Econ 582 Final Exam

Eric Zivot

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1. Instructions

This is an open book exam. You must do your own work and you cannot discuss these questions with anyone other than myself or Clarisse. The exam is due in section on Friday 6/2/00. If you have any questions about the exam or think there is a typo please email either Clarisse or myself immediately. I will post any corrections to the exam on the announcements class web page. I will also be in my office most of Thursday and available to answer questions.

1.1. Question 1: Systems of Demand Equations

Consider a translog cost function for one output and three factor inputs that does not impose constant returns to scale

$$\begin{aligned}\ln C &= \alpha + \beta_1 \ln p_1 + \beta_2 \ln p_2 + \beta_3 \ln p_3 \\ &\quad + \delta_{11} \cdot \frac{1}{2} (\ln p_1)^2 + \delta_{12} (\ln p_1 \ln p_2) + \delta_{13} (\ln p_1 \ln p_3) \\ &\quad + \delta_{22} \cdot \frac{1}{2} (\ln p_2)^2 + \delta_{23} (\ln p_2 \ln p_3) + \delta_{33} \cdot \frac{1}{2} (\ln p_3)^2 \\ &\quad + \gamma_{y1} \ln Y \ln p_1 + \gamma_{y2} \ln Y \ln p_2 + \gamma_{y3} \ln Y \ln p_3 \\ &\quad + \beta_y \ln Y + \beta_{yy} \cdot \frac{1}{2} (\ln Y)^2 + \varepsilon_c\end{aligned}$$

where C denotes total cost, p_i = input price $(i = 1, 2, 3)$ and Y = output.

The factor share equations comprise the following SUR model

$$\begin{aligned}S_{1t} &= \beta_1 + \delta_{11} \ln p_{1t} + \delta_{12} \ln p_{2t} + \delta_{13} \ln p_{3t} + \gamma_{y1} \ln Y_t + \varepsilon_{1t} \qquad (1.1) \\ S_{2t} &= \beta_2 + \delta_{12} \ln p_{1t} + \delta_{22} \ln p_{2t} + \delta_{23} \ln p_{3t} + \gamma_{y2} \ln Y_t + \varepsilon_{2t} \\ S_{3t} &= \beta_3 + \delta_{13} \ln p_{1t} + \delta_{23} \ln p_{2t} + \delta_{33} \ln p_{3t} + \gamma_{y3} \ln Y_t + \varepsilon_{3t}\end{aligned}$$

where $S_i = \frac{p_i x_i}{C} = i^{th}$ factor cost share. Assume the errors are iid with mean zero and have covariance matrix Σ.

1. The factor share equations are in the form

$$y_{mt} = x'_{mt}\beta_m + e_{mt}, \; m = 1, 2, 3 \tag{1.2}$$

   Give explicit expressions for y_{mt}, x_{mt} and β_m in terms of the variables and coefficients in (1.1).

2. Is OLS on the SUR system efficient? Why or why not?

3. The three factor cost shares must add identically to 1; that is

$$S_1 + S_2 + S_3 = 1.$$

   What restriction does this place of the model parameters?

4. Brie! y describe how to obtain efficient estimates of the parameters of (1.1) using feasible GLS.

5. Assume that the errors are normally distributed and derive the likelihood function for the model (1.1). Are there any problems associated with maximizing this likelihood function?

1.2. Question 2: Logistic Regression

Suppose you are interested in the effect of monthly income on the decision to own an automobile. You decide that the logistic distribution is the appropriate way to model the car purchase decision. That is, you specify

$$\pi_i = P(y_i = 1) = \frac{\exp(\beta x_i)}{1 + \exp(\beta x_i)}, \; i = 1, \ldots, n$$

where $y_i = 1$ if individual i owns an automobile, 0 otherwise, and x_i is the monthly income, in thousands of dollars, for individual i.

1. What is the likelihood function and log-likelihood for and iid sample of n observations on (y_i, x_i)?

2. Compute the score function of the log-likelihood, $S(\beta) = \frac{\partial \ln L(\beta)}{\partial \beta}$. Can you &nd an analytical solution for the mle?

3. Derive the marginal effect, $\frac{\partial \pi_i}{\partial x_i}$, and evaluate it at $x_i = 1$ and $x_i = 5$ for $\beta = 1$.

4. Consider testing the hypothesis that $\beta = 0$. Derive the LM statistic for this hypothesis.

5. Use the LM statistic derived in the previous question to test the hypothesis that $\beta = 0$ for the data set given below

i	1	2	3	4	5	6
y_i	1	1	0	0	1	0
x_i	5	3	3	0	4	1

1.3. Question 3: Two-Stage Estimation of the Tobit Regression Model

Consider the censored (Tobit) regression model in index form

$$\begin{aligned} y_i^* &= \mathsf{x}_i'\boldsymbol{\beta} + \varepsilon_i, \; i = 1, \ldots, N \\ \varepsilon_i &\sim iid \; N(0, \sigma^2), \end{aligned}$$

and where the observable value y_i satisfies

$$\begin{aligned} y_i &= y_i^* \text{ if } y_i^* > 0 \\ &= 0 \text{ if } y_i^* \leq 0. \end{aligned}$$

Using results on truncated and censored normal distributions we have

$$E[y_i|\mathsf{x}_i, y_i > 0] = \mathsf{x}_i'\boldsymbol{\beta} + \sigma \cdot \lambda\left(\frac{-\mathsf{x}_i'\boldsymbol{\beta}}{\sigma}\right), \tag{1.3}$$

$$E[y_i|\mathsf{x}_i] = \mathsf{x}_i'\boldsymbol{\beta} \cdot \Phi\left(\frac{\mathsf{x}_i'\boldsymbol{\beta}}{\sigma}\right) + \sigma \cdot \varphi\left(\frac{\mathsf{x}_i'\boldsymbol{\beta}}{\sigma}\right) \tag{1.4}$$

where

$$\lambda(z) = \frac{\varphi(z)}{1 - \Phi(z)}$$

is the inverse Mills ratio, $\varphi(z)$ is the density of a standard normal random variable and $\Phi(z)$ is the CDF of a standard normal random variable. Define the dummy binary variable I_i as

$$\begin{aligned} I_i &= 1 \text{ if } y_i > 0 \\ &= 0 \text{ if } y_i \leq 0. \end{aligned}$$

1. What are the problems associated with OLS estimation of the regression model
$$y_i = x_i'\beta + u_i$$
using all of the observed (censored) data?

2. Give the log-likelihood function for the censored regression model.

3. Show that you can obtain a consistent estimate of the ratio $\boldsymbol{\beta}/\sigma$ using a probit regression where the dependent variable is I_i.

4. Using the consistent estimate of $\boldsymbol{\beta}/\sigma$ from the probit regression and (1.3), give the regression equation to get estimates of $\boldsymbol{\beta}$ and σ using OLS on data for which $y_i > 0$. Will these estimates of β and σ be consistent?

5. Using the consistent estimate of $\boldsymbol{\beta}/\sigma$, describe how you can use (1.4) to compute estimates of $\boldsymbol{\beta}$ and σ using OLS on all of the observed data. Will these estimates of β and σ be consistent?

1.4. Question 4: Time Series Models

Consider modeling the dynamic behavior of the logarithm of detrended U.S. postwar quarterly real GDP, from 1947:I through 1998:II using ARMA models. Detrended real GDP is de&ned as the residual from the regression of 100 times the logarithm of real GDP on a constant and time trend.

1. A plot of detrended real GDP is given in Figure 1 (see attached). Comment on the dynamic behavior of the data.

2. The sample autocorrelation function and sample partial autocorrelation function is given in Figure 2. Based on the shapes of these functions, what ARMA(p,q) model looks appropriate for detrended real GDP? Justify your answer.

Now consider the following AR(2) model estimated by OLS for detrended real GDP (hereafter denoted y_t)

$$\begin{aligned} y_t &= \underset{(0.0690)}{0.0034} + \underset{(0.0659)}{1.3259} \cdot y_{t-1} - \underset{(0.0657)}{0.3561} \cdot y_{t-2} + \hat{\varepsilon}_t \\ R^2 &= 0.9620,\ \hat{\sigma} = 0.9847, T = 204, \end{aligned}$$

where estimated standard errors are in parentheses.

3. Based on the estimated autoregressive coefficients is the estimated AR(2) model for y_t covariance stationary? Justify your answer.

4. Compute an estimate of the mean, μ, and variance, γ_0, of y_t.

5. Using the delta method, compute an estimate of the asymptotic standard error for $\hat{\mu}$. Note that for $\beta = (c, \phi_1, \phi_2)'$ a consistent estimate of the asymptotic covariance of $\hat{\beta}$ is

$$\widehat{cov}(\hat{\beta}) = \hat{\sigma}^2(X'X)^{-1} = \begin{pmatrix} 0.0048 & 0 & 0 \\ 0 & 0.0043 & -0.0042 \\ 0 & -0.0042 & 0.0043 \end{pmatrix}.$$

6. Compute and plot an estimate of the impulse response function

$$\frac{\partial y_{t+s}}{\partial \varepsilon_t} = \psi_s$$

for $s = 1, 2, \ldots, 12$. Comment on the shape of this function. In particular, at what horizon s is the impact of a shock to ε_t the largest?