

Em 582 Lec 3

Note Title

4/8/2013

$$x_t = \phi_1 x_{t-1} + \dots + \phi_p x_{t-p} + \epsilon_t$$

$$\begin{bmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-p} \end{bmatrix} = \begin{bmatrix} \phi_1 & \phi_2 & \dots & \phi_p \\ 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-p} \end{bmatrix} + \begin{bmatrix} \epsilon_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1}B & \dots & \dots & a_{nn}B \end{bmatrix}$$

$$g_t = F g_{t-1} + v_t$$

$$\text{var}(g_t) = F \text{var}(g_{t-1}) F' + \text{var}(v_t)$$

$$\begin{matrix} P & = & F & \Gamma & F' & + & \text{var}(v_t) \\ P \times P & & P \times P & P \times P & P \times P & & K \times P \end{matrix}$$

$$\text{vec}(P) = (F \otimes F) \text{vec}(\Gamma) + \text{vec}(\text{var}(v_t))$$

$$\text{vec}(P) = (F \otimes F) \text{vec}(\Gamma) = \text{vec}(\text{var}(v_t))$$

$$\left(I_{p^2} - F \otimes F \right) \text{vec}(P) = \text{vec}(\text{var}(u_t))$$

$$\Rightarrow \text{vec}(P) = \left(I_{p^2} - F \otimes F \right)^{-1} \text{vec}(\text{var}(u_t))$$

↑

$$\text{1st column} = \begin{bmatrix} \sigma_0 \\ \sigma_1 \\ \vdots \\ \sigma_{p-1} \end{bmatrix}$$