Econ 582 Homework 1

Due: Friday, 4/5/2013.

Question 1: Is the following MA(2) process covariance stationary?

$$\begin{aligned} y_t &= (1 + 2.4L + 0.8L^2)\varepsilon_t \\ E[\varepsilon_t] &= 0,\ E(\varepsilon_t \varepsilon_\tau) = \begin{cases} 1 & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

If so, calculate its autocovariances.

Question 2: Is the following AR(2) process covariance stationary?

$$\begin{aligned} (1 - 1.1L + 0.18L^2)y_t &= \varepsilon_t \\ E[\varepsilon_t] &= 0,\ E(\varepsilon_t \varepsilon_\tau) = \begin{cases} 1 & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases} \end{aligned}$$

If so, calculate its autocovariances.

Question 3: *Properties of ARMA(1,1) model.* Consider the ARMA(1,1) model

$$\begin{aligned} y_t &= c + \phi y_{t-1} + \varepsilon_t + \theta \varepsilon_{t-1}, \\ \varepsilon_t &\sim iid(0, \sigma^2), \end{aligned}$$

or, in lag operator notation,

$$\phi(L) y_t = \theta(L) \varepsilon_t,$$

where $\phi(L) = 1 - \phi L$ and $\theta(L) = 1 + \theta L$.

1. What restrictions on the parameters ϕ and θ are required for the ARMA(1,1) model to be stationary and invertible? Intuitively, what does it mean for the model to be stationary and invertible?

2. Assuming that the model is stationary, solve for the (infinite order) moving average representation:

   $$y_t = \mu + \varepsilon_t + \psi_1 \varepsilon_{t-1} + \psi_2 \varepsilon_{t-2} + \cdots$$

   Give an algorithm for determining the moving average coefficients, ψ_j, from the parameters of the ARMA(1,1) model. If the model is stationary and ergodic, what happens to ψ_j as $j \to \infty$?

3. Determine the unconditional moments $E[y_t] = \mu$, $var(y_t) = \gamma_0$, $cov(y_t, y_{t-1})$ $= \gamma_1$ and $\rho_1 = \frac{\gamma_1}{\gamma_0}$ as functions of the parameters of the ARMA(1,1) model. In addition, show that

   $$\begin{aligned} \gamma_j &= \phi \gamma_{j-1} \\ \rho_j &= \phi \rho_{j-1} \end{aligned}$$

   for $j > 1$.

4. What happens to the model if $\phi = -\theta$?