Economics 483

Midterm Exam

This is a closed book and closed note exam. However, you are allowed one page of handwritten notes. Answer all questions and write all answers in a blue book or on separate sheets of paper. Time limit is 2 hours and 10 minutes. Total points = 100.

I. Return Calculations (15 pts)

Use the end of month price data for the S&P 500 index in the table below to answer the following questions.

Month	Price
Dec-98	1229.23
Jan-99	1279.64
Feb-99	1238.33
Mar-99	1286.37
Apr-99	1335.18
May-99	1301.84
Jun-99	1372.71
Jul-99	1328.72
Aug-99	1320.41
Sep-99	1282.71
Oct-99	1362.93
Nov-99	1388.91
Dec-99	1469.25

1. What is the simple return between December, 1998 and December, 1999? Suppose you can get this return every year for the next five years. What is the simple five year return?

2. What is the continuously compounded return between December, 1998 and December, 1999? Suppose you can get this return every year for the next five years. What is the continuously compounded five year return?

3. Why do we use continuously compounded returns instead of simple returns in our probability models of returns?

II. Random Variables and Probability (10 pts)

Let X be a continuous random variable with pdf $p(x)$. Given the following shape characteristics of $p(x)$, draw a rough sketch of the pdfs for the following cases.
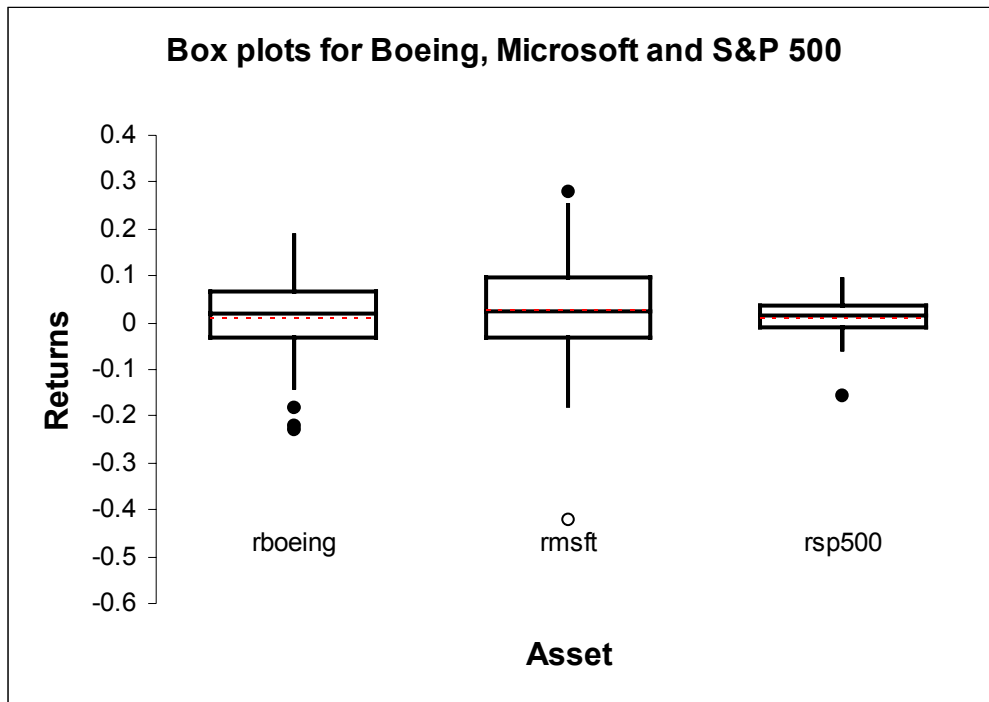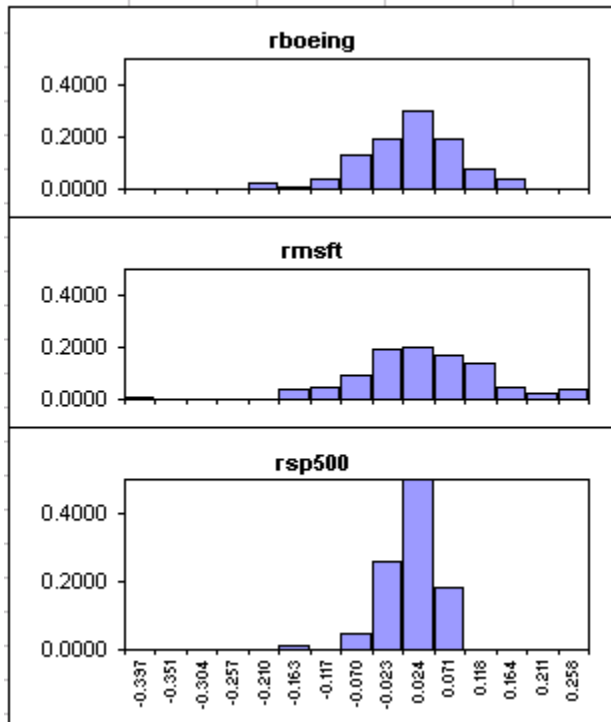
1. $E[X] = 0$, $var(X) = 1$, $skew(X) = 0$, $excess\ kurt(X) = 0$.
2. $E[X] = 0$, $var(X) = 1$, $skew(X) = -0.5$, $excess\ kurt(X) = 0$.
3. $E[X] = 0$, $var(X) = 1$, $skew(X) = 0.5$, $excess\ kurt(X) = 0$.
4. $E[X] = 0$, $var(X) = 1$, $skew(X) = 0$, $excess\ kurt(X) = 3$.
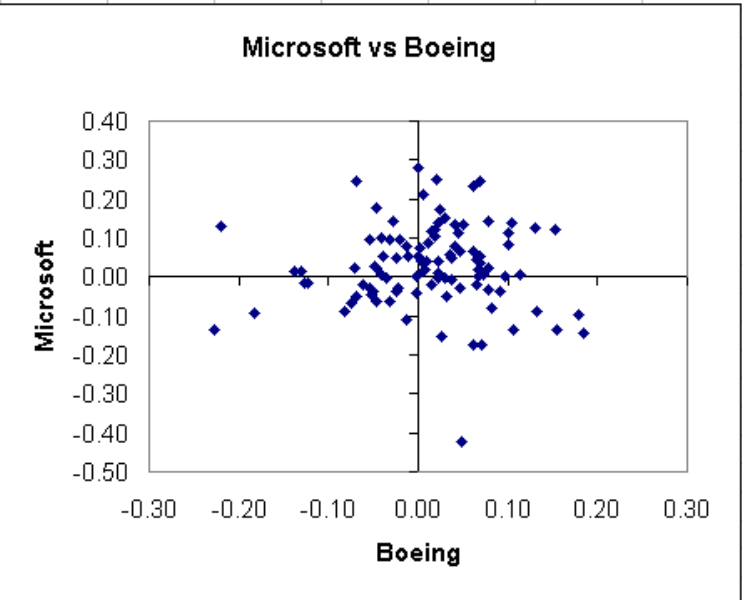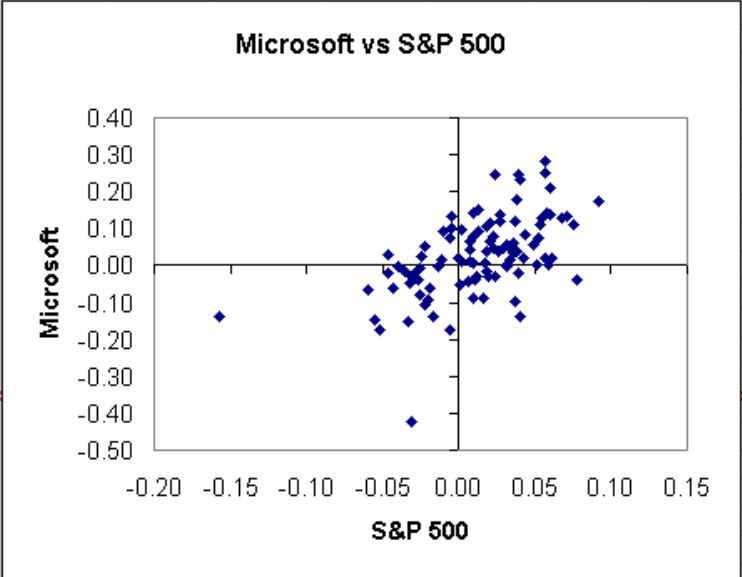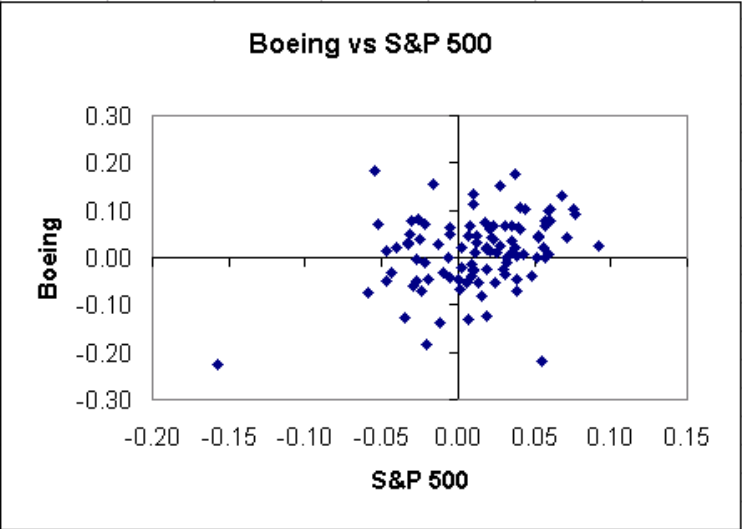5. $E[X] = 0$, $var(X) = 1$, $skew(X) = 0$, $excess\ kurt(X) = -3$.

III. Descriptive Statistics (20 pts)

Consider the monthly continuously compounded returns on Boeing, Microsoft and the S&P 500 computed over the period June 1992 – October 2000. Descriptive statistics for these returns are given in the table below and histograms, boxplots and scatterplots are presented on the following pages. Based on the descriptive statistics and graphs, answer the following questions.

1. Compare the return – risk properties of the three assets. Which asset appears to be safest asset and which asset appears to be the most risky asset?

2. Do the return distributions of the three assets look like they could be normal distributions? Justify your answers.

3. Describe the direction and strength of linear association between the three assets. Which assets appear to have the highest and lowest correlations?

4. Why is the monthly standard deviation for the S&P 500 lower than the standard deviation values for Boeing and Microsoft?

	Univariate Statistics		
	rboeing	rmsft	rsp500
Count	100	100	100
Average	0.012	0.028	0.013
Median	0.020	0.022	0.016
Standard Deviation	0.076	0.107	0.038
Variance	0.006	0.011	0.001
Skewness	-0.567	-0.511	-0.933
Excess Kurtosis	1.123	2.381	2.853

rboeing

rmsft

rsp500

Box plots for Boeing, Microsoft and S&P 500

rboeing   rmsft   rsp500

Returns

Asset

Boeing vs S&P 500

Microsoft vs S&P 500

Microsoft vs Boeing

IV. The CER Model and Monte Carlo Simulation (20 pts)

Consider the constant expected return (CER) model

$$R_{it} = \mu_i + \varepsilon_{it}, \quad i = 1, \ldots, N; \, t = 1, \ldots, T$$
$$\varepsilon_{it} \sim iid \, N(0, \, \sigma_i^2)$$

where R_{it} denotes the return on asset i and ε_{it} is a normally distributed random error term. For specificity, assume that $\mu_i = 0.025$ and $\sigma_i = 0.075$.

1. What is the interpretation of ε_{it} in the CER model?
2. Briefly explain how you could generate one Monte Carlo simulation of $T = 50$ observations from the CER model for an asset using Excel.
3. Recall, the least squares estimator of μ_i in the CER model is the sample mean

$$\hat{\mu} = \frac{1}{T} \sum_{t=1}^{T} R_{it} \, .$$

The sample mean is an unbiased estimator of μ_i; that is, $E[\hat{\mu}_i] = \mu_i$. Using the concept of Monte Carlo simulations from the CER model, briefly describe what it means for $\hat{\mu}_i$ to be an unbiased estimate of μ_i.

4. The precision of $\hat{\mu}_i$ is measured by the *standard error*, $SE(\hat{\mu}_i)$. Using the concept of Monte Carlo simulations from the CER model, briefly describe what $SE(\hat{\mu}_i)$ represents.

V. Portfolio Theory (15 pts)

Consider the problem of an investor trying to determine the best portfolio of two risky assets (stocks) and a risk-free asset (T-bill). Let the two risky assets be Boeing and Microsoft, the risk-free asset be a one-year T-bill and suppose the investment horizon is one year. It is assumed that investors like assets with high expected returns but dislike those with high risk (as measured by return standard deviation) and choose to hold efficient portfolios. Transfer the diagram below to your bluebook and answer the following questions.
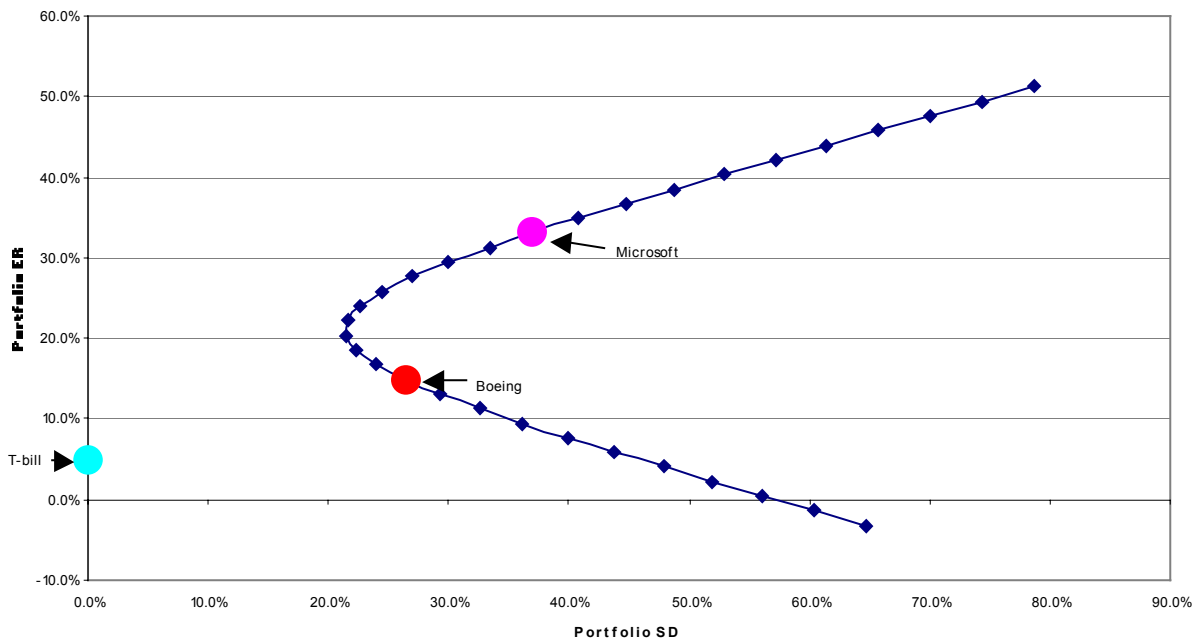
1. Identify the investment opportunity set consisting of only the two risky assets. On this diagram, identify the sets of inefficient and efficient portfolios.

2. Identify the tangency portfolio and the set of efficient combinations of T-bills and the two risky assets. On this efficient set, indicate and describe the asset allocation of the portfolios that a very risk averse investor would choose to hold and the portfolios that a very risk tolerant investor would choose to hold.

3. State the maximization problem to be solved to determine the tangency portfolio.

VI. Efficient Portfolios and Value-at-Risk (15 pts)

Consider an investor who has $100,000 to invest over the next year. Assume that the investor initially holds a portfolio consisting of 80% Boeing and 20% Microsoft. This asset is denoted "asset A" in the diagram below. Use the information in the table below to answer the following questions.
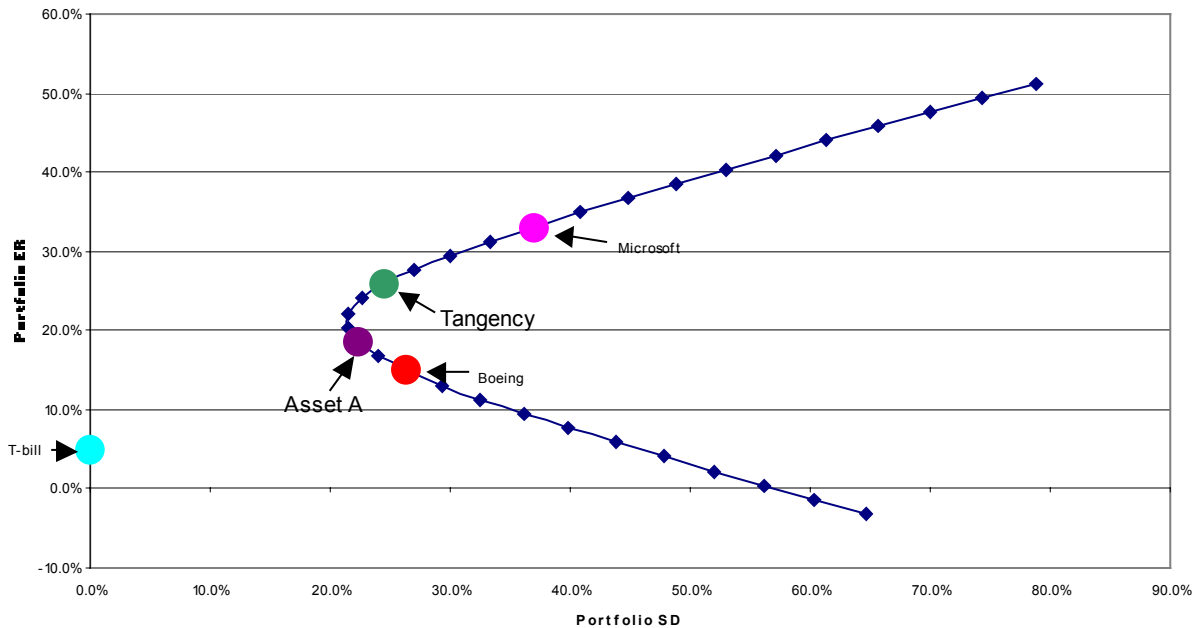
Asset	E[R]	SD(R)
Asset A	0.186	0.223
Tangency	0.259	0.246
T-Bill	0.06	0

1. What is the 5% Value-at-risk over the next year on the $100,000 portfolio consisting of just asset A? Assume that returns are continuously compounded.   (*FYI* – NORMINV(0.05, 0.186, 0.223) = -0.181)

2. Find the efficient portfolio (combination of T-bills, Boeing and Microsoft) that has the same expected return as asset A. In this efficient portfolio, how much is invested in the T-bill, Boeing and Microsoft? What is the risk (standard deviation) of this portfolio?

3. Find the efficient portfolio (combination of T-bills, Boeing and Microsoft) that has the same risk (standard deviation) as asset A. In this efficient portfolio, how much is invested in the T-bill, Boeing and Microsoft? What is the expected return of this portfolio?