

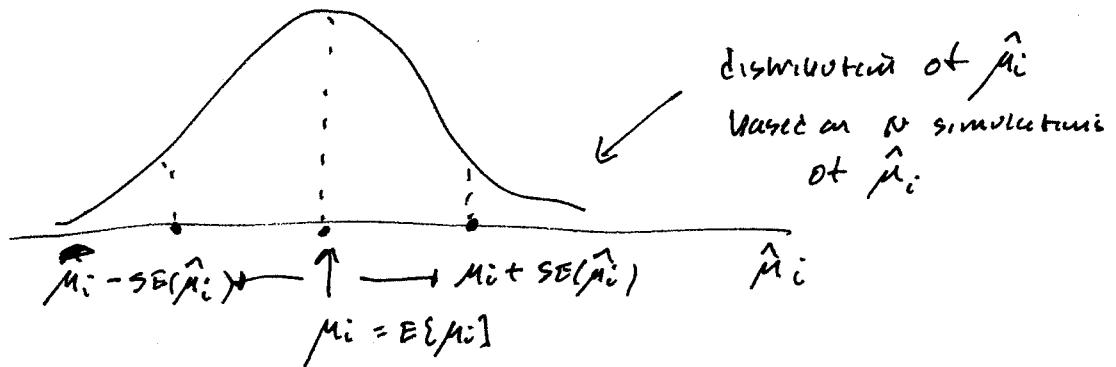
Econ 483 Final Exam Suggested Solutions

I. Monte Carlo Simulations

$$R_{it} = \mu_i + \epsilon_{it} \quad t=1, \dots, T$$

- ↳ a. ϵ_{it} represents random "news" that arrives between periods $t-1$ and t .
- ↳ b. Since $R_{it} \sim N(\mu_i, \sigma_{\epsilon_i}^2)$ $t=1, \dots, 40$ we need to simulate 40 observations from a Normal distribution with mean μ_i and variance $\sigma_{\epsilon_i}^2$.
 We can do this in Excel using Tools / Data Analysis / Random Number Generation. We specify the normal distribution, number of samples = 40, mean = μ_i and st. dev. = $\sqrt{(\sigma_{\epsilon_i}^2)}$.
- ↳ c. $\hat{\mu}_i = T^{-1} \sum_{t=1}^T R_{it}$ is unbiased, i.e. $E[\hat{\mu}_i] = \mu_i$. In a Monte Carlo Simulation context, this means that the average of $\hat{\mu}_i$ computed over many simulated samples is equal to μ_i . For example, let $\hat{\mu}_i^{(j)}$ denote the computed value of $\hat{\mu}_i$ for the j^{th} Monte Carlo simulation. Unbiasedness means that
- $$\lim_{N \rightarrow \infty} \underbrace{\frac{1}{N} \sum_{j=1}^N \hat{\mu}_i^{(j)}}_{\text{average of over simulated samples.}} = \mu_i$$

d. $SE(\hat{\mu}_i)$ represents the "standard deviation" of $\hat{\mu}_i$
 computed from the simulated values $\hat{\mu}_i^{(1)}, \dots, \hat{\mu}_i^{(n)}$



II. Matrix Algebraic portfolio Math

5 a. $R_{p,x} = \underbrace{x' R}_{\sim} \quad , \quad R_{p,y} = \underbrace{y' R}_{\sim}$

$$\underbrace{x' 1}_{\sim} = 1 \quad , \quad \underbrace{y' 1}_{\sim} = 1$$

5 b. $\mu_{p,x} = \underbrace{x' \mu}_{\sim} \quad , \quad \mu_{p,y} = \underbrace{y' \mu}_{\sim}$

5 c. $\sigma_{p,x}^2 = \underbrace{x' \Sigma x}_{\sim} \quad , \quad \sigma_{p,y}^2 = \underbrace{y' \Sigma y}_{\sim}$

5 d. $\sigma_{xy} = \underbrace{x' \Sigma y}_{\sim} = \underbrace{y' \Sigma x}_{\sim}$

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III. Single Index Model

a. $\beta_i = \frac{\text{cov}(R_{it}, R_{Mt})}{\text{var}(R_{Mt})}$ and measures the correlation
of asset i to the variability of the portfolio M .

If $\beta_i < 1$ then adding asset i to M reduces
the variability of M ; if $\beta = 1$ then adding asset i to
 M doesn't change the variability of M ; if $\beta > 1$
then adding asset i to M increases the variability of M .

b. ϵ_{it} represents random news independent of the
"market" that arrives between periods $t-1$ and t .

This is referred to as "firm specific" or "non market"
news.

c. $E[R_{it}] = \alpha_i + \beta_i E[R_{Mt}] = \alpha_i + \beta_i \cdot \mu_M$

$$\begin{aligned} \text{Var}(R_{it}) &= \beta_i^2 \text{Var}(R_{Mt}) + \text{var}(\epsilon_{it}) \\ &= \beta_i^2 \cdot \sigma_M^2 + \sigma_{\epsilon_i}^2 \end{aligned}$$

$$\text{cov}(R_{it}, R_{jt}) = \beta_i \cdot \beta_j \cdot \sigma_M^2$$

(4)

d.

$$\Sigma = \begin{pmatrix} \sigma_{11}^2 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22}^2 & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33}^2 \end{pmatrix}$$

$$= \begin{pmatrix} \beta_1^2 \sigma_m^2 + \sigma_{\epsilon_1}^2 & \beta_1 \beta_2 \sigma_m^2 & \beta_1 \beta_3 \sigma_m^2 \\ \beta_1 \beta_2 \sigma_m^2 & \beta_2^2 \sigma_m^2 + \sigma_{\epsilon_2}^2 & \beta_2 \beta_3 \sigma_m^2 \\ \beta_1 \beta_3 \sigma_m^2 & \beta_2 \beta_3 \sigma_m^2 & \beta_3^2 \sigma_m^2 + \sigma_{\epsilon_3}^2 \end{pmatrix}$$

$$= \underbrace{\sigma_m^2 \cdot \beta \cdot \beta'}_{\sim \sim} + D$$

where $\beta = (\beta_1, \beta_2, \beta_3)'$ and $D = \begin{pmatrix} \sigma_{\epsilon_1}^2 & 0 & 0 \\ 0 & \sigma_{\epsilon_2}^2 & 0 \\ 0 & 0 & \sigma_{\epsilon_3}^2 \end{pmatrix}$

e.

$$\frac{\text{Var}(R_{it})}{\text{var}(R_{it})} = \frac{\beta_i^2 \text{var}(R_{mt})}{\text{var}(R_{it})} + \frac{\text{var}(G_{it})}{\text{var}(R_{it})}$$

or

$$\lambda = \beta_i^2 \cdot \frac{\text{var}(R_{mt})}{\text{var}(R_{it})} + \frac{\text{var}(G_{it})}{\text{var}(R_{it})}$$

$$= \underbrace{\beta_i^2 \cdot \frac{\sigma_m^2}{\sigma_i^2}}_{\beta_i^2} + \frac{\sigma_{\epsilon_i}^2}{\sigma_i^2}$$

$$\beta_i^2 + 1 - \beta_i^2$$

q_0 explained by market

q_0 not explained by market

f. $R^2 = 20\% = \%$ of variability of asset i explained by market = $\%$ of risk that is not diversifiable.

$1 - R^2 = 80\% = \%$ of variability of asset i that is not explained by market = $\%$ of risk that is diversifiable.

g. For a large portfolio ~~other than~~ with assets weights x_i , the return on the portfolio is

$$R_p = \left(\sum_{i=1}^n x_i d_i \right) + \left(\sum_{i=1}^n x_i \beta_i \right) \cancel{e_i} \times R_{M,i} + \sum_{i=1}^n x_i \epsilon_i$$

$$= \alpha_p + \beta_p \cdot R_{M,i} + \epsilon_{p,i}$$

In a well diversified portfolio, $x_i \approx \frac{1}{n}$ and so

$$\epsilon_{p,i} \approx \frac{1}{n} \sum_{i=1}^n \epsilon_i \approx 0 \quad \begin{pmatrix} \text{diversification} \\ \text{eliminates specific} \\ \text{risk} \end{pmatrix}$$

Since ϵ_i is a random variable with mean zero.

Hence $\text{Var}(R_p) \approx \beta_p^2 \text{Var}(R_M)$

$$\Rightarrow R^2 \approx 1$$

(6)

h. For a typical asset, $R_i^2 \approx 30\%$.

IV. Empirical Analysis

a.

$$\hat{R}_{\text{Starbucks}} = -0.009 + 1.754 \cdot R_m$$

$$(0.017) \quad (0.386) \quad \left(\frac{\hat{\alpha} + \hat{\beta} \cdot R_m}{\text{SE}(\hat{\alpha})} \right)$$

\nearrow std. errors

$$\hat{R}_{\text{Fidelity}} = 0.000 + 1.007 \cdot R_m$$

$$(0.002) \quad (0.047)$$

b. $\hat{\beta}_{\text{Starbucks}} = 1.754 \Rightarrow$ Starbucks is riskier than S&P 500

Since adding Starbucks to S&P 500 will increase the variability of the S&P 500

$\hat{\beta}_{\text{Fidelity}} = 1.007 \Rightarrow$ Magellan Fund has the same risk as the S&P 500 since.

c. The β for Fidelity is estimated much more precisely than the β for Starbucks. $\text{SE}(\hat{\beta}_{\text{Fidelity}})$ is 8 times smaller than $\text{SE}(\hat{\beta}_{\text{Starbucks}})$ and the 95% confidence interval width is much smaller for Fidelity than for Starbucks.

(2)

	<u>Starbucks</u>	<u>Fidelity</u>
c. Market risk (β^2)	0.209	0.892
Non Market risk ($1-\beta^2$)	0.731	0.108

- The Standard Error is an estimate of the std. deviation of fit. It is the "typical distance" of a "dot" from the regression line. For Starbucks SER = 0.120 and for Fidelity SER = 0.015. Hence, the "firm specific news component" for Starbucks is almost 10 times larger than the corresponding component for Fidelity.

- e. Because Fidelity is a well diversified portfolio the firm specific risk components are "diversified" away leaving ~~only~~ only Market risk.

	<u>Starbucks</u>	<u>Fidelity</u>
f.	$t_{\beta=2} = \frac{1.754 - 2}{0.386}$ $= -0.637$	$t_{\beta=2} = \frac{1.007 - 2}{0.0417}$ $= -21.128$

(8)

we do not reject $\beta=1$ at the 5% level (

Note: $\beta=1$ is in the 95% C.I. for β). For Fidelity,

$$|t_{\beta=1}| = 0.149 < 2$$

so we do not reject $\beta=1$ at the 5% level (

Note: $\beta=1$ is in the 95% C.I. for β).

h. we use the Jarque-Bera test statistic

$$JB = \frac{58}{6} \left\{ (0.969)^2 + \frac{(2.909)^2}{4} \right\}$$

$$= 9.67 \left\{ 0.939 + 2.116 \right\}$$

$$= 29.538$$

The 5% critical value is 6. Since

$$JB = 29.538 > 6$$

we reject the null hypothesis that the residuals are normally distributed.