The CER Model and Efficient Portfolios

Let R_{it} denote the return on asset i in month t and assume that R_{it} follows CER model:

$$\begin{aligned} R_{it} &\sim iid\ N(\mu_i, \sigma_i^2), \\ i &= 1, \ldots, N \text{ (assets)} \\ t &= 1, \ldots, T \text{ (months)} \\ cov(R_{it}, R_{jt}) &= \sigma_{ij} \end{aligned}$$

We estimate the CER model parameters using sample statistics giving

$$\hat{\mu}_i, \hat{\sigma}_i^2, \hat{\sigma}_{ij}$$

Remember, the estimates $\hat{\mu}_i, \hat{\sigma}_i^2$ are $\hat{\sigma}_{ij}$ are random variables and are subject to error

Key result: Efficient portfolios are functions of $\hat{\mu}_i, \hat{\sigma}_i^2, \hat{\sigma}_{ij}$; they are random variables and are subject to error

Bootstrapping Efficient Portfolios

The bootstrap can be used to evaluate the sampling uncertainty of efficient portfolios.

Portfolio statistics to boostrap:

- Portfolio weights

- Portfolio expected returns and standard deviations

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Result: We have seen evidence that the parameters of the CER model for various assets are not constant over time:

- Rolling estimates of μ, σ, and σ_{ij} show variation over time

Implication: Since estimates of μ, σ, and σ_{ij} are inputs to efficient portfolio calculations, then time variation in $\hat{\mu}$, $\hat{\sigma}$, and $\hat{\sigma}_{ij}$ imply time variation in efficient portfolios

Rolling Efficient Portfolios

Idea: Using rolling estimates of μ and Σ compute rolling efficient portfolios

- global minimum variance portfolio
- efficient portfolio for target return
- tangency portfolio
- efficient frontier

Look at time variation in resulting portfolio weights

Rolling Global Minimum Variance Portfolio

Idea: compute estimates of portfolio weights $\mathbf{m}$ over rolling windows of length $n < T$:

$$\min_{\mathbf{m}(n)} \ \mathbf{m}_t(n)'\hat{\mathbf{\Sigma}}_t(n)\mathbf{m}_t(n) \ \text{ s.t. } \mathbf{m}_t(n)'\mathbf{1} = 1$$

$$t = n, \ldots, T$$

$$\hat{\mathbf{\Sigma}}_t(n) = \text{rolling estimate of } \mathbf{\Sigma} \text{ in month } t$$

If

$$\hat{\mathbf{\Sigma}}_n(n) \approx \hat{\mathbf{\Sigma}}_{n+1}(n) \approx \cdots \approx \hat{\mathbf{\Sigma}}_T(n)$$

then

$$\mathbf{m}_n(n) \approx \mathbf{m}_{n+1}(n) \approx \cdots \approx \mathbf{m}_T(n)$$

Rolling Efficient Portfolios

Idea: compute estimates of portfolio weights $\mathbf{x}$ over rolling windows of length $n < T$:

$$\begin{gathered}\min_{\mathbf{x}(n)} \ \mathbf{x}_t(n)'\hat{\mathbf{\Sigma}}_t(n)\mathbf{x}_t(n) \\ \text{s.t. } \mathbf{x}_t(n)'\mathbf{1} = 1, \ \mathbf{x}_t(n)'\hat{\boldsymbol{\mu}}_t(n) = \mu_p^{\text{target}} \\ t = n, \ldots, T \\ \hat{\boldsymbol{\mu}}_t(n) = \text{ rolling estimate of } \boldsymbol{\mu} \text{ in month } t \\ \hat{\mathbf{\Sigma}}_t(n) = \text{rolling estimate of } \mathbf{\Sigma} \text{ in month } t\end{gathered}$$

If

$$\begin{aligned}\hat{\boldsymbol{\mu}}_n(n) &\approx \hat{\boldsymbol{\mu}}_{n+1}(n) \approx \cdots \approx \hat{\boldsymbol{\mu}}_T(n) \\ \hat{\mathbf{\Sigma}}_n(n) &\approx \hat{\mathbf{\Sigma}}_{n+1}(n) \approx \cdots \approx \hat{\mathbf{\Sigma}}_T(n)\end{aligned}$$

then

$$\mathbf{x}_n(n) \approx \mathbf{x}_{n+1}(n) \approx \cdots \approx \mathbf{x}_T(n)$$