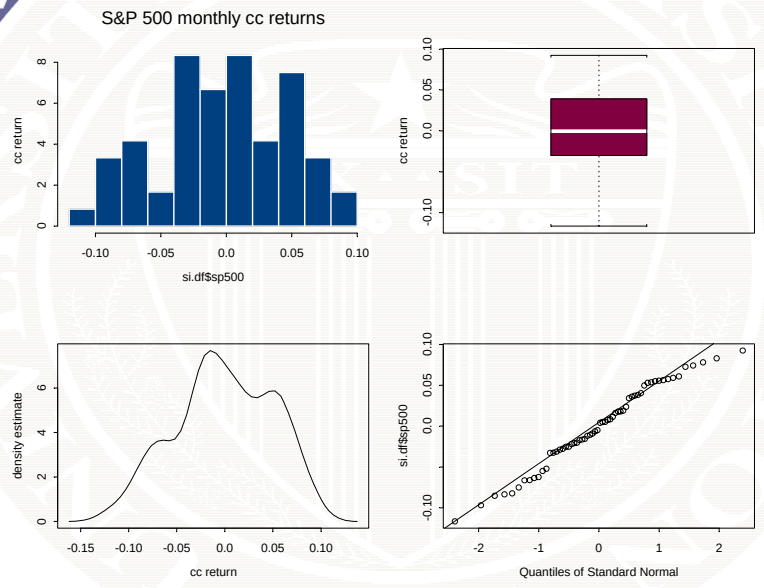
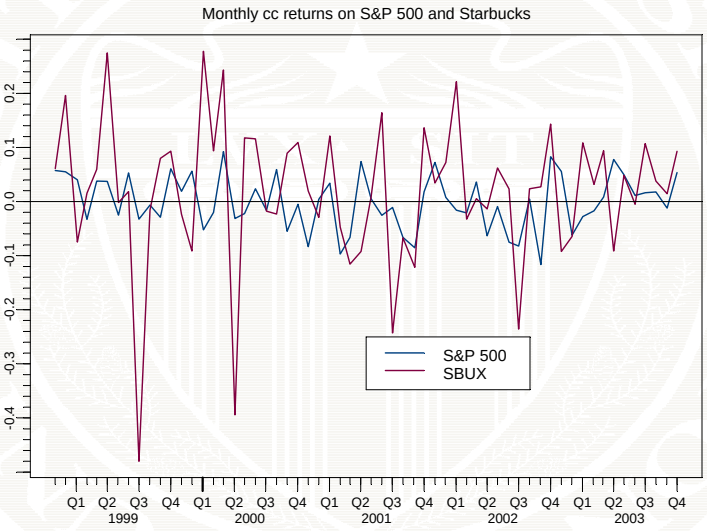
Single Index Model

Econ 424
Eric Zivot
Fall 2008
Updated: November 16, 2008

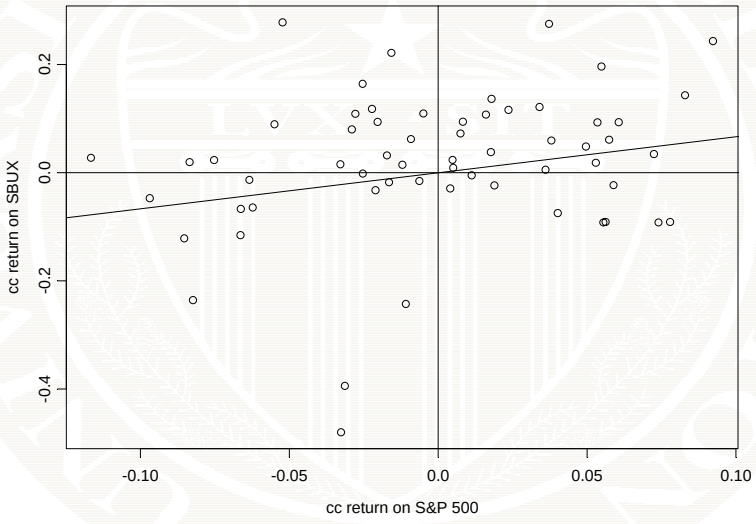


Beta = 0.67

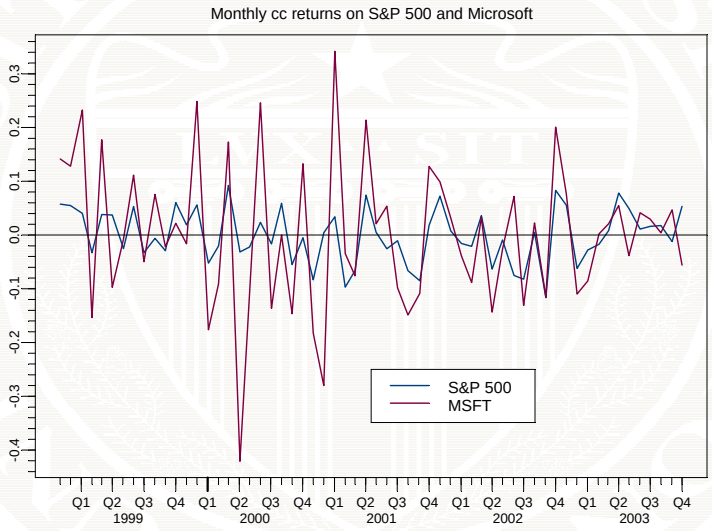


Beta = 0.67

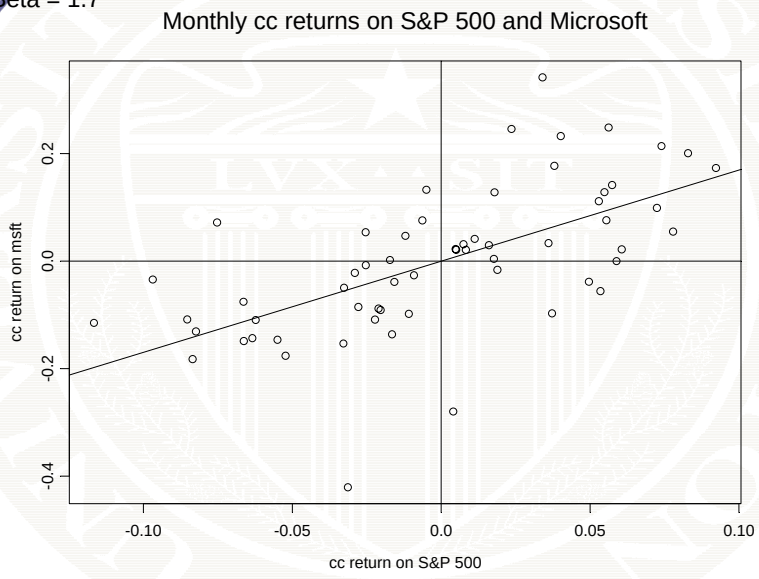
Monthly cc returns on S&P 500 and Starbucks



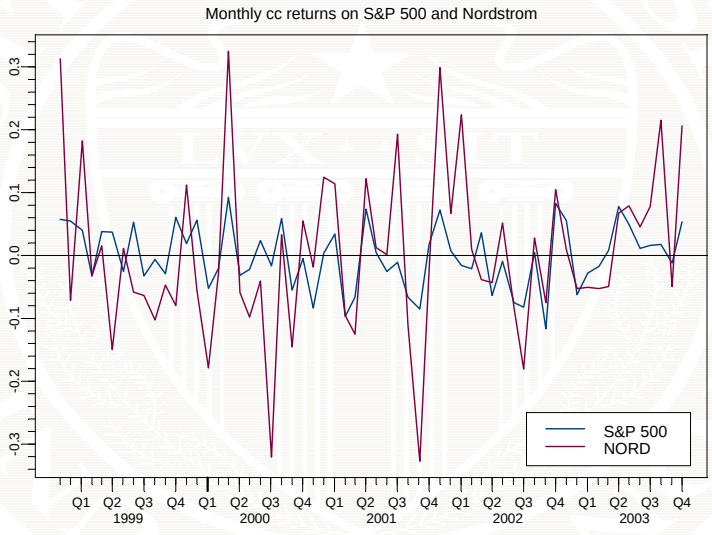
Beta = 1.7



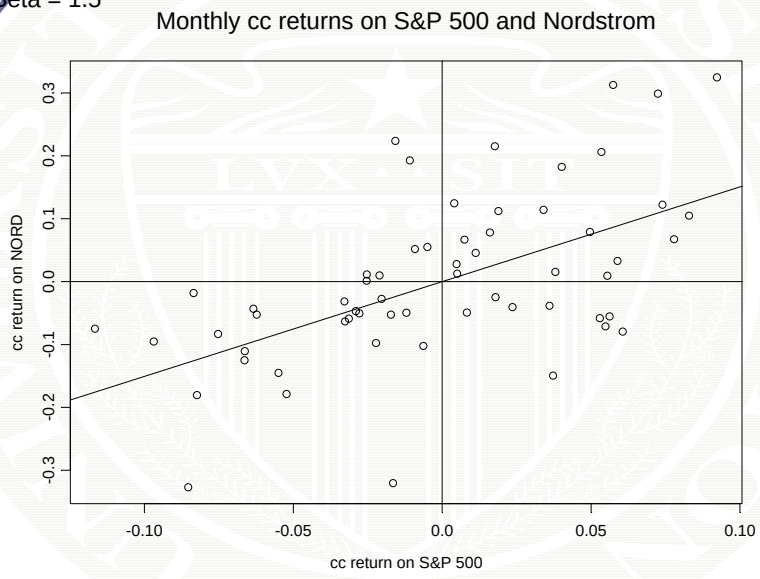
Beta = 1.7



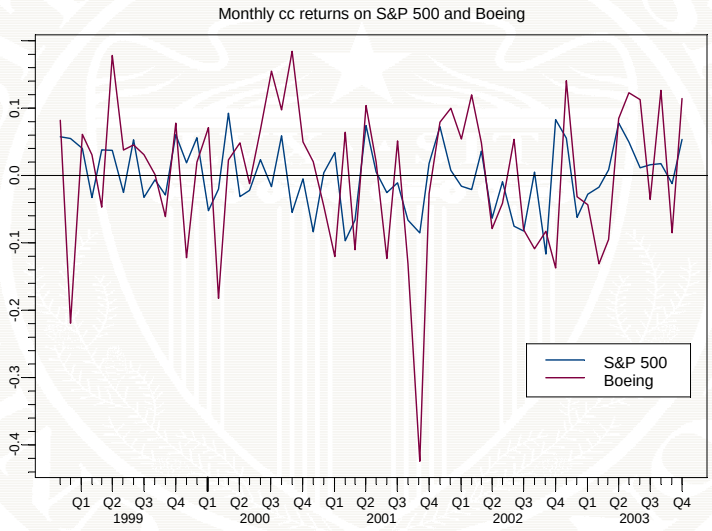
Beta = 1.5



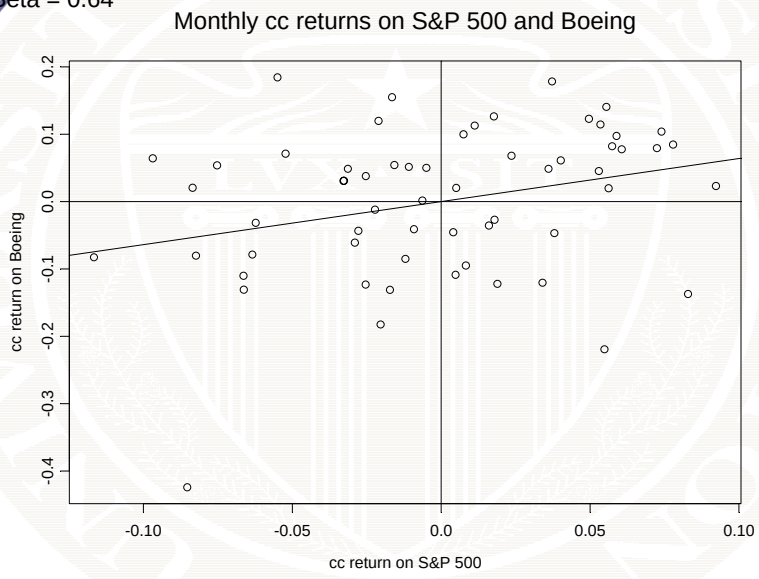
Beta = 1.5

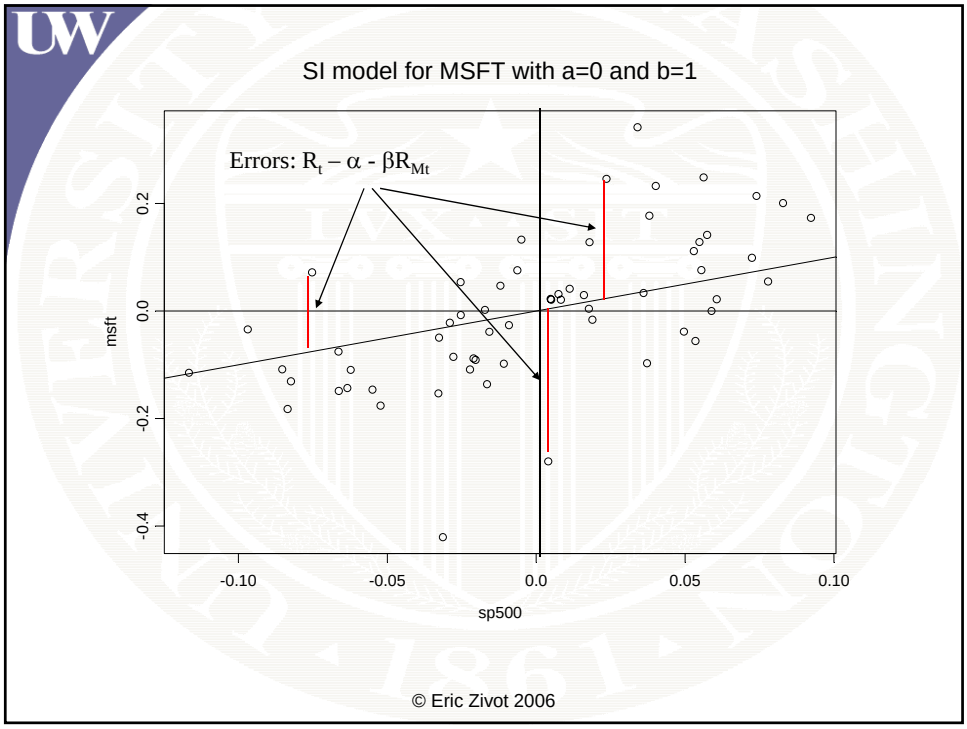


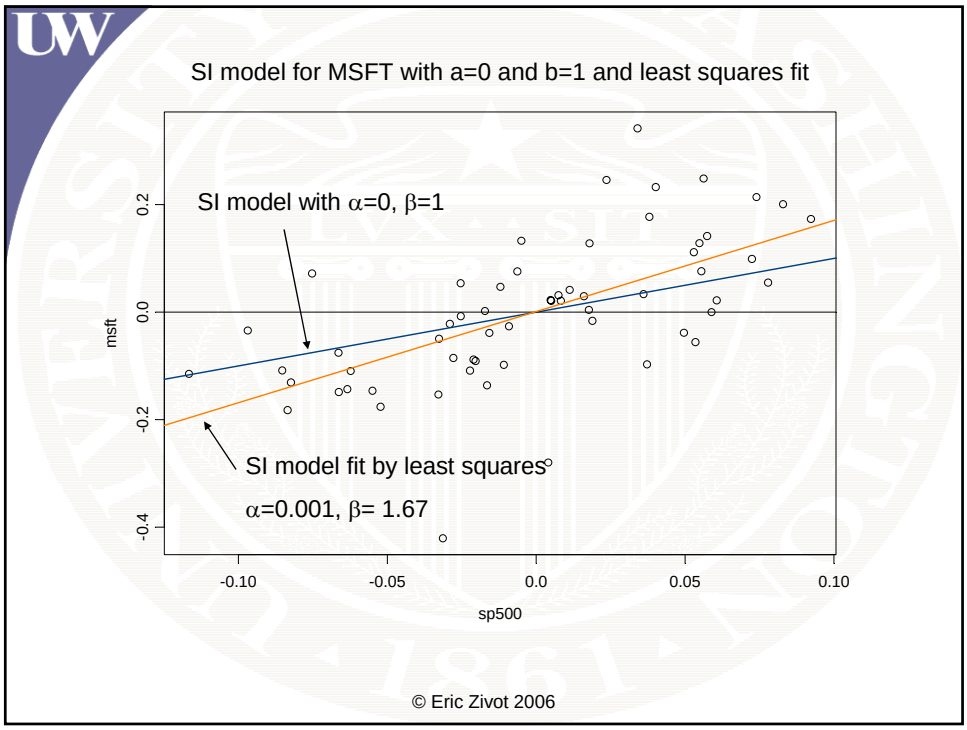
Beta = 0.64



Beta = 0.64




SI model for MSFT with a=0 and b=1
Errors: $R_t - \alpha - \beta R_{Mt}$
msft
sp500
0.2
0.0
-0.2
-0.4
-0.10
-0.05
0.0
0.05
0.10
© Eric Zivot 2006


SI model for MSFT with a=0 and b=1 and least squares fit
SI model with α=0, β=1
SI model fit by least squares
α=0.001, β= 1.67
msft
sp500
0.2
0.0
-0.2
-0.4
-0.10
-0.05
0.0
0.05
0.10
© Eric Zivot 2006

Least Squares in R

```
> msft.fit = lm(msft~sp500,data=si.df)
> class(msft.fit)
[1] "lm"

> names(msft.fit)
 [1] "coefficients"  "residuals"
"fitted.values" "effects"       "R"
 [6] "rank"          "assign"
"df.residual"   "contrasts"     "terms"
[11] "call"

> msft.fit$coef
 (Intercept)    sp500
 0.001198817 1.697067
```

α β

LM Print Method

```
> msft.fit
Call:
lm(formula = msft ~ sp500, data = si.df)

Coefficients:
 (Intercept)    sp500
 0.001198817 1.697067

Degrees of freedom: 60 total; 58 residual
Residual standard error: 0.1086581
```

$\hat{\alpha}$ $\hat{\beta}$ $\hat{\sigma}_\varepsilon$

LM Summary Method

```
> summary(msft.fit)

Call: lm(formula = msft ~ sp500, data = si.df)
Residuals:
    Min     1Q  Median      3Q     Max
 -0.369 -0.054 0.00499 0.04686 0.2828

Coefficients:
            Value Std. Error t value Pr(>|t|)
(Intercept) 0.0012 0.0140     0.0855  0.9322
      sp500 1.6971 0.2808     6.0426  0.0000

Residual standard error: 0.1087 on 58 degrees of freedom
Multiple R-Squared: 0.3863
F-statistic: 36.51 on 1 and 58 degrees of freedom, the p-
value is 1.159e-007
```

$SE(\hat{\alpha})$ $SE(\hat{\beta})$

95% Confidence Intervals

$$\hat{\beta} \pm 2 \times SE(\hat{\beta})$$

$$1.697 \pm 2 \times (0.2808)$$

$$= [1.135,\ 2.259]$$

Note: 95% confidence interval is pretty big!

=> β is not very precisely estimated for individual stocks

t-Values

In the R summary output, the t values are t-statistics for testing the hypothesis that the true coefficient is equal to zero

$$t_{\alpha=0} = \frac{\hat{\alpha}}{SE(\hat{\alpha})} = \frac{.0012}{.0140} = .0855$$

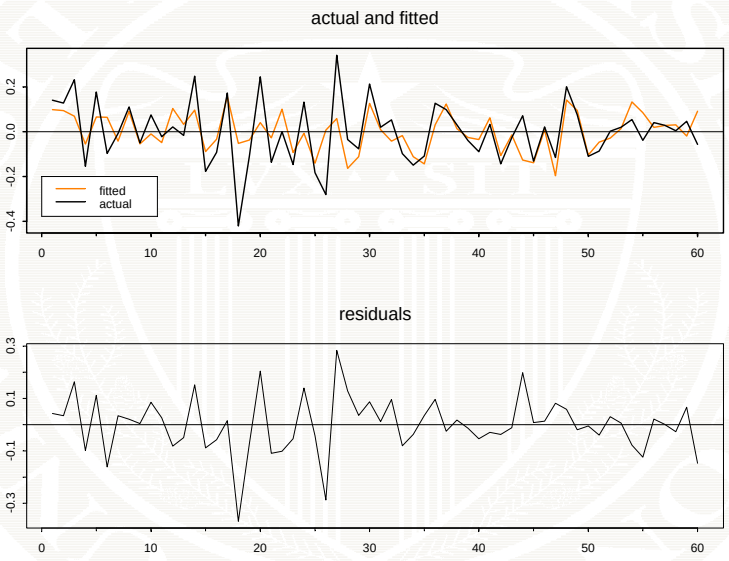
$$t_{\beta=0} = \frac{\hat{\beta}}{SE(\hat{\beta})} = \frac{1.6971}{.2808} = 6.0426$$

LM Extractor Functions

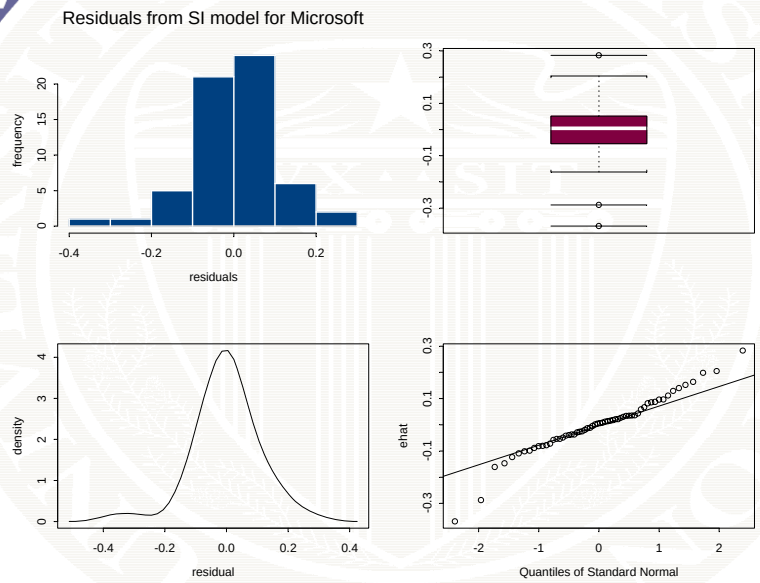
```
> coefficients(msft.fit)
 (Intercept)    sp500
 0.001198817 1.697067

> residuals(msft.fit)
          1           2           3           4           5
0.04286462 0.03407735 0.1631785 -0.09896862 0.1115909
…
 57          58          69          60
 0.0008578491 -0.0270828 0.06623053 -0.1477964

> fitted(msft.fit)
          1           2           3           4           5
 0.09868523 0.09427193 0.06940533 -0.05449057 0.06579019
…
 57          58           59          60
 0.02851057 0.03126293 -0.01919353 0.09199912
```

$\hat{\varepsilon}_t = R_t - \hat{\alpha} - \hat{\beta} R_{Mt}$

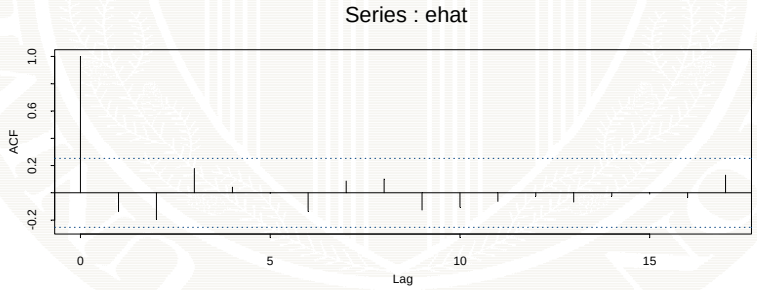
$\hat{R}_t = \hat{\alpha} + \hat{\beta} R_{Mt}$



In the Single Index model the errors should behave like White Noise



In the Single Index model the errors should be Normally Distributed

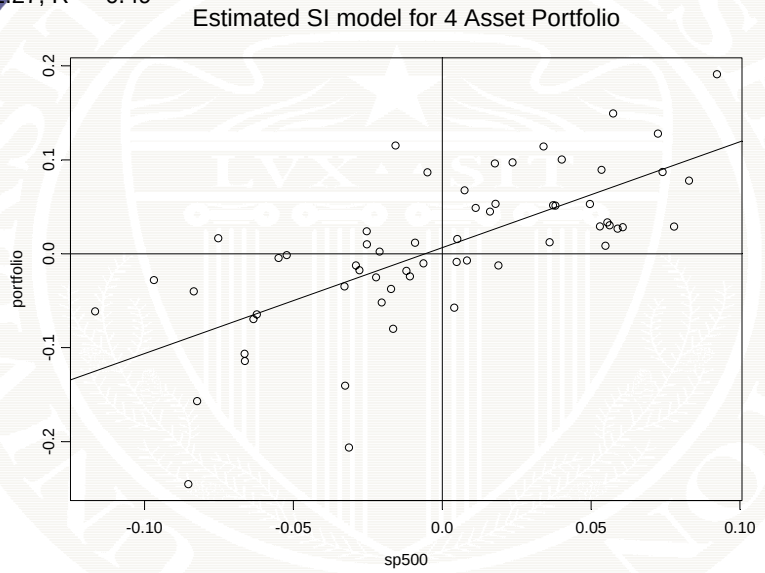


In the Single Index model the errors should behave like White Noise

SI Model for 4 Asset Portfolio

```
> port = (si.df$sbux + si.df$msft + si.df$nord +
+         si.df$boeing)/4
> new.data = data.frame(si.df,port)
> port.fit = lm(port~sp500,data=new.data)

> summary(port.fit)

Call: lm(formula = port ~ sp500, data = new.data)
Residuals:
     Min       1Q    Median      3Q    Max
 -0.1776 -0.03609 -0.002005 0.04635 0.1264

Coefficients:
            Value Std. Error t value Pr(>|t|)
(Intercept) 0.0065 0.0075     0.8616  0.3924
      sp500 1.1276 0.1510     7.4668  0.0000

Residual standard error: 0.05842 on 58 degrees of freedom
Multiple R-Squared: 0.4901
```

$\beta = 1.27$, $R^2 = 0.49$



Portfolio Beta

```
# show beta of portfolio = weighted avg of individual betas

> beta.sbux = coef(lm(sbux~sp500,data=si.df))[2]
> beta.msft = coef(lm(msft~sp500,data=si.df))[2]
> beta.nord = coef(lm(nord~sp500,data=si.df))[2]
> beta.boeing = coef(lm(boeing~sp500,data=si.df))[2]

> (beta.sbux + beta.msft + beta.nord + beta.boeing)/4
    sp500
 1.127567

> coef(port.fit)[2]
    sp500
 1.127567
```

Single Index Model Fit

Asset	β	SE(β)	σ_ε	R^2
Boeing	0.638	0.273	0.106	0.09
Msft	1.697	0.281	0.109	0.39
Nord	1.508	0.283	0.110	0.33
Sbux	0.667	0.342	0.132	0.06
port	**1.128**	**0.151**	**0.058**	**0.49**

Single stocks vs. portfolio

- Portfolio β is closer to 1
- Portfolio β is estimated more precisely (SE is smaller)
- Portfolio σ_ε is smaller (diversification effect)
- Portfolio R^2 is higher (diversification effect)

Estimating Covariances

```
> beta.vec
     SBUX     MSFT     NORD    BOEING
 0.6665845 1.697067 1.507994 0.6386241

> sig2.sp500
[1] 0.002537042

> cov.market = sig2.sp500*(beta.vec%*%t(beta.vec))
> cov.market
           SBUX        MSFT        NORD      BOEING
[1,] 0.001127296 0.002869999 0.002550248 0.001080011
[2,] 0.002869999 0.007306770 0.006492712 0.002749615
[3,] 0.002550248 0.006492712 0.005769350 0.002443276
[4,] 0.001080011 0.002749615 0.002443276 0.001034709
```


Estimating Covariances

```
> D.mat =
diag(c(sig2e.sbux,sig2e.msft,sig2e.nord,sig2e.boeing))
> D.mat
          [,1]      [,2]      [,3]      [,4]
[1,] 0.01719034 0.00000000 0.00000000 0.00000000
[2,] 0.00000000 0.01160648 0.00000000 0.00000000
[3,] 0.00000000 0.00000000 0.01179479 0.00000000
[4,] 0.00000000 0.00000000 0.00000000 0.01100993

> cov.si = cov.market + D.mat
> cov.si
           SBUX        MSFT        NORD      BOEING
[1,] 0.018317634 0.002869999 0.002550248 0.001080011
[2,] 0.002869999 0.018913247 0.006492712 0.002749615
[3,] 0.002550248 0.006492712 0.017564143 0.002443276
[4,] 0.001080011 0.002749615 0.002443276 0.012044643
```

Estimating Covariances

```
# compare with sample covariance matrix
> print(cov.hat,digits=4)
          sbux     msft     nord    boeing
  sbux 0.0183176 0.0055003 0.002735 0.0001221
  msft 0.0055003 0.0189132 0.006987 0.0001189
  nord 0.0027354 0.0069870 0.017564 0.0037662
boeing 0.0001221 0.0001189 0.003766 0.0120446

> print(cov.si,digits=4)
        SBUX     MSFT     NORD   BOEING
[1,] 0.01832 0.002870 0.002550 0.001080
[2,] 0.00287 0.018913 0.006493 0.002750
[3,] 0.00255 0.006493 0.017564 0.002443
[4,] 0.00108 0.002750 0.002443 0.012045
```


Estimating Correlations

```
> print(cor.hat,digits=4)
          sbux     msft   nord   boeing
  sbux 1.000000 0.295506 0.1525 0.008218
  msft 0.295506 1.000000 0.3833 0.007876
  nord 0.152500 0.383348 1.0000 0.258940
boeing 0.008218 0.007876 0.2589 1.000000

> print(cor.si,digits=4)
         sbux   msft   nord  boeing
  sbux 1.00000 0.1542 0.1422 0.07271
  msft 0.15419 1.0000 0.3562 0.18218
  nord 0.14218 0.3562 1.0000 0.16798
boeing 0.07271 0.1822 0.1680 1.00000
```