

Econ 483 Fall 2003

Midterm Solns

(c)

I

1. $P_{A,t-1} = 27$, $P_{A,t} = 32$

$P_{C,t-1} = 15$, $P_{C,t} = 12$

$$R_{A,t} = \frac{32 - 27}{27} = 0.185, R_{C,t} = \frac{12 - 15}{15} = -0.20$$

2. $r_{A,t} = \ln(1 + 0.185) = 0.170$, $r_{C,t} = \ln(1 - 0.20) = -0.200$
 $= -0.223$

3. $R_{C,t} = \frac{(12 + 3) - 15}{15} = 0.8$

Dividend yield = $3/15 = 0.20$

4. initial wealth = $27 + 15 = 42$

$x_A = 27/42 = 0.64$, $x_C = 15/42 = 0.36$

$$R_p = x_A R_A + x_C R_C = (0.64)(0.185) + (0.36)(-0.20) \\ = 0.0464$$

Check: initial wealth = 42
end of period wealth = 44

$\left. \begin{array}{l} \text{8.0\% in wealth} \\ = \frac{44 - 42}{42} = 0.047 \end{array} \right\}$

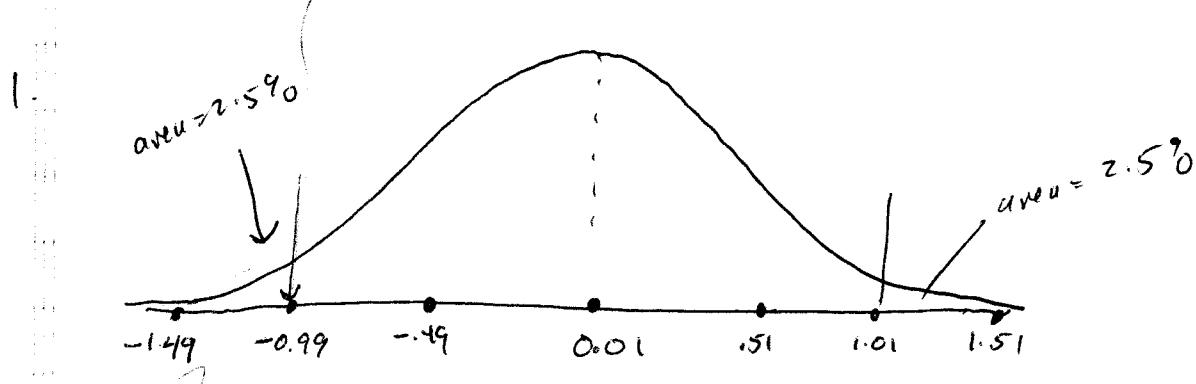
difference due
to rounding.

5. $r_p = \ln(1 + 0.0464) = 0.045$

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II Normal Distribution

$$X \sim N(0.01, (0.50)^2)$$



1. $\mu + \sigma = 0.01 + 0.50 = 0.51$, $\mu - \sigma = 0.01 - 0.50 = -0.49$

$\mu + 2\sigma = 0.01 + 1 = 1.01$, $\mu - 2\sigma = 0.01 - 1 = -0.99$

$\mu + 3\sigma = 0.01 + 1.5 = 1.51$, $\mu - 3\sigma = 0.01 - 1.5 = -1.49$

2. Because roughly 99% of the area is between $\mu \pm 3\sigma$, the 0.5% quantile is roughly $\mu - 3\sigma = -1.49$

3. Given the above sketch, about 2.5% probability is less than -0.99 so a guess for $\Pr(X < -1)$ is about 2%.

4. No, because the smallest value for a simple return is -1 (-100%) : This distribution allows values ~~less than -1~~ less than -1 with a non-negative probability.

5. Yes, because $r = \ln(1+\mu)$ or $\mu = e^r - 1$. A value for $r < -1$ implies a value for $\mu > -1$ e.g. let $r = -2$. Then $\mu = e^{-2} - 1 = -0.864$.

III

1. Graphically, the returns look quite normal
 - roughly symmetric histograms & densities and fairly linear qq-plots.

However, the histograms & boxplots indicate a slight negative skewness (long left tail)

The numerical summaries show negative sample skewness. The excess kurtosis values are small, which indicates that the tails of the distribution are close to normal.

Amazon appears more risky than Dow Jones because its SD value, 0.225, is 4 times larger than the SD value for Dow Jones, 0.051.

2. The plug-in principle estimates are

	Amazon	Dow Jones
$\hat{\mu}_1$	0.018	0.004
$\hat{\sigma}_1^2$	0.051	0.003
$\hat{\sigma}_1$	0.226	0.051
$\hat{\sigma}_{ij}$	0.005	0.005
$\hat{\rho}_{ij}$	0.469	0.469

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	Amazon	Dow Jones
$\hat{\mu}$	0.018	0.004
$\hat{SE}(\hat{\mu})_{boot}$	0.030	0.006
$\hat{\sigma}$	0.226	0.051
$\hat{SE}(\hat{\sigma})_{boot}$	0.021	0.005

The values of $\hat{SE}(\hat{\mu})_{boot}$ are large compared to the estimate $\hat{\mu}$ for both Amazon & Dow Jones. The $\hat{SE}(\hat{\mu})_{boot}$ values are about $2 \times$ the $\hat{\mu}$ values.

The approximate 95% confidence intervals are $\hat{\mu} \pm 2 \cdot \hat{SE}(\hat{\mu})_{boot}$. These intervals will contain both positive and negative values for μ . Hence, there is a lot of uncertainty about the values for μ .

The values of $\hat{SE}(\hat{\sigma})_{boot}$ are much smaller than the estimates of σ for both Amazon & Dow Jones. (about 10x smaller!). The approximate 95% confidence intervals, $\hat{\sigma} \pm 2 \cdot \hat{SE}(\hat{\sigma})_{boot}$, are very narrow and imply that σ is well estimated for both Amazon & Dow. Jones.

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4. Analytic Formulas for SE

$$\hat{SE}(\hat{\mu}) = \hat{\sigma}/\sqrt{n}, \quad \hat{SE}(\hat{\delta}) = \hat{\delta}/\sqrt{2n}$$

	Amazon	Dow Jones
$\hat{SE}(\hat{\mu})_{boot}$	0.030	0.006
$\hat{SE}(\hat{\mu})$	$\frac{0.226}{\sqrt{61}} = 0.029$	$\frac{0.051}{\sqrt{61}} = 0.0065$
$\hat{SE}(\hat{\delta})_{boot}$	0.021	0.005
$\hat{SE}(\hat{\delta})$	$\frac{0.226}{\sqrt{2 \cdot 61}} = 0.020$	$\frac{0.051}{\sqrt{2 \cdot 61}} = 0.0046$

The bootstrap \hat{SE} values are very close to the analytic formulas. As a result the bootstrap distributions of $\hat{\mu} + \hat{\delta}$ are symmetric.

5. \$100,000 initial investment in Dow Jones.

$$\begin{aligned}\hat{q}_{0.05} &= \hat{\mu} + \hat{\delta}(-1.645) \\ &= 0.004 + (0.051)(-1.645) \\ &= -0.0768\end{aligned}$$

$$\exp(\hat{q}_{0.05}) - 1 = -0.0768$$

$$\hat{Var}_{0.05} = \$100,000 (-0.0768) = \underline{\underline{-\$7680}}$$

i.e. 5% chance to lose ~~\$7680~~ ~~00~~

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VI Portfolio Theory

$$R_p = 0.5 R_A + 0.5 R_B$$

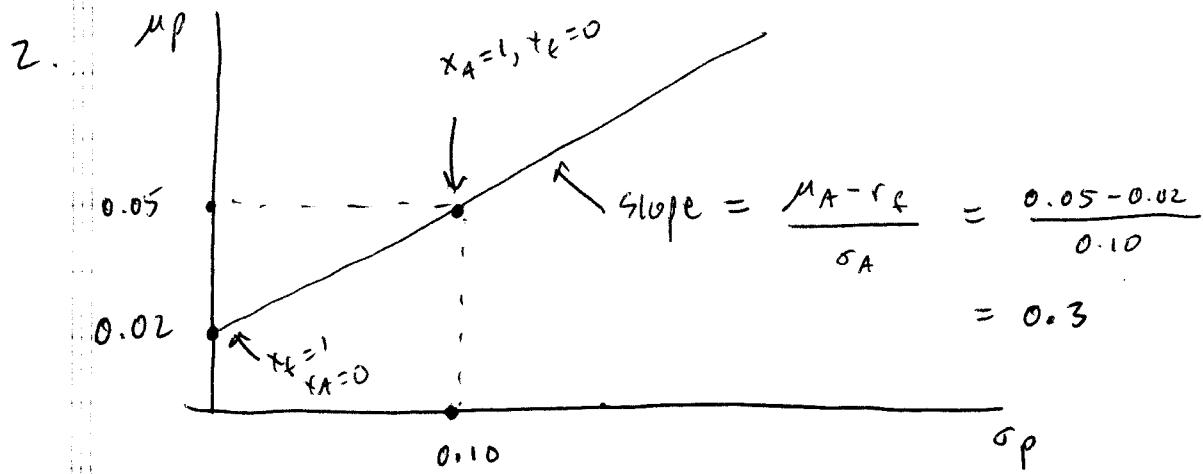
$$\mu_p = 0.5(0.05) + 0.5(0.07) = 0.06 \quad \underline{=}$$

~~0.0025~~ ~~0.01~~

$$\sigma_p^2 = (.5)^2 (.10)^2 + (.5)^2 (.20)^2 + 2(.5)(.5)(\rho)$$

$$= 0.0125$$

$$\sigma_p = 0.112$$



$$\mu_p = r_f + x_B(\mu_B - r_f)$$

Solve: $0.10 = \mu_p = 0.02 + x_B(0.07 - 0.02)$

$$\Rightarrow x_B = \frac{0.10 - 0.02}{0.07 - 0.02} = \frac{0.08}{0.05}$$

$$= \underline{1.6}$$

4. $\sigma_p = x_B \cdot \sigma_B$

Solve $0.10 = \sigma_p = x_B \cdot 0.20$

$$\Rightarrow x_B = \frac{0.10}{0.20} = 0.5$$

5. The global minimum variance portfolio solves

$$\min_{x_A, x_B} \sigma_p^2 = x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_{AB}$$

The analytic soln is

$$x_A = \frac{\sigma_B^2 - \sigma_{AB}}{\sigma_A^2 + \sigma_B^2 - 2\sigma_{AB}} \quad , \quad x_B = 1 - x_A$$

$$= \frac{(0.20)^2 - 0}{(0.10)^2 + (0.20)^2 - 0}$$

$$= 0.80$$

$$x_B = 1 - x_A = 1 - 0.80 = 0.20$$

$$\mu_p^{\min} = (0.80)(0.05) + (0.20)(0.07)$$

$$= \underline{0.054}$$