University of Washington
Department of Economics

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Economics 483

Final Exam

This is a closed book and closed note exam. However, you are allowed one page of handwritten notes. Answer all questions and write all answers in a blue book. Total points = 100.

I. Monte Carlo Simulation (20 pts)

Consider the constant expected return (CER) model

$$R_{it} = \mu_i + \varepsilon_{it}, \quad t = 1,\ldots,T$$
$$\varepsilon_{it} \sim iid \text{ N}(0,\ \sigma^2_{\varepsilon_i})$$

where R_{it} denotes the return on asset i and ε_{it} is a normally distributed random error term.

a. What is the interpretation of ε_{it} in the CER model?
b. Briefly explain how you could generate one Monte Carlo simulation of $T = 60$ observations from the CER model using Excel.
c. Recall, the estimator of μ_i in the CER model is the sample mean

$$\hat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} R_{it}\ .$$

The sample mean is an unbiased estimator of μ_i; that is, E[$\hat{\mu}_i$] = μ_i. Using the concept of Monte Carlo simulations from the CER model, briefly describe what it means for $\hat{\mu}_i$ to be an unbiased estimate of μ_i.
d. The precision of $\hat{\mu}_i$ is measured by the *standard error*, $SE(\hat{\mu}_i)$. Using the concept of Monte Carlo simulations from the CER model, briefly describe what $SE(\hat{\mu}_i)$ represents.

II. Matrix Algebra and Portfolio Math (20 points)

Let R_i denote the continuously compounded return on asset i ($i = 1, \ldots, N$) with $E[R_i] = \mu_i$, var(R_i) = σ_i^2 and cov(R_i, R_j) = σ_{ij}. Consider forming two portfolios of these N assets. Let x_i denote the share of wealth invested in asset i in the first portfolio and let y_i denote the share of wealth invested in asset i in the second portfolio such that $\sum_{i=1}^{N} x_i = 1$ and $\sum_{i=1}^{N} y_i = 1$. Define the ($N \times$ 1) vectors $\boldsymbol{R} = (R_1,\ldots,R_N)'$, $\boldsymbol{\mu} = (\mu_1,\ldots,\mu_N)'$, $\boldsymbol{x} = (x_1,\ldots,x_N)'$, $\boldsymbol{y} = (y_1,\ldots,y_N)'$, $\mathbf{1} = (1,\ldots,1)'$ and the ($N \times N$) covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}.$$

Using simple matrix algebra, answer the following questions.

a. What are the expressions for the returns on the two portfolios and what are the expressions for the constraint that the portfolio weights sum to 1?
b. What are the expressions for the expected returns on the two portfolios?
c. What are the expressions for the variances of the two portfolios?
d. What is the expression for the covariance between the two portfolios?

III. Single Index Model (30 pts)

Consider the single index model regression

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it},\ t = 1,\ldots,T$$
$$\varepsilon_{it} \sim iid\ \mathrm{N}(0,\ \sigma^2_{\varepsilon_i}) \text{ and } R_{Mt} \text{ is independent of } \varepsilon_{it} \text{ for all } i \text{ and } t$$

where R_{it} denotes the return on asset i and R_{Mt} denotes the return on the market portfolio proxy. Let μ_i and μ_{M} denote the expected returns on the asset and the market, respectively, and let $\sigma^2_{\varepsilon_i}$ and σ^2_M denote the variances of the asset and the market, respectively. Finally, let σ_{iM} denote the covariance between the asset and the market.

a. What is the interpretation of β in the single index model? What does it mean for β to be less than one, equal to one, or greater than 1?

b. What is the interpretation of ε_{it} in the single index regression?

c. Using the single index regression, compute $\mathrm{E}[R_{it}]$, $\mathrm{var}(R_{it})$ and $\mathrm{cov}(R_{it}, R_{jt})$.

d. Consider the special case of three assets (i = 1,2,3). Give an expression for the covariance matrix, Σ, based on the single index model.

e. Using the expression for $\mathrm{var}(R_{it})$, what is the proportion of the variance of the asset due to the variability in the market return (the asset=s R-square) and what is the proportion unexplained by variability in the market?

f. If an asset has an R-square value of 20%, how much of the asset's risk is diversifiable and how much is non-diversifiable?

g. It is often stated that the proportion of the variability of a well diversified portfolio explained by the variability in the market (the portfolio R-square) is much higher than the R-square of an individual asset. Using the single index model, briefly explain why this is so.

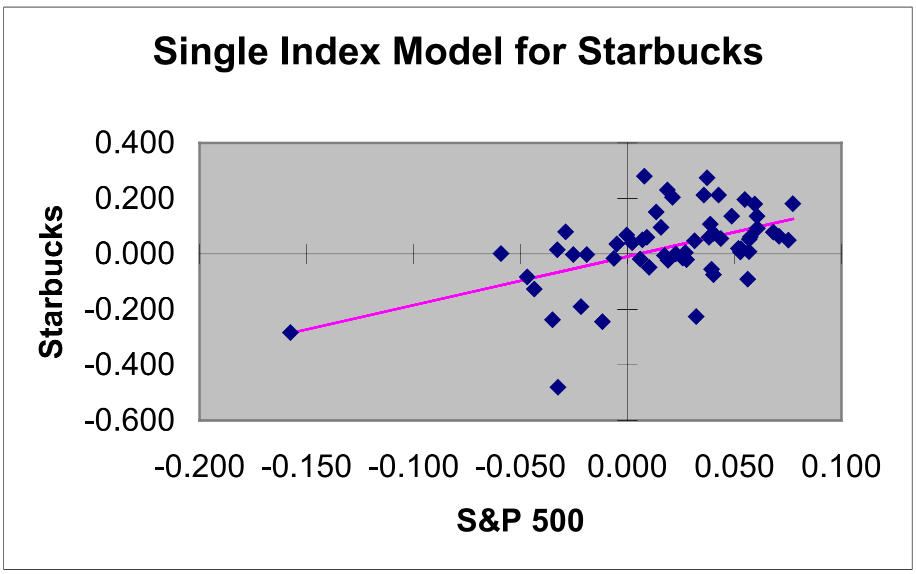
h. Accordingly to William Sharpe, Nobel prize winner in economics and father of the single index model, what percentage of a typical stock=s variability is explained by the market?

IV. Empirical Analysis of the single index model (30 points)

The following represents Excel's linear regression output from estimating the single index model for Starbucks and the Fidelity Magellan mutual fund (a well diversified portfolio) using monthly continuously compounded return data over the period February 1995 – December 1999. In the regressions, the market index is the S&P 500 value weighted index of 500 assets.

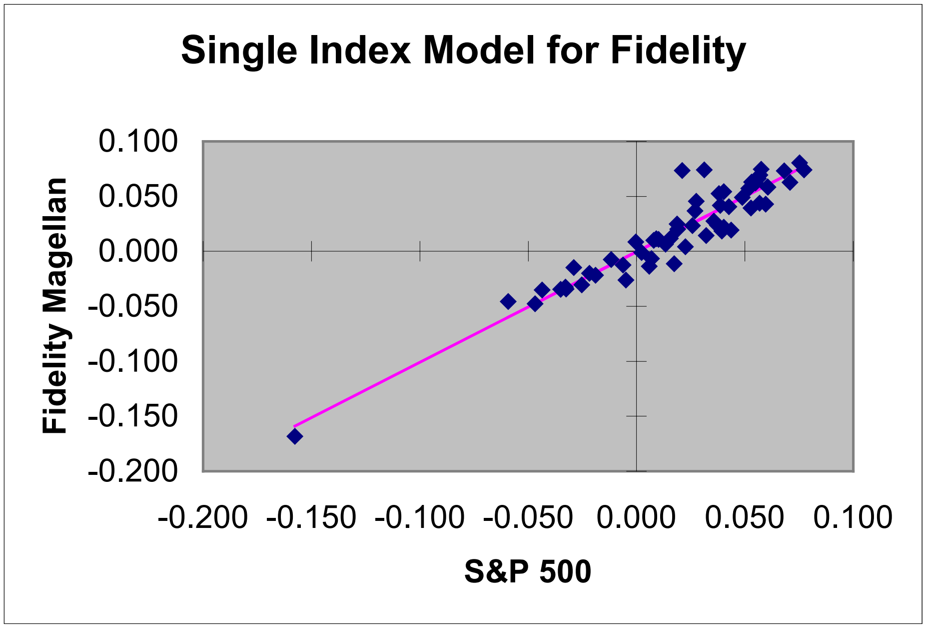
Regression Statistics Starbucks	
Multiple R	0.519
R Square	**0.269**
Adjusted R Square	0.256
Standard Error	**0.120**
Observations	58

	Coefficients	***Standard Error***	*t Stat*	*P-value*	***Lower 95%***	***Upper 95%***
Intercept	**-0.009**	**0.017**	-0.528	0.600	**-0.044**	**0.026**
S&P 500	**1.754**	**0.386**	4.540	0.000	**0.980**	**2.528**



Regression Statistics Fidelity Magellan Fund	
Multiple R	0.945
R Square	**0.892**
Adjusted R Square	0.890
Standard Error	**0.015**
Observations	58

	Coefficients	***Standard Error***	*t Stat*	*P-value*	***Lower 95%***	***Upper 95%***
Intercept	**0.000**	**0.002**	-0.185	0.854	**-0.005**	**0.004**
S&P 500	**1.007**	**0.047**	21.541	0.000	**0.913**	**1.100**



a. For Starbucks and the Fidelity Magellan Fund, what are the estimated values of α and β and what are the estimated standard errors for these estimates?

b. Based on the estimated values of β, what can you say about the risk characteristics of Starbucks and the Fidelity Magellan Fund relative to the S&P 500 index?

c. Is β for the Fidelity Magellan Fund estimated more precisely than β for Starbucks? Why or why not?

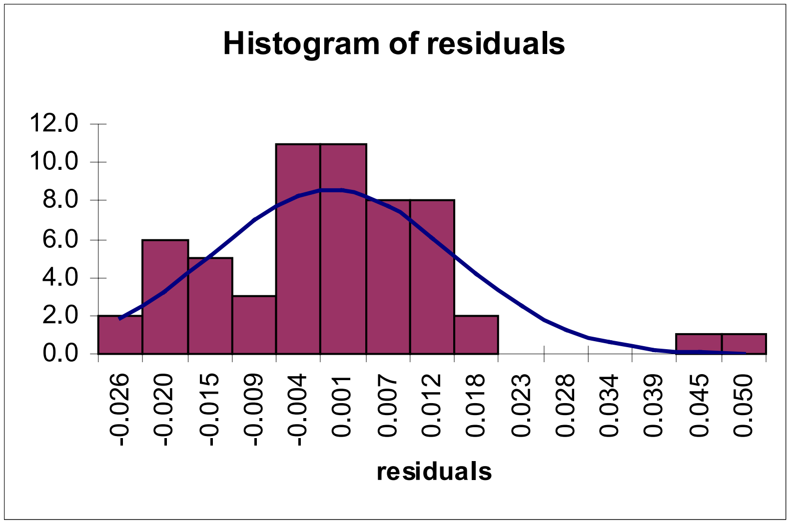
d. For each regression, what is the proportion of market or systematic risk and what is the proportion of firm specific or unsystematic risk? For each regression, what does the Standard Error represent?

e. Why should the Fidelity Magellan Fund have a greater proportion of systematic risk and a smaller standard error than Starbucks?

f. For Starbucks and the Fidelity Magellan Fund, test the null hypothesis that $\beta = 2$ against the alternative hypothesis that $\beta \neq 2$ using a 5% significance level. What do you conclude?

g. For Starbucks and the Fidelity Magellan Fund, test the null hypothesis that $\beta = 1$ against the alternative hypothesis that $\beta \neq 1$ using a 5% significance level. What do you conclude?

h. The histogram and descriptive statistics for the residuals from the single index model regression for the Fidelity Magellan Fund are given below. Using these statistics, test the null hypothesis that the residuals are normally distributed against the alternative that the residuals are not normally distributed using a 5% significance level.



mean	std. deviation	skewness	kurtosis - 3
0.000	0.014	0.969	2.909