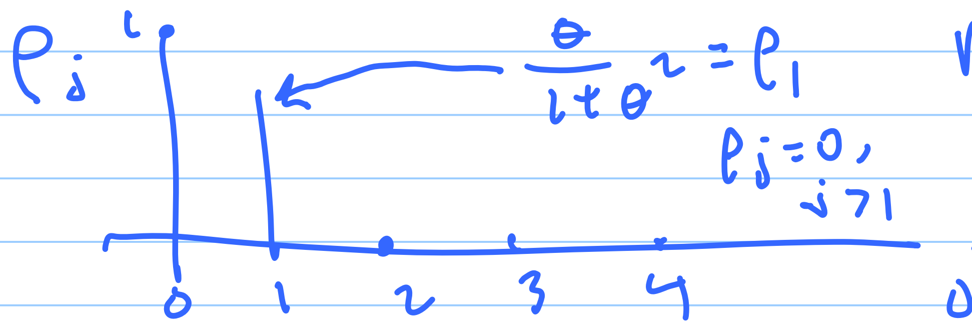
Econ 424 Lec 6

Note Title 10/19/2009

Friday O.H. CSSCR small computer lab
SAV 121

ACF = autocorrelation function

= plot of $\rho_j = cor(Y_t, Y_{t-j})$
against j



MA(1) $\theta > 0$

AR(1)

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t \qquad |\phi| < 1$$

$\epsilon_t \sim \text{iid } N(0, \sigma_\epsilon^2)$

$\{Y_t\}$ is covariance stationary.

$E[Y_t] = ?$

$$Y_t = \mu + \phi Y_{t-1} - \phi\mu + \epsilon_t$$

$$E[Y_t] = \mu + \phi \underbrace{E[Y_{t-1}]}_{E[Y_t] \text{ by stationarity}} - \phi\mu + \underbrace{E(\epsilon_t)}_{=0}$$

$$\Rightarrow E[Y_t](1-\phi) = \mu(1-\phi)$$

$$\Rightarrow E[Y_t] = \frac{\mu(1-\phi)}{1-\phi} = \mu$$

Use similar calculations to determine $var(Y_t)$, $cov(Y_t, Y_{t-j})$ etc. See notes for more details

RW is special case of AR(1)!

$$Y_t - \mu = \phi(Y_{t-1} - \mu) + \epsilon_t$$

Let $\phi = 1$

$$\Rightarrow Y_t - \cancel{\mu} = Y_{t-1} - \cancel{\mu} + \epsilon_t$$

$$\Rightarrow Y_t = Y_{t-1} + \epsilon_t \quad : \text{RW}$$

Note: $\{Y_t\}$ is not stationary!

$$R_{p,x} = x_A R_A + x_B R_B$$

$$\sigma_{p,x}^2 = (x_A \;\; x_B) \begin{pmatrix} \sigma_A^2 & \sigma_{AB} \\ \sigma_{AB} & \sigma_B^2 \end{pmatrix} \begin{pmatrix} x_A \\ x_B \end{pmatrix} = x' \Sigma x$$

$$x' \Sigma x = (x' \Sigma) x \quad \text{or} \quad x' (\Sigma x)$$

$$\underbrace{\underset{1\times 2}{x'} \; \underset{2\times 2}{\Sigma}}_{1\times 2} = (x_A \;\; x_B) \begin{pmatrix} \sigma_A^2 & \sigma_{AB} \\ \sigma_{AB} & \sigma_B^2 \end{pmatrix} = \underset{1\times 2}{\begin{pmatrix} x_A \sigma_A^2 + x_B \sigma_{AB} \\ x_A \sigma_{AB} + x_B \sigma_B^2 \end{pmatrix}}$$

$$
\begin{aligned}
\underbrace{\underbrace{(x'\Sigma)}_{1\times 2}\underbrace{x}_{2\times 1}}_{1\times 1} &= \begin{pmatrix} x_A \sigma_A^2 + x_B \sigma_{AB} \\ x_A \sigma_{AB} + x_B \sigma_B^2 \end{pmatrix} \begin{pmatrix} x_A \\ x_B \end{pmatrix} \\
&= x_A^2 \sigma_A^2 + x_A x_B \sigma_{AB} \\
&\quad + x_A x_B \sigma_{AB} + x_B^2 \sigma_B^2 \\
&= x_A^2 \sigma_A^2 + x_B^2 \sigma_B^2 + 2 x_A x_B \sigma_{AB} \\
&= \sigma_{p,x}^2
\end{aligned}
$$