Portfolio Performance Measurement

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December 8, 2009
1 Investment Styles

1.1 Passive Management

- Believe that markets are in equilibrium
  - Assets are correctly priced

- Hold securities for relatively long periods with small infrequent changes
• Hold surrogates for market portfolio known as index funds
  
  – Low cost diversified portfolios (e.g. Vanguard Index Funds)
  
  – motivated by portfolio theory and CAPM: efficient portfolios are combinations of T-Bills and a market index portfolio

• Do not try to create portfolios to “actively” beat the returns on index funds
1.2 Active Management

- Markets are not always in equilibrium
  - Some securities are “mis-priced”

- Buy under-priced (positive “alpha”) assets and sell over-priced (negative “alpha”) assets

- Active managers often “tweak” a benchmark (index) portfolio

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<thead>
<tr>
<th>Security</th>
<th>weight in benchmark</th>
<th>weight in active port</th>
<th>active position</th>
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<tbody>
<tr>
<td>MSFT</td>
<td>.05</td>
<td>.10</td>
<td>+.05</td>
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<tr>
<td>GM</td>
<td>.02</td>
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• Active management strategies

  – individual stock selection

  – sector selection (e.g. utility, technology)

  – asset class selection (stocks, bonds, real estate)

• Most mutual funds are actively managed.

  – management fees can vary substantially from fund to fund

  – fee is often a percentage of assets under management
2 Evaluating Investment Performance

Q: Is it worthwhile to “pay” for active management of portfolios?

Key Concepts

• Actively managed portfolios should be compared with passive (index) benchmarks of a similar risk class

• Superior past performance could be luck or could be skill

• Often very little historical data to evaluate managed portfolios
  – Statistical analysis is difficult
2.1 Risk Adjusted Measures of Performance

Observe returns on active portfolio and benchmark over some time horizon (e.g. 5 years of monthly data)

- Does the managed portfolio exhibit superior performance adjusted for risk?

- How to rank different actively managed portfolios?
Measures of risk

- Market risk (portfolio beta, $\beta_p$, from SI model or CAPM)

- Total risk (portfolio standard deviation, $\sigma_p$)

**Ex Post (Historical) measures**

\[
\hat{\mu}_p = \frac{1}{T} \sum_{t=1}^{T} R_{p,t}, \quad \hat{r}_f = \frac{1}{T} \sum_{t=1}^{T} r_{f,t}
\]

\[
\hat{\sigma}_p = \left( \frac{1}{T - 1} \sum_{t=1}^{T} (R_{p,t} - \hat{\mu}_p)^2 \right)^{1/2}
\]

\[
\hat{\beta}_p = \frac{\text{cov}(R_{p,t}, R_{M,t})}{\text{var}(R_{M,t})}
\]
Types of Performance Measures

• Average return difference adjusted for risk
  
  \[
  \text{ave return on active portfolio} - \text{ave return on risk adjusted benchmark}
  \]

• Risk adjusted reward/risk ratio
  
  \[
  \frac{\text{average excess return}}{\text{risk measure}}
  \]
2.1.1 Performance Measures Based on Market Risk

Idea: Under CAPM, market risk is captured by $\beta$ and expected returns are captured by the Security Market Line (SML)

$$\mu_{p,CAPM} = r_f + \beta_p(\mu_M - r_f)$$
Jensen’s alpha

Risk-adjusted return difference

\[ \hat{\alpha}_p = \hat{\mu}_p - \hat{\mu}_{p,\text{CAPM}} \]

Computation: use linear regression to estimate the excess returns SI model

\[ R_{p,t} - r_f = \alpha^*_p + \beta_p (R_{Mt} - r_f) + \varepsilon_{pt}, \varepsilon_{pt} \sim iid \ N(0, \sigma^2_{\varepsilon}) \]

Statistical evaluation:

\[ H_0 : \alpha^*_p = 0 \text{ (no superior performance) vs. } H_1 : \alpha^*_p \neq 0 \]
Information Ratio

\[ \widehat{IR}_p = \frac{\hat{\alpha}_p^*}{\hat{\sigma}_\varepsilon} \]

Statistical evaluation: Use bootstrap to compute standard error and confidence interval
2.1.2 Performance Measures Based on Total Risk

Idea: Efficient portfolios are combination of T-bills and tangency portfolio. Under CAPM, the tangency portfolio is the market portfolio

Sharpe ratio

\[ SR_p = \frac{\hat{\mu}_p - \hat{r}_f}{\hat{\sigma}_p} \]

= excess return per unit portfolio risk

Statistical evaluation:

\[ H_0 : SR_p = SR_M \text{ (no superior performance) vs } H_1 : SR_p \neq SR_M \]

Evaluate \( H_0 \) using bootstrap
R Package for Performance Evaluation

PerformanceAnalytics