Introduction to Computational Finance and Financial Econometrics

Return Calculations

Eric Zivot
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Outline

1. The time value of money
   - Future value
   - Multiple compounding periods
   - Effective annual rate

2. Asset return calculations
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2. Asset return calculations
Future value

- $V$ invested for $n$ years at simple interest rate $R$ per year
- Compounding of interest occurs at end of year

\[ FV_n = V \cdot (1 + R)^n, \]

where $FV_n$ is future value after $n$ years
Example

Consider putting $1000 in an interest checking account that pays a simple annual percentage rate of 3%. The future value after $n = 1, 5$ and 10 years is, respectively,

\[ FV_1 = 1000 \cdot (1.03)^1 = 1030, \]

\[ FV_5 = 1000 \cdot (1.03)^5 = 1159.27, \]

\[ FV_{10} = 1000 \cdot (1.03)^{10} = 1343.92. \]
Future value

FV function is a relationship between four variables: \( FV_n, V, R, n \). Given three variables, you can solve for the fourth:

- **Present value:**
  \[
  V = \frac{FV_n}{(1 + R)^n}.
  \]

- **Compound annual return:**
  \[
  R = \left( \frac{FV_n}{V} \right)^{1/n} - 1.
  \]

- **Investment horizon:**
  \[
  n = \frac{\ln(FV_n/V)}{\ln(1 + R)}.
  \]
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1. The time value of money
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2. Asset return calculations
Multiple compounding periods

- Compounding occurs $m$ times per year

\[ FV^m_n = V \cdot \left(1 + \frac{R}{m}\right)^{m \cdot n}, \]

\[ \frac{R}{m} = \text{periodic interest rate}. \]

- Continuous compounding

\[ FV^\infty_n = \lim_{m \to \infty} V \cdot \left(1 + \frac{R}{m}\right)^{m \cdot n} = Ve^{R \cdot n}, \]

\[ e^1 = 2.71828. \]
If the simple annual percentage rate is 10% then the value of $1000 at the end of one year ($n = 1$) for different values of $m$ is given in the table below.

<table>
<thead>
<tr>
<th>Compounding Frequency</th>
<th>Value of $1000 at end of 1 year ($R = 10%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annually ((m = 1))</td>
<td>1100.00</td>
</tr>
<tr>
<td>Quarterly ((m = 4))</td>
<td>1103.81</td>
</tr>
<tr>
<td>Weekly ((m = 52))</td>
<td>1105.06</td>
</tr>
<tr>
<td>Daily ((m = 365))</td>
<td>1105.16</td>
</tr>
<tr>
<td>Continuously ((m = \infty))</td>
<td>1105.17</td>
</tr>
</tbody>
</table>
1. The time value of money
   - Future value
   - Multiple compounding periods
   - Effective annual rate

2. Asset return calculations
Annual rate $R_A$ that equates $FV^m$ with $FV^n$; i.e.,

$$V \left(1 + \frac{R}{m}\right)^{m \cdot n} = V(1 + R_A)^n.$$  

Solving for $R_A$

$$\left(1 + \frac{R}{m}\right)^m = 1 + R_A \Rightarrow R_A = \left(1 + \frac{R}{m}\right)^m - 1.$$
Continuous compounding

\[ Ve^{R \cdot n} = V (1 + R_A)^n \]

\[ \Rightarrow e^R = (1 + R_A) \]

\[ \Rightarrow R_A = e^R - 1. \]
Compute effective annual rate with semi-annual compounding

The effective annual rate associated with an investment with a simple annual rate $R = 10\%$ and semi-annual compounding ($m = 2$) is determined by solving

$$(1 + R_A) = \left(1 + \frac{0.10}{2}\right)^2$$

$\Rightarrow R_A = \left(1 + \frac{0.10}{2}\right)^2 - 1 = 0.1025.$
### Effective annual rate

<table>
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<th>$R_A$</th>
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<tr>
<td>Annually ($m = 1$)</td>
<td>1100.00</td>
<td>10%</td>
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<td>1103.81</td>
<td>10.38%</td>
</tr>
<tr>
<td>Weekly ($m = 52$)</td>
<td>1105.06</td>
<td>10.51%</td>
</tr>
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Outline

1 The time value of money

2 Asset return calculations
   - Simple returns
   - Continuously compounded (cc) returns
Outline

1. The time value of money

2. Asset return calculations
   - Simple returns
     - Multi-period returns
     - Portfolio returns
     - Adjusting for dividends
     - Adjusting for inflation
     - Annualizing returns
     - Average returns
   - Continuously compounded (cc) returns
Simple returns

- \( P_t \) = price at the end of month \( t \) on an asset that pays no dividends
- \( P_{t-1} \) = price at the end of month \( t - 1 \)

\[
R_t = \frac{P_t - P_{t-1}}{P_{t-1}} = \% \Delta P_t = \text{net return over month } t,
\]

\[
1 + R_t = \frac{P_t}{P_{t-1}} = \text{gross return over month } t.
\]
One month investment in Microsoft stock

Buy stock at end of month \( t - 1 \) at \( P_{t-1} = \$85 \) and sell stock at end of next month for \( P_t = \$90 \). Assuming that Microsoft does not pay a dividend between months \( t - 1 \) and \( t \), the one-month simple net and gross returns are

\[
R_t = \frac{90 - 85}{85} = \frac{90}{85} - 1 = 1.0588 - 1 = 0.0588, \\
1 + R_t = 1.0588.
\]

The one month investment in Microsoft yielded a 5.88% per month return.
Multi-period returns

Simple two-month return

\[ R_t(2) = \frac{P_t - P_{t-2}}{P_{t-2}} \]

\[ = \frac{P_t}{P_{t-2}} - 1. \]

Relationship to one month returns

\[ R_t(2) = \frac{P_t}{P_{t-2}} - 1 = \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}} - 1 \]

\[ = (1 + R_t) \cdot (1 + R_{t-1}) - 1. \]
Multi-period returns

Here

\[ 1 + R_t = \text{one-month gross return over month } t, \]
\[ 1 + R_{t-1} = \text{one-month gross return over month } t - 1, \]
\[ \implies 1 + R_t(2) = (1 + R_t) \cdot (1 + R_{t-1}). \]

two-month gross return = the product of two one-month gross returns

Note: two-month returns are not additive:

\[ R_t(2) = R_t + R_{t-1} + R_t \cdot R_{t-1} \]
\[ \approx R_t + R_{t-1} \text{ if } R_t \text{ and } R_{t-1} \text{ are small} \]
Example

*Two-month return on Microsoft*

Suppose that the price of Microsoft in month $t - 2$ is $80$ and no dividend is paid between months $t - 2$ and $t$. The two-month net return is

$$R_t(2) = \frac{90 - 80}{80} = \frac{90}{80} - 1 = 1.1250 - 1 = 0.1250,$$

or 12.50% per two months. The two one-month returns are

$$R_{t-1} = \frac{85 - 80}{80} = 1.0625 - 1 = 0.0625$$

$$R_t = \frac{90 - 85}{85} = 1.0588 - 1 = 0.0588,$$

and the geometric average of the two one-month gross returns is

$$1 + R_t(2) = 1.0625 \times 1.0588 = 1.1250.$$
Multi-period returns

Simple $k$-month Return

$$R_t(k) = \frac{P_t - P_{t-k}}{P_{t-k}} = \frac{P_t}{P_{t-k}} - 1$$

$$1 + R_t(k) = (1 + R_t) \cdot (1 + R_{t-1}) \cdots (1 + R_{t-k+1})$$

$$= \prod_{j=0}^{k-1} (1 + R_{t-j})$$

Note

$$R_t(k) \neq \sum_{j=0}^{k-1} R_{t-j}$$
Portfolio returns

- Invest $V$ in two assets: A and B for 1 period
- $x_A = \text{share of } V \text{ invested in A}; V \times x_A = \$ \text{ amount}$
- $x_B = \text{share of } V \text{ invested in B}; V \times x_B = \$ \text{ amount}$
- Assume $x_A + x_B = 1$
- Portfolio is defined by investment shares $x_A$ and $x_B$
Portfolio returns

At the end of the period, the investments in A and B grow to

\[ V(1 + R_{p,t}) = V[x_A(1 + R_{A,t}) + x_B(1 + R_{B,t})] \]

\[ = V[x_A + x_B + x_A R_{A,t} + x_B R_{B,t}] \]

\[ = V[1 + x_A R_{A,t} + x_B R_{B,t}] \]

\[ \Rightarrow R_{p,t} = x_A R_{A,t} + x_B R_{B,t} \]

The simple portfolio return is a share weighted average of the simple returns on the individual assets.
Example

Portfolio of Microsoft and Starbucks stock

Purchase ten shares of each stock at the end of month $t-1$ at prices

$$P_{msft,t-1} = $85, \quad P_{sbux,t-1} = $30,$$

The initial value of the portfolio is

$$V_{t-1} = 10 \times $85 + 10 \times 30 = $1,150.$$  

The portfolio shares are

$$x_{msft} = 850/1150 = 0.7391, \quad x_{sbux} = 300/1150 = 0.2609.$$  

The end of month $t$ prices are $P_{msft,t} = $90 and $P_{sbux,t} = $28.
Assuming Microsoft and Starbucks do not pay a dividend between periods $t - 1$ and $t$, the one-period returns are

$$R_{msft,t} = \frac{90 - 85}{85} = 0.0588$$

$$R_{sbux,t} = \frac{28 - 30}{30} = -0.0667$$

The return on the portfolio is

$$R_{p,t} = (0.7391)(0.0588) + (0.2609)(-0.0667) = 0.02609$$

and the value at the end of month $t$ is

$$V_t = 1,150 \times (1.02609) = 1,180$$
In general, for a portfolio of \( n \) assets with investment shares \( x_i \) such that \( x_1 + \cdots + x_n = 1 \)

\[
1 + R_{p,t} = \sum_{i=1}^{n} x_i (1 + R_{i,t})
\]

\[
R_{p,t} = \sum_{i=1}^{n} x_i R_{i,t}
\]

\[
= x_1 R_{1t} + \cdots + x_n R_{nt}
\]
Adjusting for dividends

\[ D_t = \text{dividend payment between months } t - 1 \text{ and } t \]

\[
R_{t}^{\text{total}} = \frac{P_t + D_t - P_{t-1}}{P_{t-1}} = \frac{P_t - P_{t-1}}{P_{t-1}} + \frac{D_t}{P_{t-1}}
\]

= capital gain return + dividend yield (gross)

\[ 1 + R_{t}^{\text{total}} = \frac{P_t + D_t}{P_{t-1}} \]
Total return on Microsoft stock

Buy stock in month $t - 1$ at $P_{t-1} = $85 and sell the stock the next month for $P_t = $90. Assume Microsoft pays a $1 dividend between months $t - 1$ and $t$. The capital gain, dividend yield and total return are then

$$R_{t}^{total} = \frac{\$90 + \$1 - \$85}{\$85} = \frac{\$90 - \$85}{\$85} + \frac{\$1}{\$85}$$

$$= 0.0588 + 0.0118$$

$$= 0.0707$$

The one-month investment in Microsoft yields a 7.07% per month total return. The capital gain component is 5.88%, and the dividend yield component is 1.18%.
The computation of real returns on an asset is a two step process:

- Deflate the nominal price $P_t$ of the asset by an index of the general price level $CPI_t$
- Compute returns in the usual way using the deflated prices

$$P_{t, \text{Real}} = \frac{P_t}{CPI_t}$$

$$R_{t, \text{Real}} = \frac{P_{t, \text{Real}} - P_{t-1, \text{Real}}}{P_{t-1, \text{Real}}} = \frac{P_t}{CPI_t} - \frac{P_{t-1}}{CPI_{t-1}}$$

$$= \frac{P_t}{P_{t-1}} \cdot \frac{CPI_{t-1}}{CPI_t} - 1$$
Alternatively, define inflation as

\[ \pi_t = \% \Delta CPI_t = \frac{CPI_t - CPI_{t-1}}{CPI_{t-1}} \]

Then

\[ R^\text{Real}_t = \frac{1 + R_t}{1 + \pi_t} - 1 \]
**Example**

*Compute real return on Microsoft stock*

Suppose the CPI in months $t - 1$ and $t$ is 1 and 1.01, respectively, representing a 1% monthly growth rate in the overall price level. The real prices of Microsoft stock are

$$P_{t-1}^{\text{Real}} = \frac{$85}{1} = $85, \quad P_{t}^{\text{Real}} = \frac{$90}{1.01} = $89.1089$$

The real monthly return is

$$R_{t}^{\text{Real}} = \frac{$89.10891 - $85}{$85} = 0.0483$$
Example cont.

The nominal return and inflation over the month are

\[ R_t = \frac{$90 - $85}{$85} = 0.0588, \quad \pi_t = \frac{1.01 - 1}{1} = 0.01 \]

Then the real return is

\[ R_t^{\text{Real}} = \frac{1.0588}{1.01} - 1 = 0.0483 \]

Notice that simple real return is almost, but not quite, equal to the simple nominal return minus the inflation rate

\[ R_t^{\text{Real}} \approx R_t - \pi_t = 0.0588 - 0.01 = 0.0488 \]
Annualizing returns

Returns are often converted to an annual return to establish a standard for comparison.

**Example:** Assume same monthly return $R_m$ for 12 months:

\[
\text{Compound annual gross return (CAGR)} = 1 + R_A = 1 + R_t(12) = (1 + R_m)^{12} \\
\text{Compound annual net return} = R_A = (1 + R_m)^{12} - 1
\]

Note: We don’t use $R_A = 12R_m$ because this ignores compounding.
Annualized return on Microsoft

Suppose the one-month return, \( R_t \), on Microsoft stock is 5.88%. If we assume that we can get this return for 12 months then the compounded annualized return is

\[
R_A = (1.0588)^{12} - 1 = 1.9850 - 1 = 0.9850
\]

or 98.50% per year. Pretty good!
Example

Annualized return on Microsoft

Suppose the one-month return, $R_t$, on Microsoft stock is 5.88%. If we assume that we can get this return for 12 months then the compounded annualized return is

$$R_A = (1.0588)^{12} - 1 = 1.9850 - 1 = 0.9850$$

or 98.50% per year. Pretty good!
For investments over a given horizon, it is often of interest to compute a measure of average return over the horizon.

Consider a sequence of monthly investments over the year with monthly returns

\[ R_1, R_2, \ldots, R_{12} \]

The annual return is

\[ R_A = R(12) = (1 + R_1)(1 + R_2) \cdots (1 + R_{12}) - 1 \]

Q: What is the average monthly return?
Two possibilities:

1. Arithmetic average (can be misleading)

\[
\bar{R} = \frac{1}{12} (R_1 + \cdots + R_{12})
\]

2. Geometric average (better measure of average return)

\[
(1 + \bar{R})^{12} = (1 + R_A) = (1 + R_1)(1 + R_2) \cdots (1 + R_{12})
\]

\[
\Rightarrow \bar{R} = (1 + R_A)^{1/12} - 1
\]

\[
= [(1 + R_1)(1 + R_2) \cdots (1 + R_{12})]^{1/12} - 1
\]
Consider a two period investment with returns

$$R_1 = 0.5, \quad R_2 = -0.5$$

$1$ invested over two periods grows to

$$FV = 1 \times (1 + R_1)(1 + R_2) = (1.5)(0.5) = 0.75$$

for a 2-period return of

$$R(2) = 0.75 - 1 = -0.25$$

Hence, the 2-period investment loses $25\%$
Example cont.

The arithmetic average return is

$$\bar{R} = \frac{1}{2}(0.5 + -0.5) = 0$$

This is misleading because the actual investment lost money over the 2 period horizon. The compound 2-period return based on the arithmetic average is

$$(1 + \bar{R})^2 - 1 = 1^2 - 1 = 0$$

The geometric average is

$$[(1.5)(0.5)]^{1/2} - 1 = (0.75)^{1/2} - 1 = -0.1340$$

This is a better measure because it indicates that the investment eventually lost money. The compound 2-period return is

$$(1 + \bar{R})^2 - 1 = (0.867)^2 - 1 = -0.25$$
1. The time value of money

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Continuously compounded (cc) returns

\[ r_t = \ln(1 + R_t) = \ln \left( \frac{P_t}{P_{t-1}} \right) \]

\( \ln(\cdot) = \text{natural log function} \)

Note:

\[ \ln(1 + R_t) = r_t : \text{given } R_t \text{ we can solve for } r_t \]

\[ R_t = e^{r_t} - 1 : \text{given } r_t \text{ we can solve for } R_t \]

\( r_t \) is always smaller than \( R_t \)
Digression on natural log and exponential functions

- \( \ln(0) = -\infty \), \( \ln(1) = 0 \)
- \( e^{-\infty} = 0 \), \( e^0 = 1 \), \( e^1 = 2.7183 \)
- \( \frac{d \ln(x)}{dx} = \frac{1}{x} \), \( \frac{d e^x}{dx} = e^x \)
- \( \ln(e^x) = x \), \( e^{\ln(x)} = x \)
- \( \ln(x \cdot y) = \ln(x) + \ln(y) \); \( \ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y) \)
- \( \ln(x^y) = y \ln(x) \)
- \( e^x e^y = e^{x+y} \), \( e^x e^{-y} = e^{x-y} \)
- \( (e^x)^y = e^{xy} \)
$$e^{rt} = e^{\ln(1+R_t)} = e^{\ln(P_t/P_{t-1})}$$

$$= \frac{P_t}{P_{t-1}}$$

$$\implies P_{t-1} \cdot e^{rt} = P_t$$

$$\implies r_t = \text{cc growth rate in prices between months } t - 1 \text{ and } t$$
If $R_t$ is small then

$$r_t = \ln(1 + R_t) \approx R_t$$

**Proof.** For a function $f(x)$, a first order Taylor series expansion about $x = x_0$ is

$$f(x) = f(x_0) + \frac{d}{dx} f(x_0)(x - x_0) + \text{remainder}$$

Let $f(x) = \ln(1 + x)$ and $x_0 = 0$. Note that

$$\frac{d}{dx} \ln(1 + x) = \frac{1}{1 + x}, \quad \frac{d}{dx}\ln(1 + x_0) = 1$$

Then

$$\ln(1 + x) \approx \ln(1) + 1 \cdot x = 0 + x = x$$
Computational trick

\[ r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) \]

\[ = \ln(P_t) - \ln(P_{t-1}) \]

\[ = p_t - p_{t-1} \]

\[ = \text{difference in log prices} \]

where

\[ p_t = \ln(P_t) \]
Example

Let $P_{t-1} = 85$, $P_t = 90$ and $R_t = 0.0588$. Then the cc monthly return can be computed in two ways:

$$r_t = \ln(1.0588) = 0.0571$$

$$r_t = \ln(90) - \ln(85) = 4.4998 - 4.4427 = 0.0571.$$

Notice that $r_t$ is slightly smaller than $R_t$. 
Multi-period returns

\[ r_t(2) = \ln(1 + R_t(2)) \]
\[ = \ln \left( \frac{P_t}{P_{t-2}} \right) \]
\[ = p_t - p_{t-2} \]

Note that

\[ e^{r_t(2)} = e^{\ln(P_t/P_{t-2})} \]
\[ \Rightarrow P_{t-2}e^{r_t(2)} = P_t \]

\[ \implies r_t(2) = \text{cc growth rate in prices between months } t - 2 \text{ and } t \]
Result

cc returns are additive

\[ r_t(2) = \ln \left( \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}} \right) \]

\[ = \ln \left( \frac{P_t}{P_{t-1}} \right) + \ln \left( \frac{P_{t-1}}{P_{t-2}} \right) \]

\[ = r_t + r_{t-1} \]

where \( r_t = \) cc return between months \( t - 1 \) and \( t \), \( r_{t-1} = \) cc return between months \( t - 2 \) and \( t - 1 \)
**Example**

*Compute cc two-month return*

Suppose $P_{t-2} = 80$, $P_{t-1} = 85$ and $P_t = 90$. The cc two-month return can be computed in two equivalent ways: (1) take difference in log prices

$$r_t(2) = \ln(90) - \ln(80) = 4.4998 - 4.3820 = 0.1178.$$  

(2) sum the two cc one-month returns

$$r_t = \ln(90) - \ln(85) = 0.0571$$

$$r_{t-1} = \ln(85) - \ln(80) = 0.0607$$

$$r_t(2) = 0.0571 + 0.0607 = 0.1178.$$  

Notice that $r_t(2) = 0.1178 < R_t(2) = 0.1250$.  

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\[ r_t(k) = \ln(1 + R_t(k)) = \ln\left( \frac{P_t}{P_{t-k}} \right) \]

\[ = \sum_{j=0}^{k-1} r_{t-j} \]

\[ = r_t + r_{t-1} + \cdots + r_{t-k+1} \]
Portfolio returns

\[ R_{p,t} = \sum_{i=1}^{n} x_i R_{i,t} \]

\[ r_{p,t} = \ln(1 + R_{p,t}) = \ln(1 + \sum_{i=1}^{n} x_i R_{i,t}) \neq \sum_{i=1}^{n} x_i r_{i,t} \]

\Rightarrow \text{portfolio returns are not additive}

Note: If \( R_{p,t} = \sum_{i=1}^{n} x_i R_{i,t} \) is not too large, then \( r_{p,t} \approx R_{p,t} \) otherwise, \( R_{p,t} > r_{p,t} \).
Example

Compute cc return on portfolio

Consider a portfolio of Microsoft and Starbucks stock with

\[ x_{msft} = 0.25, \ x_{sbux} = 0.75, \]

\[ R_{msft,t} = 0.0588, \ R_{sbux,t} = -0.0503 \]

\[ R_{p,t} = x_{msft}R_{msft,t} + x_{sbux,t}R_{sbux,t} = -0.02302 \]

The cc portfolio return is

\[ r_{p,t} = \ln(1 - 0.02302) = \ln(0.977) = -0.02329 \]

Note

\[ r_{msft,t} = \ln(1 + 0.0588) = 0.05714 \]

\[ r_{sbux,t} = \ln(1 - 0.0503) = -0.05161 \]

\[ x_{msft}r_{msft} + x_{sbux}r_{sbux} = -0.02442 \neq r_{p,t} \]
Adjusting for inflation

The cc one period real return is

\[ r_t^{\text{Real}} = \ln(1 + R_t^{\text{Real}}) \]

\[ 1 + R_t^{\text{Real}} = \frac{P_t}{P_{t-1}} \cdot \frac{CPI_{t-1}}{CPI_t} \]

It follows that

\[ r_t^{\text{Real}} = \ln \left( \frac{P_t}{P_{t-1}} \cdot \frac{CPI_{t-1}}{CPI_t} \right) = \ln \left( \frac{P_t}{P_{t-1}} \right) + \ln \left( \frac{CPI_{t-1}}{CPI_t} \right) \]

\[ = \ln(P_t) - \ln(P_{t-1}) + \ln(CPI_{t-1}) - \ln(CPI_t) \]

\[ = r_t - \pi_t^{cc} \]

where

\[ r_t = \ln(P_t) - \ln(P_{t-1}) = \text{nominal cc return} \]

\[ \pi_t^{cc} = \ln(CPI_t) - \ln(CPI_{t-1}) = \text{cc inflation} \]
Example

Compute cc real return

Suppose:

\[ R_t = 0.0588 \]

\[ \pi_t = 0.01 \]

\[ R_t^{\text{Real}} = 0.0483 \]

The real cc return is

\[ r_t^{\text{Real}} = \ln(1 + R_t^{\text{Real}}) = \ln(1.0483) = 0.047. \]

Equivalently,

\[ r_t^{\text{Real}} = r_t - \pi_t^{\text{cc}} = \ln(1.0588) - \ln(1.01) = 0.047 \]
faculty.washington.edu/ezivot/