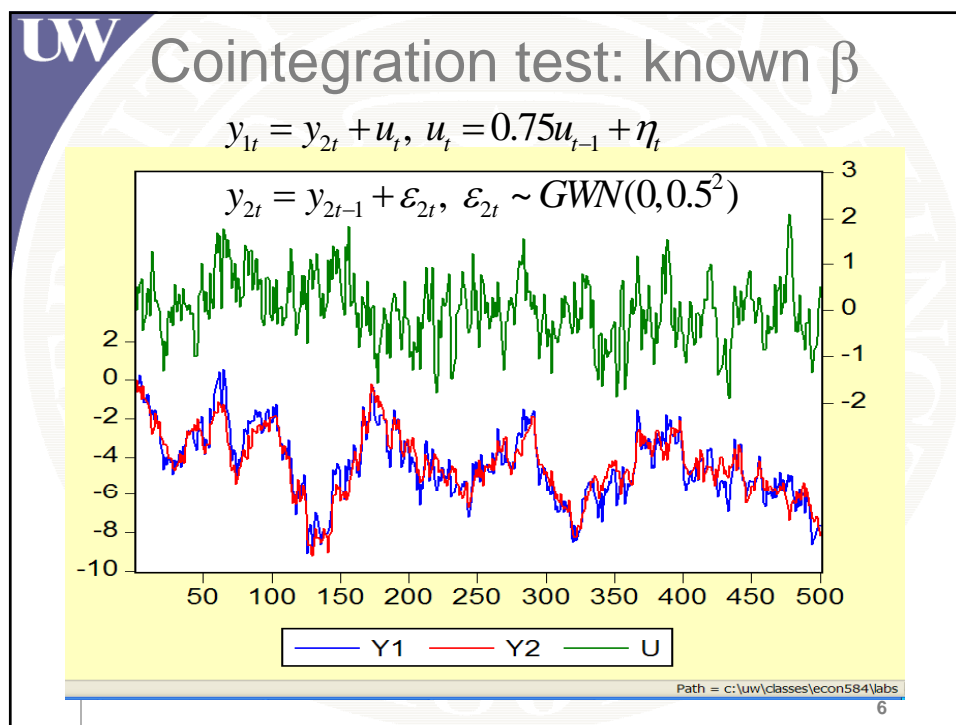
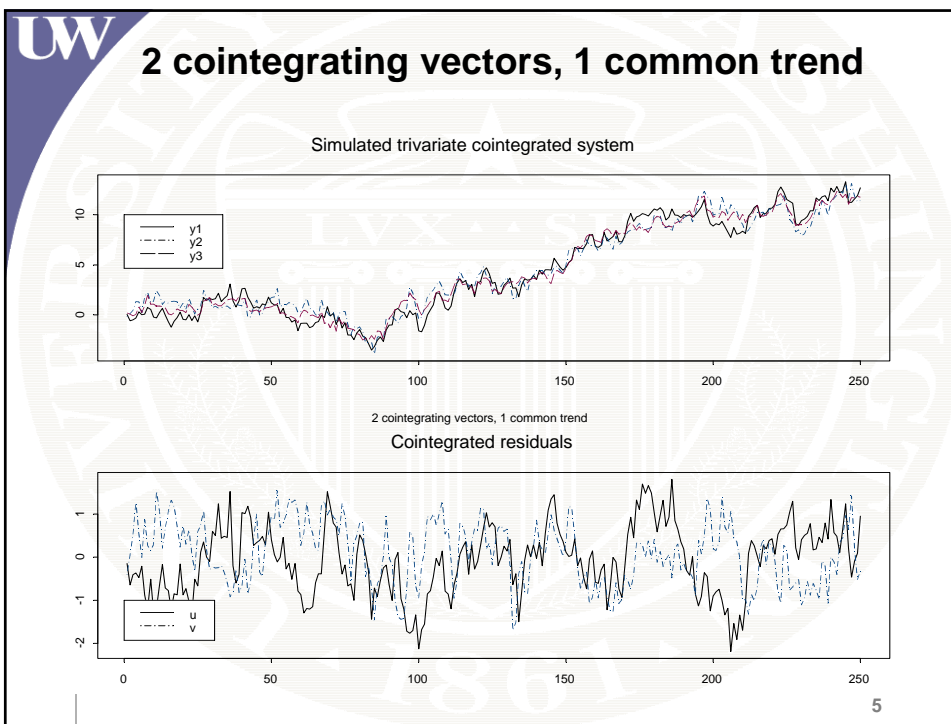
Spurious Regression



$y_{it} = y_{it-1} + \varepsilon_{it}, \varepsilon_{it} \sim iid N(0,1)$
 $cov(\varepsilon_{1t}, \varepsilon_{2t}) = 0$

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ADF test on u_t

Null Hypothesis: U has a unit root
 Exogenous: Constant
 Lag Length: 0 (Automatic based on SIC, MAXLAG=5)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.097037	0.0000
Test critical values:		
1% level	-3.443228	
5% level	-2.867112	
10% level	-2.569800	

*MacKinnon (1996) one-sided p-values.

Reject H_0 : y_1 and y_2 are not cointegrated at 1% level

Cointegration Test: Estimate β

Step 1: estimate β by OLS

Dependent Variable: Y1
 Method: Least Squares
 Date: 05/24/06 Time: 13:30
 Sample: 1 500
 Included observations: 500

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.015124	0.084955	-0.178019	0.8588
Y2	0.993235	0.017938	55.36959	0.0000

R-squared	0.860261	Mean dependent var	-4.382984
Adjusted R-squared	0.859981	S.D. dependent var	1.884345
S.E. of regression	0.705106	Akaike info criterion	2.143056
Sum squared resid	247.5931	Schwarz criterion	2.159914
Log likelihood	-533.7639	F-statistic	3065.792
Durbin-Watson stat	0.572230	Prob(F-statistic)	0.000000

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Step 2: ADF test on OLS residual

$$\Delta \hat{u}_t = \pi \hat{u}_{t-1} + \eta_t$$

Unit Root Test

Test type: Augmented Dickey-Fuller

Test for unit root in:

- Level
- 1st difference
- 2nd difference

Include in test equation:

- Intercept
- Trend and intercept
- None

Lag length:

- Automatic selection: Modified Akaike
- Maximum lags: 17
- User specified: 0

OK Cancel

No intercept

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Use PO Critical Values

Null Hypothesis: UHAT has a unit root
Exogenous: None
Lag Length: 0 (Fixed)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-9.105974	0.0000
Test critical values:		
1% level	-2.569604	
5% level	-1.941459	
10% level	-1.616273	

*MacKinnon (1996) one-sided p-values.

N-1 = 1, constant in 1st step regression

PO 5% critical value is -3.36

Do not use!

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Stock-Watson DOLS

$$y_{t1} = \alpha + \beta y_{2t} + \psi_1 \Delta y_{2t+1} + \psi_0 \Delta y_{2t} + \psi_{-1} \Delta y_{2t-1} + \varepsilon_t$$

Dependent Variable: Y1
 Method: Least Squares
 Date: 05/24/06 Time: 14:05
 Sample (adjusted): 3 499
 Included observations: 497 after adjustments
 Newey-West HAC Standard Errors & Covariance (lag truncation=5)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.019383	0.167521	-0.115705	0.9079
Y2	0.991895	0.035268	28.12417	0.0000
D(Y2(1))	0.103777	0.067753	1.531687	0.1262
D(Y2)	0.064669	0.064762	0.998574	0.3185
D(Y2(-1))	0.080376	0.069807	1.151412	0.2501
R-squared	0.858116	Mean dependent var	-4.393828	
Adjusted R-squared	0.856963	S.D. dependent var	1.864151	
S.E. of regression	0.705027	Akaike info criterion	2.148847	
Sum squared resid	244.5548	Schwarz criterion	2.191187	
Log likelihood	-528.9885	F-statistic	743.9081	
Durbin-Watson stat	0.583469	Prob(F-statistic)	0.000000	

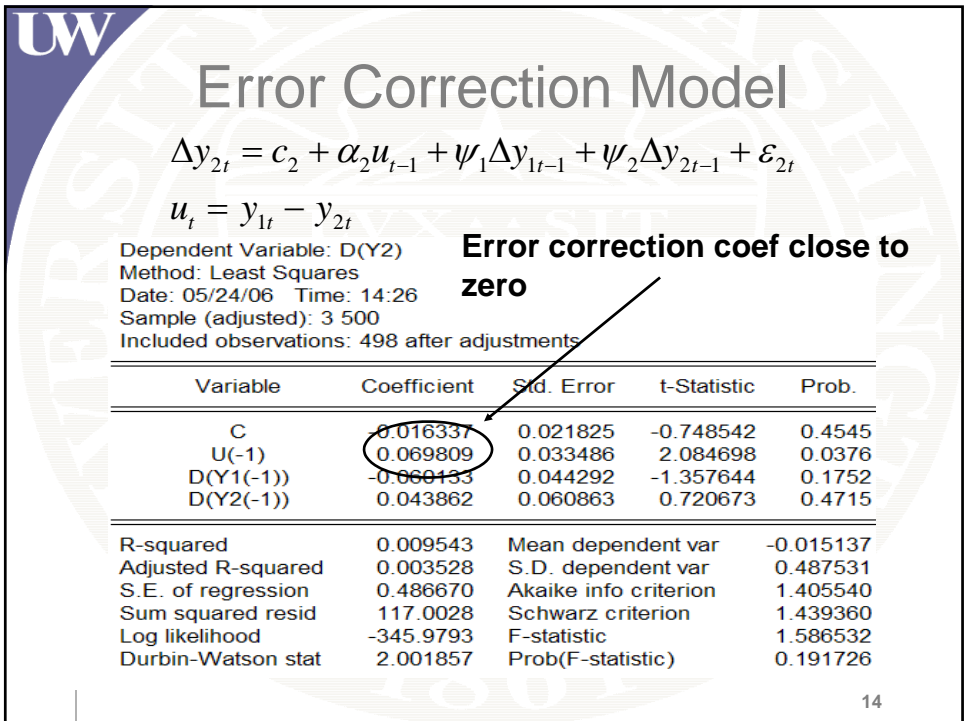
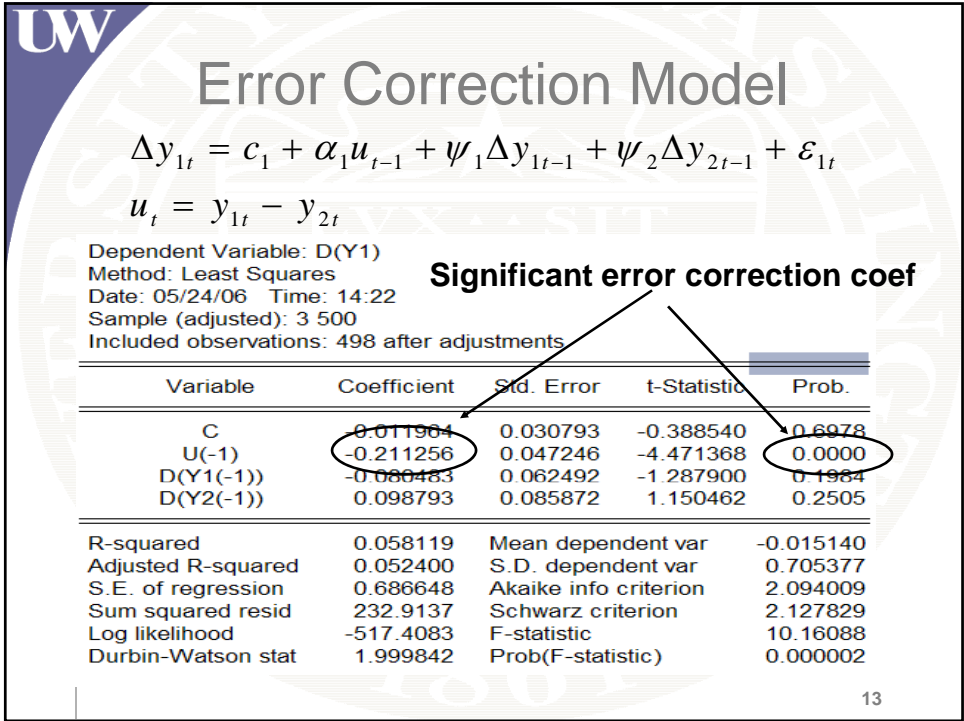
Stock-Watson DGLS

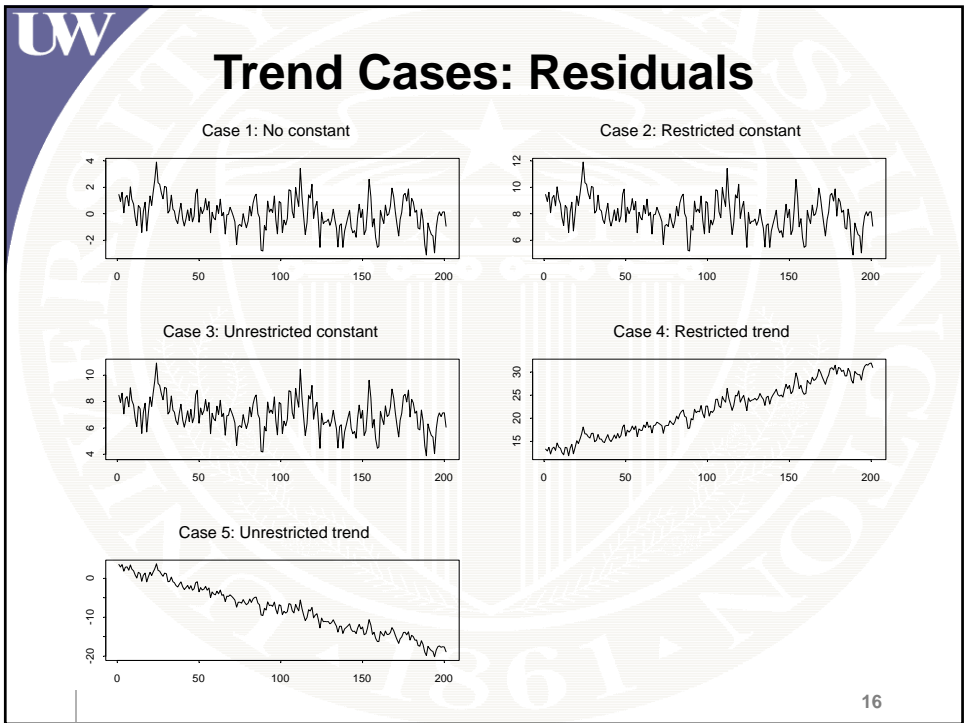
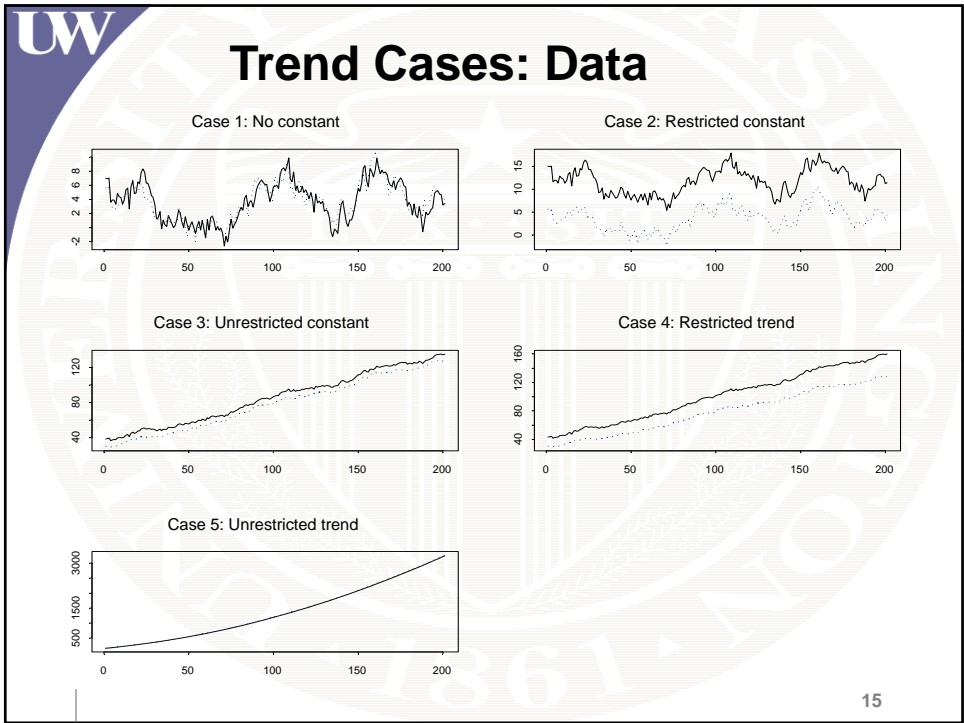
$$y_{t1} = \alpha + \beta y_{2t} + \psi_1 \Delta y_{2t+1} + \psi_0 \Delta y_{2t} + \psi_{-1} \Delta y_{2t-1} + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

Dependent Variable: Y1
 Method: Least Squares
 Date: 05/24/06 Time: 14:09
 Sample (adjusted): 4 499
 Included observations: 496 after adjustments
 Convergence achieved after 4 iterations

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.121389	0.206434	-0.588028	0.5568
Y2	0.970262	0.042988	22.57072	0.0000
D(Y2(1))	0.002622	0.045515	0.057618	0.9541
D(Y2)	0.007644	0.054670	0.139816	0.8889
D(Y2(-1))	0.044936	0.045376	0.990307	0.3225
AR(1)	0.714675	0.031811	22.46608	0.0000
R-squared	0.929317	Mean dependent var	-4.402307	
Adjusted R-squared	0.928595	S.D. dependent var	1.856414	
S.E. of regression	0.496064	Akaike info criterion	1.447801	
Sum squared resid	120.5792	Schwarz criterion	1.498687	
Log likelihood	-353.0547	F-statistic	1288.465	
Durbin-Watson stat	2.034233	Prob(F-statistic)	0.000000	
Inverted AR Roots	.71			





Test for Cointegration

$$y_{1t} = y_{2t} + u_t, \quad u_t = 0.75u_{t-1} + \varepsilon_{1t}$$

$$y_{2t} = y_{2t-1} + \varepsilon_{2t}$$

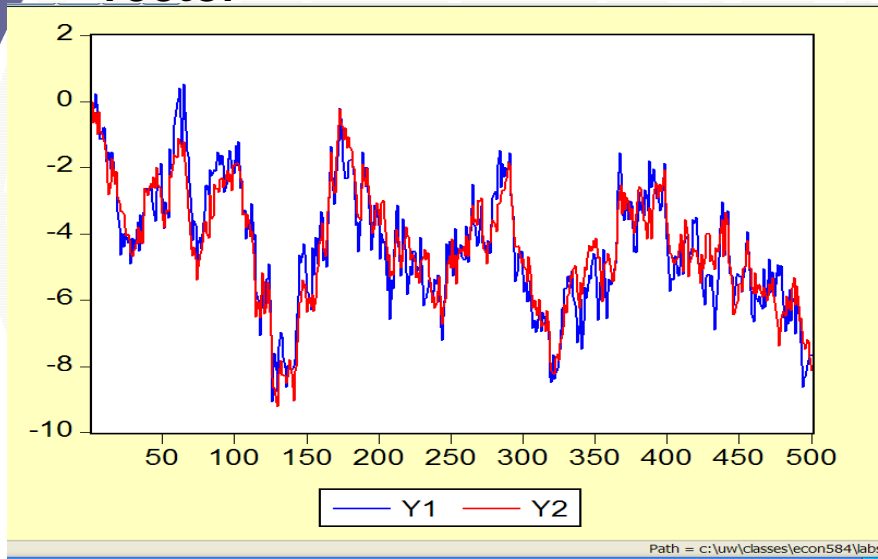
$$\varepsilon_{1t} \sim iid N(0, 0.5^2)$$

$$\varepsilon_{2t} \sim iid N(0, 0.5^2)$$

$$\beta = (1, -1)'$$

Note: No drift and residuals have mean zero

Bivariate VAR: 1 cointegrating vector



Johansen LR Tests based on Levels VAR(1) and No Deterministic Terms

Johansen Cointegration Test ✕

Cointegration Test Specification

Deterministic trend assumption of test

Assume no deterministic trend in data:

1) No intercept or trend in CE or test VAR
 2) Intercept (no trend) in CE - no intercept in VAR

Allow for linear deterministic trend in data:

3) Intercept (no trend) in CE and test VAR
 4) Intercept and trend in CE - no trend in VAR

Allow for quadratic deterministic trend in data:

5) Intercept and trend in CE - linear trend in VAR

Summary:

6) Summarize all 5 sets of assumptions

* Critical values may not be valid with exogenous variables; do not include C or Trend.

Exog variables*

Lag intervals

0 0

Lag spec for differenced endogenous

Critical Values

MHM
 Osterwald-Lenum

Size

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Johansen LR Tests

Date: 05/31/06 Time: 13:36
 Sample (adjusted): 2 500
 Included observations: 499 after adjustments
 Trend assumption: No deterministic trend
 Series: Y1 Y2
 Lags interval (in first differences): No lags

Trace test finds 1 cointegrating vector

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.145974	79.02705	12.32090	0.0001
At most 1	0.000578	0.288257	4.129906	0.6524

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.145974	78.73880	11.22480	0.0001
At most 1	0.000578	0.288257	4.129906	0.6524

Max-eigenvalue test indicates 1 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

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MLE of Cointegrating Vector

Unrestricted Cointegrating Coefficients (normalized by $b'S_{11}b=I$):

	Y1	Y2	
	-1.420152	1.412525	← Eigenvectors
	-0.018335	0.229824	

Unrestricted Adjustment Coefficients (alpha):

	D(Y1)	D(Y2)
	0.165470	-0.013342
	-0.036046	-0.011496

1 Cointegrating Equation(s): Log likelihood -703.6957

Normalized cointegrating coefficients (standard error in parentheses)

	Y1	Y2	
	1.000000	-0.994629	← Normalized on Y1
		(0.01615)	

Adjustment coefficients (standard error in parentheses)

	D(Y1)	D(Y2)
	-0.234992	(0.04356)
	0.051190	(0.03094)

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Johansen Test with Misspecified Trend

Johansen Cointegration Test

Cointegration Test Specification **No lagged differences**

Deterministic trend assumption of test

Assume no deterministic trend in data:

1) No intercept or trend in CE or test VAR

2) Intercept (no trend) in CE - no intercept in VAR

Allow for linear deterministic trend in data:

3) Intercept (no trend) in CE and test VAR

4) Intercept and trend in CE - no trend in VAR

Allow for quadratic deterministic trend in data:

5) Intercept and trend in CE - linear trend in VAR

Summary:

6) Summarize all 5 sets of assumptions

Exog variables*

Lag intervals

0 0

Lag spec for differenced endogenous

Critical Values

MHM

Size

Osterwald-Lenum

* Critical values may not be valid with exogenous variables; do not include C or Trend.

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Misspecifying trend leads to wrong conclusion!

Test finds 2 cointegrating vectors!

Date: 05/31/06 Time: 13:33
 Sample (adjusted): 2 500
 Included observations: 499 after adjustments
 Trend assumption: No deterministic trend (restricted constant)
 Series: Y1 Y2
 Lags interval (in first differences): No lags

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.145989	90.23008	20.26184	0.0000
At most 1 *	0.022748	11.48237	9.164546	0.0179

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.145989	78.74771	15.89210	0.0000
At most 1 *	0.022748	11.48237	9.164546	0.0179

Max-eigenvalue test indicates 2 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

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Test for Cointegration

$$y_{4t} = 0.5y_{2t} + 0.5y_{3t} + u_t, u_t = 0.75u_{t-1} + \varepsilon_{1t}$$

$$\varepsilon_{1t} \sim iid N(0, 0.5^2)$$

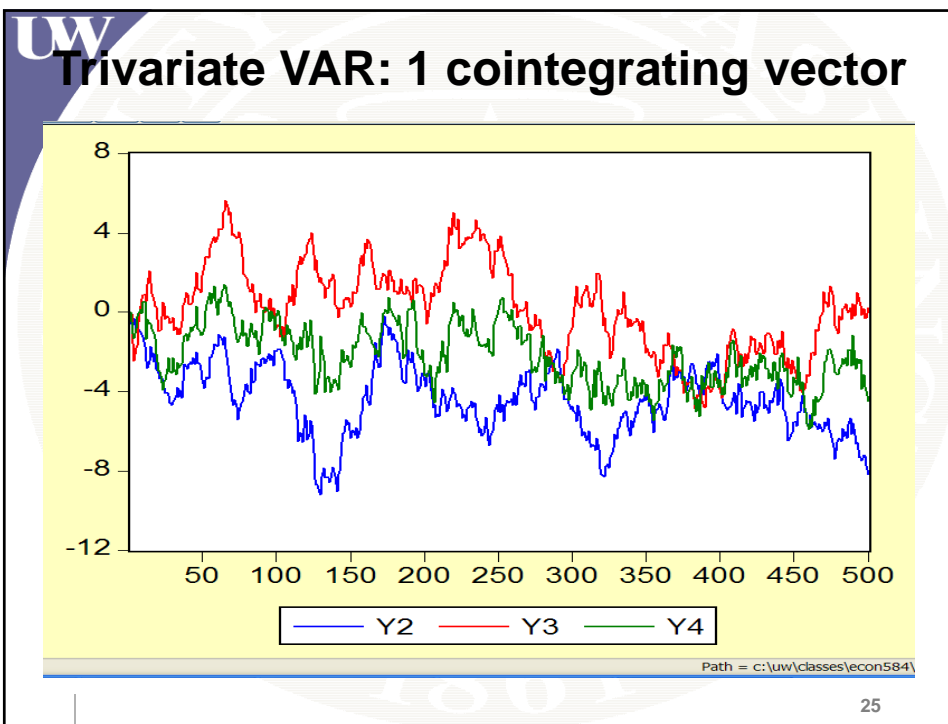
$$y_{2t} = y_{2t-1} + \varepsilon_{2t}, \varepsilon_{2t} \sim iid N(0, 0.5^2)$$

$$y_{3t} = y_{3t-1} + \varepsilon_{3t}, \varepsilon_{3t} \sim iid N(0, 0.5^2)$$

$$Y_t = (y_{4t}, y_{2t}, y_{3t})'$$

$$\beta = (1, -0.5, -0.5)'$$

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Johansen LR Tests

Date: 05/31/06 Time: 14:17
 Sample (adjusted): 2 500
 Included observations: 499 after adjustments
 Trend assumption: No deterministic trend
 Series: Y2 Y3 Y4
 Lags interval (in first differences): No lags

Trace test finds 1 cointegrating vector

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.111588	65.77018	24.27596	0.0000
At most 1	0.012827	6.728387	12.32090	0.3530
At most 2	0.000574	0.286414	4.129906	0.6535

Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.111588	59.04179	17.79730	0.0000
At most 1	0.012827	6.441973	11.22480	0.3022
At most 2	0.000574	0.286414	4.129906	0.6535

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MLE of Cointegrating Vector

Unrestricted Cointegrating Coefficients (normalized by $b'S11*b=1$):

Y4	Y2	Y3	Eigenvectors
-1.308485	0.664332	0.623918	
-0.019231	-0.001391	-0.436581	
0.013616	-0.219099	-0.021765	

Unrestricted Adjustment Coefficients (alpha):

D(Y4)	0.157658	0.020169	0.007805
D(Y2)	-0.026663	0.004464	0.011484
D(Y3)	0.024620	0.055706	-0.000975

1 Cointegrating Equation(s): Log likelihood -1036.903

Normalized cointegrating coefficients (standard error in parentheses)

Y4	Y2	Y3	Normalize on Y4
1.000000	-0.507711 (0.02052)	-0.476824 (0.04309)	

Adjustment coefficients (standard error in parentheses)

D(Y4)	-0.206294 (0.03398)
D(Y2)	0.034888 (0.02855)
D(Y3)	-0.032214 (0.02922)

Test for Cointegration

$$y_{1t} = y_{2t} + u_t, \quad u_t = 0.75u_{t-1} + \varepsilon_{1t}$$

$$y_{5t} = y_{2t} + v_t, \quad v_t = 0.75v_{t-1} + \varepsilon_{5t}$$

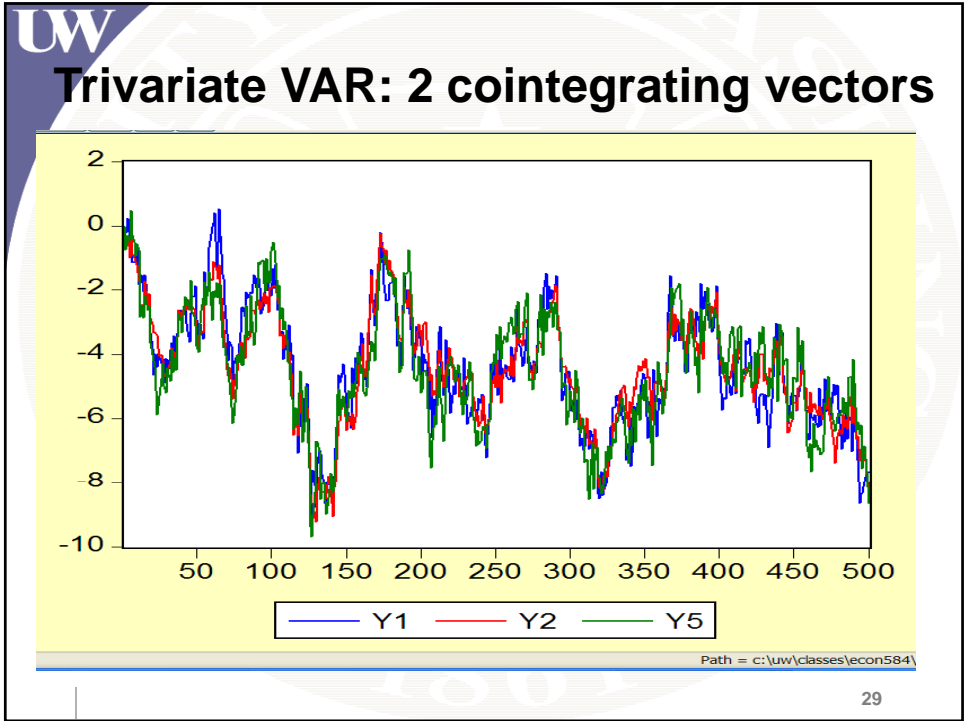
$$\varepsilon_{1t} \sim iid N(0, 0.5^2), \quad \varepsilon_{5t} \sim iid N(0, 0.5^2)$$

$$y_{2t} = y_{2t-1} + \varepsilon_{2t}, \quad \varepsilon_{2t} \sim iid N(0, 0.5^2)$$

$$Y_t = (y_{1t}, y_{5t}, y_{2t})'$$

$$\beta_1 = (1, 0, -1)'$$

$$\beta_2 = (0, 1, -1)'$$



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Johansen LR Tests

Date: 05/31/06 Time: 14:30
 Sample (adjusted): 2 500
 Included observations: 499 after adjustments
 Trend assumption: No deterministic trend
 Series: Y1 Y5 Y2
 Lags interval (in first differences): No lags

Trace test finds 2 cointegrating vectors

Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.159565	140.6944	24.27596	0.0001
At most 1 *	0.101909	53.95064	12.32090	0.0000
At most 2	0.000634	0.316276	4.129906	0.6358

Trace test indicates 2 cointegrating eqn(s) at the 0.05 level
 * denotes rejection of the hypothesis at the 0.05 level
 **MacKinnon-Haug-Michelis (1999) p-values

Unrestricted Cointegration Rank Test (Maximum Eigenvalue)

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	0.05 Critical Value	Prob.**
None *	0.159565	86.74372	17.79730	0.0000
At most 1 *	0.101909	53.63437	11.22480	0.0000
At most 2	0.000634	0.316276	4.129906	0.6358

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MLE of Cointegrating Vectors

Unrestricted Cointegrating Coefficients (normalized by $b'S11*b=I$):

Y1	Y5	Y2
-1.359439	0.634932	0.712040
0.474157	1.159686	-1.640103
0.016830	0.019750	-0.248253

Unrestricted Adjustment Coefficients (alpha):

D(Y1)	0.169668	-0.034704	0.013869
D(Y5)	-0.113758	-0.100263	0.014284
D(Y2)	-0.021412	0.035916	0.011866

1 Cointegrating Equation(s): Log likelihood -1055.831

Normalized cointegrating coefficients (standard error in parentheses)

Y1	Y5	Y2
1.000000	-0.467054	-0.523775
	(0.09867)	(0.10098)

Adjustment coefficients (standard error in parentheses)

D(Y1)	-0.230653	
	(0.04163)	
D(Y5)	0.154647	
	(0.04259)	
D(Y2)	0.029108	
	(0.02967)	

Assume 1
cointegrating vector

MLE of Cointegrating Vectors

2 Cointegrating Equation(s): Log likelihood -1029.014

Normalized cointegrating coefficients (standard error in parentheses)

Y1	Y5	Y2
1.000000	0.000000	-0.994416
		(0.01647)
0.000000	1.000000	-1.007681
		(0.02111)

Adjustment coefficients (standard error in parentheses)

D(Y1)	-0.247109	0.067481
	(0.04403)	(0.04044)
D(Y5)	0.107106	-0.188503
	(0.04464)	(0.04099)
D(Y2)	0.046138	0.028057
	(0.03134)	(0.02878)

Eviews solves
for triangular
representation
by default