University of Washington | Winter 2002
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Economics 483

Midterm Exam

This is a closed book and closed note exam. However, you are allowed one page of handwritten notes. Answer all questions and write all answers in a blue book or on separate sheets of paper. Time limit is 2 hours and 10 minutes. Total points = 100.

I. Return Calculations (20 pts)

Use the end of month price data for the S&P 500 index in the table below to answer the following questions.

Date	Close price
December-00	129.975
January-01	135.754
February-01	122.805
March-01	115.923
April-01	125.827
May-01	125.122
June-01	122.14
July-01	120.895
August-01	113.722
September-01	104.44
October-01	105.8
November-01	114.05
December-01	114.3

1. What is the simple 1 month return between December, 2000 and December, 2001? Suppose you can get this return every month for the next year. What is the simple 1 year return? Compare this return to the actual 1 year simple return between December, 2000 and December, 2001.

2. Suppose the consumer price index (CPI) for December 2000 is equal to 100 and that the CPI for December 2001 is 105. Compute the simple 1 year real return between December 2000 and December 2001.

3. What is the continuously compounded 1 month return between December, 2000 and December, 2001? Suppose you can get this return every month for the next year. What is the continuously compounded annual year return? Compare this return to the actual 1 year continuously compounded return between December, 2000 and December, 2001.

4. Suppose the consumer price index (CPI) for December 2000 is equal to 100 and that the CPI for December 2001 is 105. Compute the continuously compounded annual real return between December 2000 and December 2001.

II. Random Variables and Probability (32 pts)

A. Normal distribution

Let X be a normally distributed random variable with mean $\mu = 0.05$ and variance $\sigma^2 = (0.10)^2$.

1. Sketch the pdf (probability curve) of X. On your sketch indicate the location of $\mu,\ \mu+\sigma,$ and $\mu-\sigma$.
2. What are the skewness and kurtosis values for X?
3. What is the approximate 2.5% quantile of the pdf for X?
4. Briefly explain how you would compute $\Pr(-1.5 \le X < 0.5)$ using Excel.
5. Consider the new random variable $W = 1000 + 1000 \cdot X$. Compute $E[W],\ \text{var}(W)$ and $SD(W)$.

B. Value-at-Risk

Consider an investment of \$100,000 in the S&P 500 index for a period of *one day*. Let R_t denote the continuously compounded *daily* return on the S&P 500 index and assume that $R_t \sim N(0.0004, (0.00125)^2)$. That is, $E[R_t] = 0.04\%$ and $SD(R_t) = 0.125\%$.

6. Compute the daily 1% and 5% value-at-risk (VaR). FYI, the 1% and 5% quantiles of the $N(0.0004, (0.00125)^2)$ pdf are –0.0025 and –0.0017, respectively.

C. Aggregating returns

Assume that $R_t \sim iid\ N(0.0004, (0.00125)^2)$ for every day over the next month and assume there are 30 days in a month. Let R_m denote the monthly continuously compounded return.

7. Express the monthly continuously compounded return R_m in terms of the daily continuously compounded returns R_t.
8. Compute $E[R_m],\ \text{var}(R_m),$ and $SD(R_m)$

III. Descriptive Statistics and the CER Model (28 pts)

Consider the monthly continuously compounded returns on Washington Mutual stock and the S&P 500 computed using end of month closing prices over the period December 1991 – December 2001. Descriptive statistics for these returns are given in the table below and histograms, boxplots and scatterplots are presented on the following pages. Based on the descriptive statistics and graphs, answer the following questions.

1. Compare the return – risk properties of the two assets. Which asset appears to be safest asset and which asset appears to be the most risky asset? Justify your answer.

2. Do the return distributions of the two assets look like they could be normal distributions? Use the univariate statistics in the table below and the boxplots to justify your answers.

3. Describe the direction and strength of linear association between the two assets.

4. For the constant expected return model

$$R_{it} = \mu_i + \varepsilon_{it},\ \varepsilon_{it} \sim iid\ N(0, \sigma_i^2)$$
$$\text{cov}(R_{it}, R_{jt}) = \sigma_{ij},\ corr(R_{it}, R_{jt}) = \rho_{ij}$$

give the method of moments estimates for $\mu_i, \sigma_i^2, \sigma_i, \sigma_{ij}$ and ρ_{ij} for the S&P 500 index and for Washington Mutual.

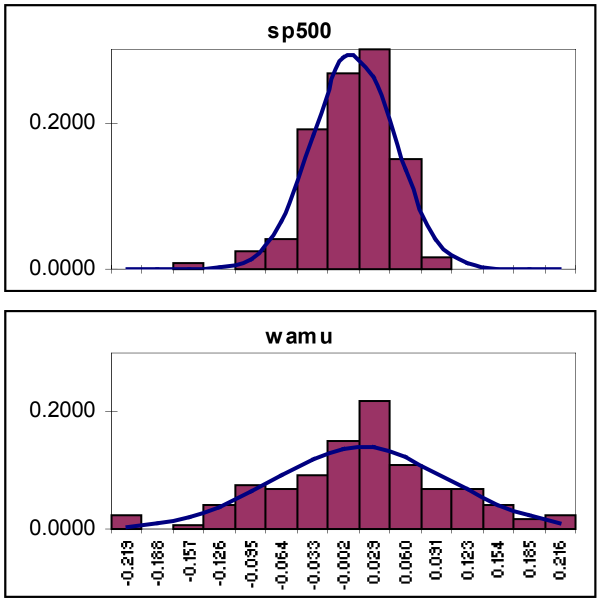
5. For each asset, use the approximate analytical formulas to compute estimated standard error values for $\mu_i, \sigma_i,$ and ρ_{ij}. That is compute $SE(\hat{\mu}_i), SE(\hat{\sigma}_i),$ and $SE(\hat{\rho}_{ij})$. Comment on the size of these estimated standard errors.

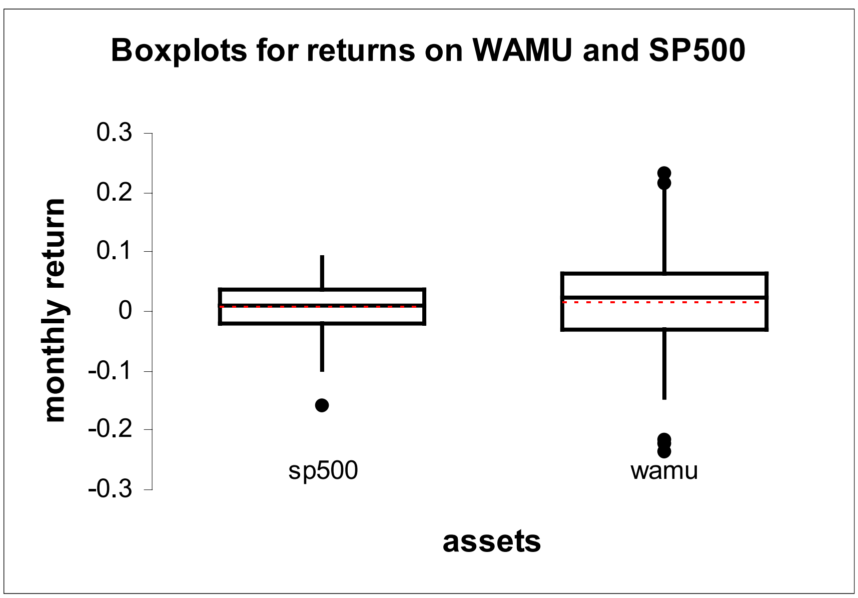
6. For each asset, compute an approximate 95% confidence interval for μ_i. Use the widths of the intervals to evaluate the precision of estimate $\hat{\mu}_i$.

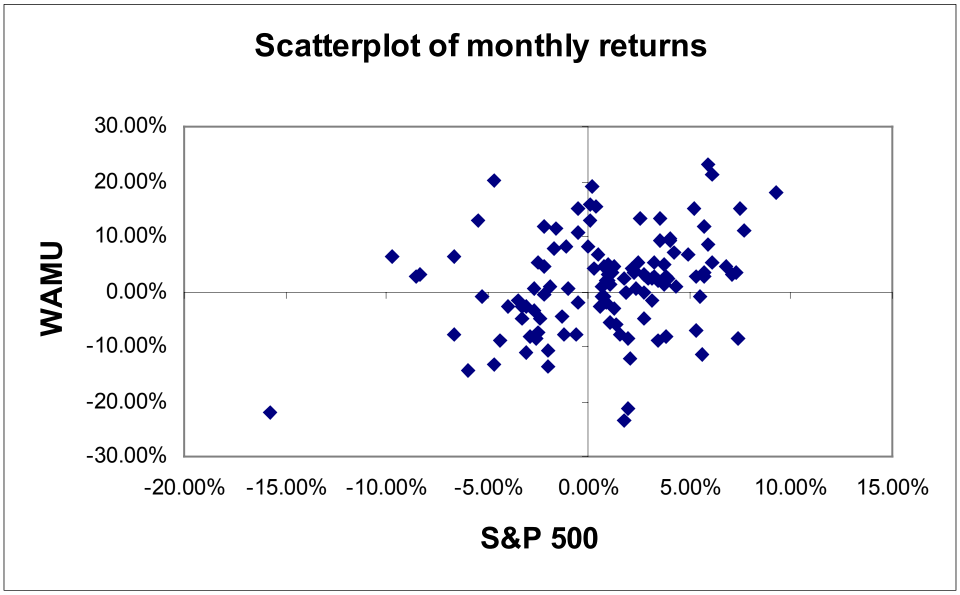
7. The monthly returns on the S&P 500 index and 24-month rolling estimates of $\hat{\mu}_{sp500}$ and $\hat{\sigma}_{sp500}$ are illustrated in figure 1. Based on this figure, what can you say about the appropriateness of the CER model for the returns on the S&P 500 index over the ten year period December 1991 – December 2001?

Univariate Statistics

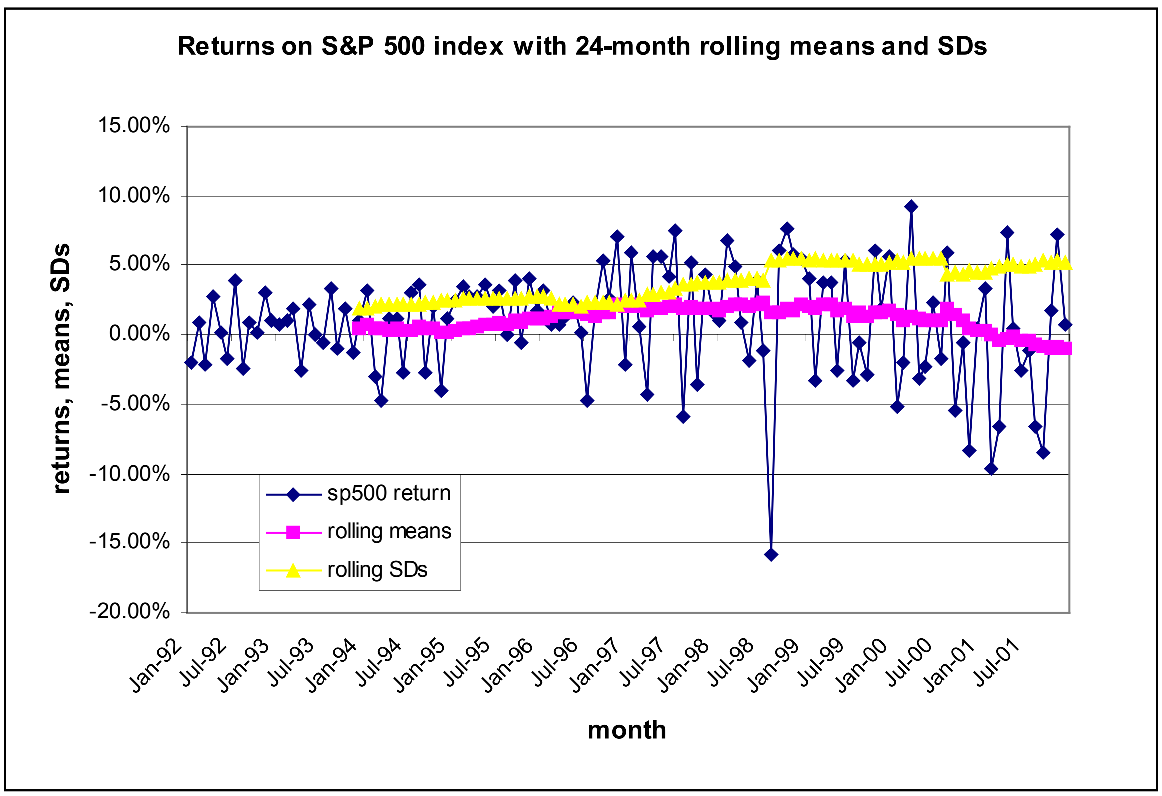
	sp500	wamu
Count	120	120
Average	0.84%	1.56%
Minimum	-15.76%	-23.49%
Maximum	9.23%	23.13%
Range	24.99%	46.61%
Standard Deviation	4.06%	8.80%
Variance	0.0017	0.0077
Skewness	-0.803	-0.171
Kurtosis	1.602	0.444
Covariance	0.0011	
Correlation	0.3003	


sp500
0.2000
0.0000
wamu
0.2000
0.0000
-0.219
-0.188
-0.157
-0.126
-0.095
-0.064
-0.033
-0.002
0.029
0.060
0.091
0.123
0.154
0.185
0.216


Boxplots for returns on WAMU and SP500
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
monthly return
sp500
wamu
assets



Scatterplot of monthly returns
30.00%
20.00%
10.00%
0.00%
-10.00%
-20.00%
-30.00%
WAMU
-20.00%
-15.00%
-10.00%
-5.00%
0.00%
5.00%
10.00%
15.00%
S&P 500



Returns on S&P 500 index with 24-month rolling means and SDs
15.00%
10.00%
5.00%
0.00%
-5.00%
-10.00%
-15.00%
-20.00%
returns, means, SDs
sp500 return
rolling means
rolling SDs
Jan-92
Jul-92
Jan-93
Jul-93
Jan-94
Jul-94
Jan-95
Jul-95
Jan-96
Jul-96
Jan-97
Jul-97
Jan-98
Jul-98
Jan-99
Jul-99
Jan-00
Jul-00
Jan-01
Jul-01
month

Figure 1

IV. The CER Model and Monte Carlo Simulation (20 pts)

Consider the constant expected return (CER) model

$$R_t = \mu + \varepsilon_t,\ \ t = 1,\ldots,T$$
$$\varepsilon_{it} \sim iid\ N(0,\ \sigma^2)$$

where R_{it} denotes the continuously compounded return on asset i and ε_t is a normally distributed random error term. For specificity, assume that $\mu = 0.01$ and $\sigma = 0.05$.

1. What are the assumptions of the CER model?
2. What is the interpretation of ε_t in the CER model?
3. The method of moments estimator of σ in the CER model is the square root of the sample variance

$$\hat{\sigma} = \sqrt{\frac{1}{T-1}\sum_{t=1}^{T}(R_t - \hat{\mu})^2}\ ,$$

where $\hat{\mu} = T^{-1}\sum_{t=1}^{T} R_t$ is the sample mean. Using the concept of Monte Carlo simulations from the CER model, briefly describe how you could determine if $\hat{\sigma}$ is an unbiased estimate of σ.
4. The precision of $\hat{\sigma}$ is measured by the *standard error*, $SE(\hat{\sigma})$. What is the approximate formula for computing $SE(\hat{\sigma})$? Using the concept of Monte Carlo simulations from the CER model, briefly describe how you could compute $SE(\hat{\sigma})$ exactly.