Econ 424 Lecture 5

Note Title 10/14/2009

Topics

- Finish Prob Review
- Time Series Concepts
- Matrix Algebra Review

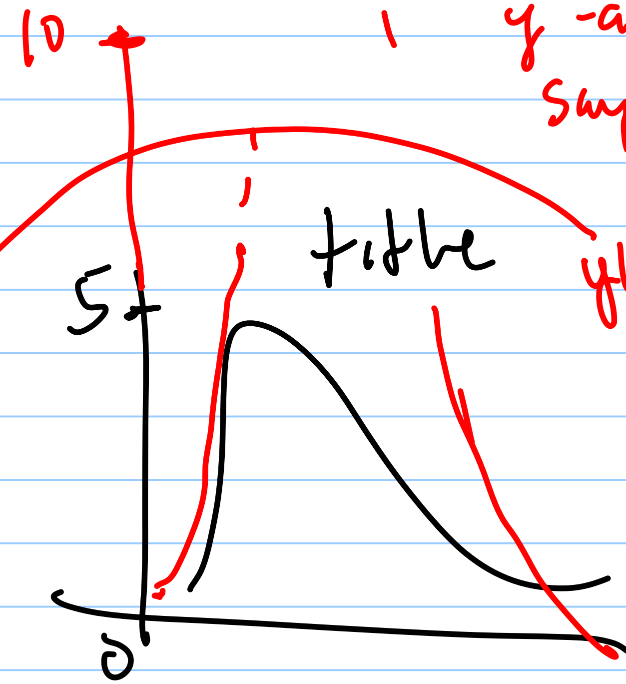
Plotting in R

plot(·) – function

plot(x, y, type = "l",)

optional argument
graphics parameters

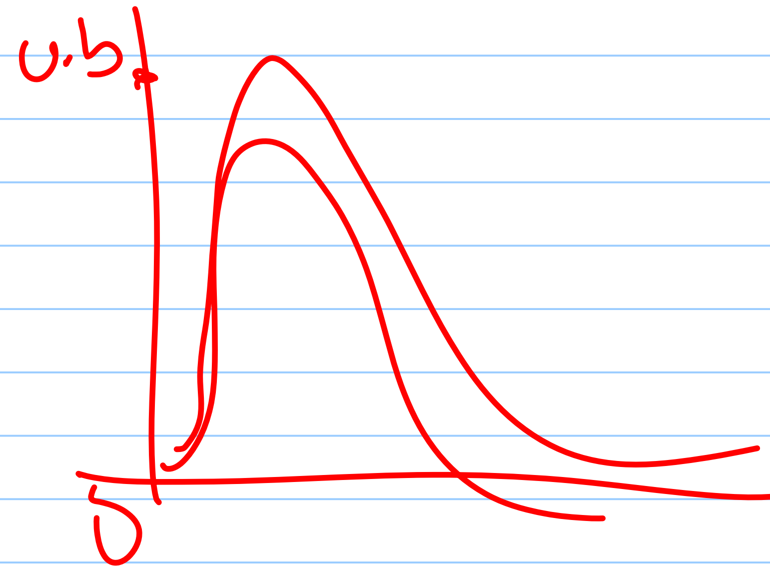
main = "title"



points(x, z, type="l")

to set y-axis b/w say 0 and 10 use
ylim = c(0,10)

```
x.vals = seq(0, 10, length=100)
x = dchisq(x.vals, df=1)
y = dchisq(x.vals, df=2)
u.b = max(y, x)
ylim = c(0, u.b)
```



Ex: Gaussian White Noise

$\{Y_t\} = \{Y_1, Y_2, \ldots, Y_t, \ldots\}$

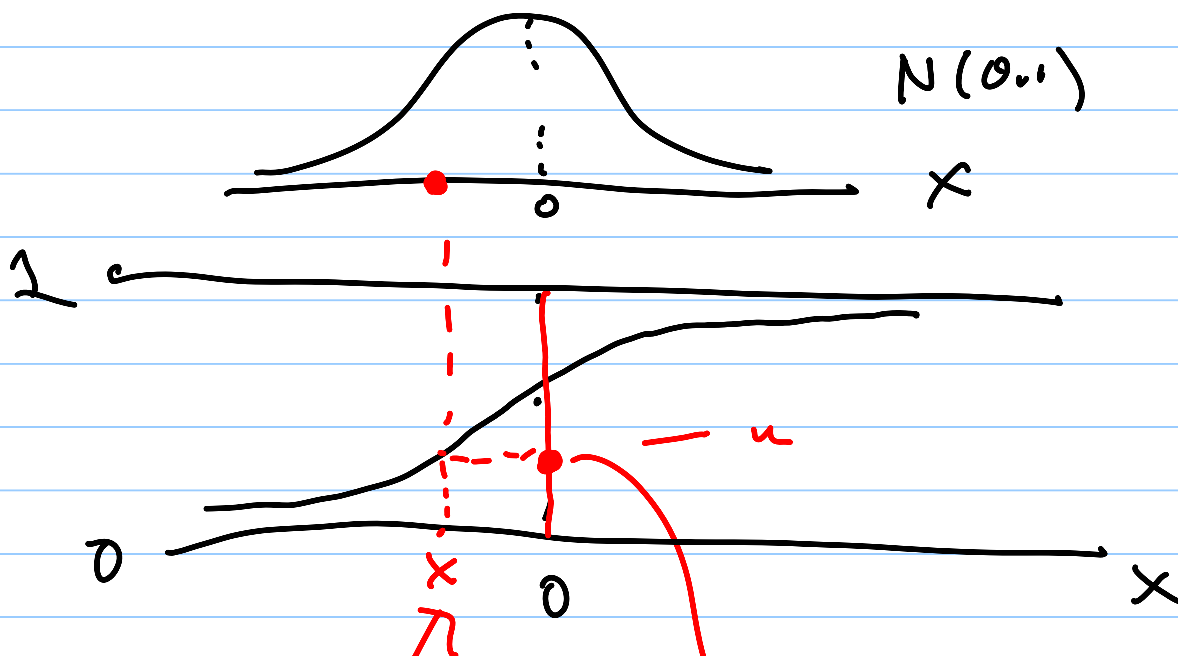
$Y_t \sim \text{iid } N(0, \sigma^2)$

iid: indep, identically distributed

- $E[Y_t] = 0$ indep of t
- $\text{Var}(Y_t) = \sigma^2$ indep of t
- $\text{Cov}(Y_t, Y_{t-j}) = 0 \quad \forall j \neq 0$

covariance stationary.

Digression on Random number generation



CDF $= \Pr(X \le x)$
$= \Phi(x)$

$x = \Phi^{-1}(u)$

computer simulates
$u \sim U[0,1]$

Ex: Random Walk Model

$$Y_t = Y_{t-1} + \epsilon_t, \quad \epsilon_t \sim WN(0, \sigma^2)$$

$t=1$: $Y_1 = Y_0 + \epsilon_1$, Assume Y_0 is fixed number.

$t=2$: $Y_2 = Y_1 + \epsilon_2$

$= Y_0 + \epsilon_1 + \epsilon_2$

.

.

$t = 10 = Y_{10} = Y_0 + \epsilon_1 + \epsilon_2 + \cdots + \epsilon_{10}$

$\epsilon_t \sim WN(0, \sigma^2)$

Is this model stationary??

$$E[Y_{10}] = Y_0 + \underbrace{E[\epsilon_1]}_{0} + \cdots + \underbrace{E[\epsilon_{10}]}_{0}$$

$= Y_0$

$E[Y_t] = Y_0$ indep of t.

$Var(Y_{10}) = Var(Y_0 + \epsilon_1 + \cdots + \epsilon_{10})$

$= Var(\epsilon_1 + \cdots + \epsilon_{10})$

$= \sigma^2 + \cdots + \sigma^2 = 10 * \sigma^2$

$Var(Y_t) = t * \sigma^2$ depends on t

$\Rightarrow$ Not Stationary!

Generating RW data

$$Y_1 = Y_0 + \epsilon_1$$

$$Y_2 = Y_0 + \epsilon_1 + \epsilon_2$$

$$\vdots$$

$$Y_t = Y_0 + \epsilon_1 + \dots + \epsilon_t$$

For i=1 to 250

Y[i] = Y[i-1] + z[i]

z = rnorm(250)

Y = cumsum(z)

Set $Y_0 = 0$

$$\begin{bmatrix} z_1 \\ z_1 + z_2 \\ z_1 + z_2 + z_3 \\ \vdots \\ z_1 + z_2 \cdots z_{250} \end{bmatrix}$$

Ex: MA(1)

$$Y_t = \mu + \epsilon_t + \theta \epsilon_{t-1}, \quad \epsilon_t \sim \text{iid } N(0, \sigma_\epsilon^2)$$

Q: Is this cov stat?? A: Yes.

$$E[Y_t] = \mu + \underbrace{E[\epsilon_t]}_{0} + \theta \underbrace{E[\epsilon_{t-1}]}_{0}$$

$$= \mu \quad \text{indep of } t!$$

$$\text{var}(Y_t) = \text{var}(\mu + \epsilon_t + \theta \epsilon_{t-1})$$

$$= \text{var}(\epsilon_t) + \theta^2 \text{var}(\epsilon_{t-1})$$

$$= \sigma_\epsilon^2 + \theta^2 \sigma_\epsilon^2 = \sigma_\epsilon^2 (1+\theta^2)$$

indep of t !

$$\gamma_1 = \mathrm{Cov}\left(Y_t, Y_{t-1}\right) = E\left[(Y_t - \mu)(Y_{t-1} - \mu)\right]$$

Recall, $Y_t = \mu + \epsilon_t + \theta \epsilon_{t-1}$

$Y_{t-1} = \mu + \epsilon_{t-1} + \theta \epsilon_{t-2}$

$\Rightarrow Y_t - \mu = \epsilon_t + \theta \epsilon_{t-1}$

$Y_{t-1} - \mu = \epsilon_{t-1} + \theta \epsilon_{t-2}$

$$\mathrm{Cov}(y_t, y_{t-1}) = E\left[(\epsilon_t + \theta\epsilon_{t-1})(\epsilon_{t-1} + \theta\epsilon_{t-2})\right]$$

$$= E\left[\epsilon_t\epsilon_{t-1} + \theta\epsilon_t\epsilon_{t-2} + \theta\epsilon_{t-1}^2 + \theta^2\epsilon_{t-1}\epsilon_{t-2}\right]$$

$$= \underbrace{E\left[\epsilon_t\epsilon_{t-1}\right]}_{= 0} + \theta \underbrace{E\left[\epsilon_t\epsilon_{t-2}\right]}_{= 0} + \theta \underbrace{E\left[\epsilon_{t-1}^2\right]}_{= \theta \cdot \sigma_\epsilon^2}$$

$$+ \theta^2 \underbrace{E\left[\epsilon_{t-1}\epsilon_{t-2}\right]}_{= 0}$$

$$\boxed{\mathrm{Cov}(Y_t, Y_{t-1}) = \gamma_1 = \theta\sigma_\epsilon^2 \neq 0}$$

Note: $E[\epsilon_{t-1}^2] = \mathrm{Var}(\epsilon_{t-1}) = \sigma_\epsilon^2$

$$\mathrm{Var}(\epsilon_t) = E\left[\left(\epsilon_t - E[\epsilon_t]\right)^2\right]$$

$$= E[\epsilon_t^2] \quad \text{bc} \quad E[\epsilon_t] = 0$$

$$\rho_1 = \frac{\gamma_1}{\sigma^2} = \frac{\theta \cdot \cancel{\sigma_\varepsilon^2}}{\cancel{\sigma_\varepsilon^2}(1+\theta^2)}$$

$$= \frac{\theta}{1+\theta^2}$$

<u>Note:</u> $-\frac{1}{2} \leq \frac{\theta}{1+\theta^2} \leq \frac{1}{2}$

$$\gamma_2 = \text{cov}(Y_t, Y_{t-2})$$

$$= E[(Y_t - \mu)(Y_{t-2} - \mu)]$$

$$= E[(\epsilon_t + \theta\epsilon_{t-1})(\epsilon_{t-2} + \theta\epsilon_{t-3})]$$

$$= E[\epsilon_t\epsilon_{t-2} + \theta\epsilon_t\epsilon_{t-3} + \theta\epsilon_{t-1}\epsilon_{t-2} + \theta^2\epsilon_{t-1}\epsilon_{t-3}]$$

$= 0$ b/c ϵ_t is indep of ϵ_s for $t \neq s$

$\gamma_j = 0$ for $j > 1$