University of Washington
Department of Economics

Economics 582
Spring 2013

Assignment 2

Introduction to Time Series and Forecasting
Due Wednesday, April 17, 2013

Reading

- Greene, Chapter 21.
- Cochrane, *Time Series for Macroeconomics and Finance* (see webpage)
- Hayashi, chapters 2 and 6
- Eviews help topics: AR Specification, Forecasting from a single equation

Data

- rgdpq.csv

I. Analytic Exercises

1. Consider the AR(2) process

$$y_t = 2.5 + 1.1y_{t-1} - 0.18y_{t-2} + \varepsilon_t\ , \ \ \varepsilon_t \sim \text{i.i.d. } (0, 1)$$

(a) Using Eviews, Matlab or R, generate 250 simulated observations from the above process.

(b) Show that the AR(2) is stable/stationary by computing the roots of $\phi(z) = 0$ and the eigenvalues of the companion matrix F. What is the relationship between the roots of $\phi(z) = 0$ and the eigenvalues of the companion matrix F?

(c) Calculate the unconditional mean of the process.

(d) Calculate and plot the first 10 autocorrelations. Compare these with the sample ACF computed from your simulated data.

(d) Determine the Wold representation of the series (MA(∞) representation) and thus the sequence of dynamic multipliers required for the impulse response function. Compute the first 10 MA weights (dynamic multipliers) and plot the impulse response function.

2. Consider the generic AR(2) process

$$y_t = c + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t,\ \varepsilon_t \sim iid\ N(0, \sigma^2)$$

(a) Under what conditions is the process covariance stationary? State these conditions using the companion F matrix as well as the AR(2) characteristic equation.

(b) Show that the covariance stationarity conditions can be expressed as the following restrictions on the AR parameters

$$\begin{aligned} &\phi_2 + \phi_1 < 1 \\ &\phi_2 - \phi_1 < 1 \\ &-1 < \phi_2 < 1 \end{aligned}$$

(c) Show that roots of the AR(2) characteristic equation will be real when $\phi_1^2 + 4\phi_2 > 0$ and will be complex when $\phi_1^2 + 4\phi_2 < 0$.

(d) Derive the likelihood function for the AR(2) model based on a sample of size T conditional on the first two initial values.

(e) Determine the unconditional distribution of the first two initial values assuming the process is covariance stationary.

II. Computer Exercises and Hints

For the empirical computer exercises, direct the computer output (e.g., cut and paste into a word processor) and answer all questions on the computer output. Also, please feel free to comment liberally on the computer output. Note any unusual results or simply make comments to yourself about what the results tell you.

Exercise 1: Identify AR(p) models for log real GDP

(a) Plot the log level of real GDP over the period 1947.1 to 2006.4. Comment on any unusual features of the data. Compute, plot and interpret the correlogram for the level of real GDP.

(b) Estimate an AR(1) model for the level of log real GDP in two different ways. First estimate the autoregression

$$Y_t = c + \varphi Y_{t\text{-}1} + \varepsilon_t\,,$$

and then estimate the mean-adjusted model

$$Y_t - \mu = \varphi(Y_{t\text{-}1} - \mu) + \varepsilon_t.$$

Interpret the constant terms in the two regressions. What does your estimate of φ tell you about the stability of the model?

(c) Detrend log real GDP by forming the residuals of a regression of log real GDP on a constant and a time trend. Compute, plot and interpret the corrrelogram (ACF) and partial correlogram (PACF) for the detrended series. What AR(p) models seem most appropriate?

(d) Compute, plot and interpret the ACF and PACF for the first difference of log real GDP. What AR(p) models seem most appropriate?

(e) For detrended log real GDP and the first difference of real GDP, compute the AIC and BIC information criteria for all AR(p) models with p=0,1,2,3,4. Which models are suggested by these model selection statistics?

Computer Hints for Exercise 1

Start EVIEWS in the usual way.

Loading the data

The first thing we must do is to create a workfile which specifies the type of data that will be analyzed. We will be working with postwar quarterly real GDP over the period 1947.1 to 2010.4 in the file rgdpq.csv. There are two series in the file: rgdp and date (postwar quarterly real gdp). Create the work file: **File/New/Workfile**. Choose Quarterly frequency, set the Start date to **1947.1**, the End date to **2010.4**, click **OK** and follow the prompts.

To transform the data into logs, click [**Genr**], type the equation **lrgdp = log(rgdp)** and click **OK**. The series **lrgdp** should be added to the workfile.

Analysis of the level of lrgdp

Make a timeplot of the level of log real GDP. Highlight the series lrgdp and double click to bring up the spreadsheet view of the data (note: in EVIEWS all series are treated as objects and each object has a number of views). Click [**View**] and select **Line Graph** to change the view to a time plot. Right click on the plot and select **Copy to clipboard**. Paste the graph into your word document or whatever you use as a word processor.

Next, compute the sample autocorrelation function or correlogram of lrgdp. Click [**View**], select **Correlogram**, choose **16** lags and then OK.

Finally, estimate a simple AR(1) model for lrgdp. There are two ways to estimate an AR model in EVIEWS - using lagged values of the dependent variable and using AR(.) terms in an equation. We do the regression with lagged values first. From the main menu, select **Quick/Estimate/Equation**. Type **lrgdp c lrgdp(-1)**, choose **ordinary least squares** as the estimation method and then click **OK**. This produces the equation object window with the estimation results displayed. Either copy and paste the output to your word processor or click [**Print**] on the window toolbar and re-direct the output to an .rtf file of your choice. Next we repeat the regression using AR terms. Click [**Estimate**], type **lrgdp c AR(1)**, click **OK** then [**Print**]. Close the equation window (click the close box in the upper left hand corner of the equation window).

Analysis of detrended log real GDP.

First, create detrended log real GDP by regressing lrgdp on a constant and a time trend. To create a time trend variable, click [**Genr**] and type **t = @trend(1947.1).** This creates the variable t that is equal to zero in 1947.1 and increases by one unit in every time period (see **Help/search/trend**). Next, regress lrgdp on a constant and trend. From the main menu, select **Quick/Estimate Equation** and type the equation **lrgdp c t**. Click **OK**, [**Print**] the regression results and close the equation window. To save the residuals in a new series click [**Genr**] and type **dtlrgdp = resid** (the residuals from the last regression are automatically stored in the series resid). Plot the detrended series as well as the correlogram

Next, create a new series containing the first difference of lrdgp. Click [**Genr**] and type **dlrgdp = D(lrgdp). P**lot the first difference of lrgdp and its correlogram.

Compute information criteria for AR(p) models: p=0,1,2,3,4.

The estimation output in EVIEWS for AR models includes the AIC and BIC (schwarz) statistics. First, we estimate the models for detrended lrgdp. To compute these statistics for the nine AR models with p=0,1,2,3,4 do the following. Select **Quick/Estimate Equation** and type the AR(0) equation: **dtlrgdp c.** Next, fix the sample size so that each model is estimated over the same set of data and uses the same sample size. To do this, type **1948:1 2006:4** in the sample box and then click **OK** to estimate the model From the estimation results write down the AIC and BIC statistics (do not print). Click [**Estimate**] to bring back the equation window and type in the AR(1) model **dtlrgdp c AR(1)**. Click **OK**, note the results and repeat the process for the remaining models.

Do the same process for the first difference of lrgdp. Start with the AR(0) model: [**Estimate**], **d(lrgdp) c**, **OK**, (write results) go to next model and repeat.

Exercise 2: Estimate preferred AR(p) model for detrended log real GDP

(a) Estimate your preferred AR(q) model for detrended log real GDP. Check the residuals for serial correlation and normality. Determine if the model is stable, compare the sample ACF to the theoretical ACF of your chosen AR(p) model, and compute and plot the impulse response function. Interpret your results.

Computer Hints for Exercise 2

Estimate AR(p) and analyze residuals

First, estimate your preferred model for **dtlrgdp**. Based on your choice of p, select **Quick/Estimate Equation** and type **dtlrgdp c AR(1) AR(2) … AR(p), OK**, [**Print**]. Next, click [**Resids**], [**Print**]. To compute residual diagnostics, click [**View**] and select **Residual Tests/Correlogram**, choose **16** lags and [**Print**]. Next, click [**View**], select **Residual/Tests/Serial Correlation-LM Test**, [**Print**], and [**View**], select **Residual Tests/Histogram -Normality Test** and [**Print**]. To investigate the ARMA structure, [**View**], ARMA structure **Roots** (for eigenvalues of F matrix), **Correlogram** (for residual ACF) and **Impulse Response** (for Wold representation and impulse response weights).

Exercise 3: Estimate AR(2) model for first difference log real GDP

(a) Estimate AR(2) model for the first difference of log real GDP. Check the residuals for serial correlation and normality. Determine if the model is stable, compare the sample ACF to the theoretical ACF of your chosen AR(p) model, and compute and plot the impulse response function. Interpret your results.

Computer Hints for Exercise 3

Estimate AR(2) and analyze residuals

Select **Quick/Estimate Equation** and type **d(lrgdp) c AR(1) AR(2), OK**, [**Print**]. Next, click [**Resids**], [**Print**]. Name the equation for later use: [**Name**], **Eq2**, **OK**. To compute residual diagnostics, click [**View**] and select **Residual Tests/Correlogram**, choose **16** lags and [**Print**]. Next, click [**View**], select **Residual/Tests/Serial Correlation-LM Test**, [**Print**]. Finally, click [**View**], select **Residual Tests/Histogram -Normality Test** and [**Print**]. To investigate the ARMA structure, [**View**], ARMA structure **Roots** (for eigenvalues of F matrix), **Correlogram** (for residual ACF) and **Impulse Response** (for Wold representation and impulse response weights).

Exercise 4: Compute and compare long-term forecasts of log real GDP

(a) Estimate the level of log real GDP by regressing log real GDP on a constant, time trend and two autoregressive terms over the period 1947.1 - 2006.4. Using this model, compute and plot the forecasts of log real GDP from 2007.4 – 2010.4 along with 95% confidence intervals for the forecasts and the actual value of log real GDP. Interpret your results.

(b) Estimate the AR(2) model for the first difference of log real GDP over the period 1947.1 - 2006.4. Using this model, compute and plot the forecasts of the level of log real GDP for 2007.1 – 2010.4 along with 95% confidence intervals for the forecasts and the actual value of log real GDP. Interpret your results.

Computer Hints for Exercise 4

Compute time trend regression with AR(p) errors

Compute the levels time trend regression using the sample period,: Click **Quick/Estimate Equation** and type the formula **lrgdp c t AR(1) AR(2)** using the sample period **1947.1 2006.4.** To produce forecasts over the period 2007.1 – 2010.4, click [**Forecast**], name the forecasts series **lrgdpf1,** name the standard errors **sef1**, and specify the **forecast period** as **2007.1 – 2010.4.** Click **OK** and [**Print**]. Close the equation window.

To compute a 95% confidence interval for the forecasts do the following. First, compute the upper bound for the CI: Click [**Genr**], type **uf1 = lrgdpf1 + 1.96*sef1**, **OK**. Next compute the lower bound: Click [**Genr**], type **lf1 = lrgdpf1 - 1.96*sef1**, **OK**.

To plot the forecasts, 95% confidence interval and actual values of log real GDP over the period 2007.1 - 2010.4 do the following. Change the sample period: [**Sample**],

2007.1 2010.4, **OK**. Then, highlight the four series **lrgdp, lrgdpf1, lf1, uf1** and double click to bring up the group window. Click [**View**], select **Graph** (you might want to play around with the graph options). Close the group window.

Change the sample period back to 1947.1 - 2006.4: [**Sample**], **1947.1 2006.4**, **OK**.

Compute and plot forecasts from ARIMA(2,1,0) model

Click **Quick/Estimate Equation** and type the formula **d**(**lrgdp) c AR(1) AR(2)** using the sample period **1947.1 2006.4.** To produce forecasts over the period 2007.1 - 2010.4, click [**Forecast**], name the forecasts series **lrgdp2** and name the standard errors **sef2**, and specify the **forecast period** as **2007.1 2006.4.** Click **OK** and [**Print**]. Close the equation window (Notice that when a model is estimated with the dependent variable transformed with the **D()** operator, the forecasts produced by EVIEWS are for the level of the series and not the first difference).

To compute a 95% confidence interval for the forecasts do the following. First, compute the upper bound for the CI: Click [**Genr**], type **uf2 = lrgdpf2 + 1.96*sef2**, **OK**. Next compute the lower bound: Click [**Genr**], type **lf2 = lrgdpf2 - 1.96*sef2**, **OK**.

To plot the forecasts, 95% confidence interval and actual values of log real GDP over the period 2007.1 - 2010.4 do the following. Change the sample period: [**Sample**], **2007.1 2010.4**, **OK**. Then, highlight the four series **lrgdp, lrgdpf2, lf2, uf2** and double click to bring up the group window. Click [**View**], select **Graph** and [**Print**]. Close the group window.

That's it for Lab #2. Save the workfile, [**Save**], and exit EVIEWS, **File/Exit**. Go have a beer.