

Capital Asset Pricing Model

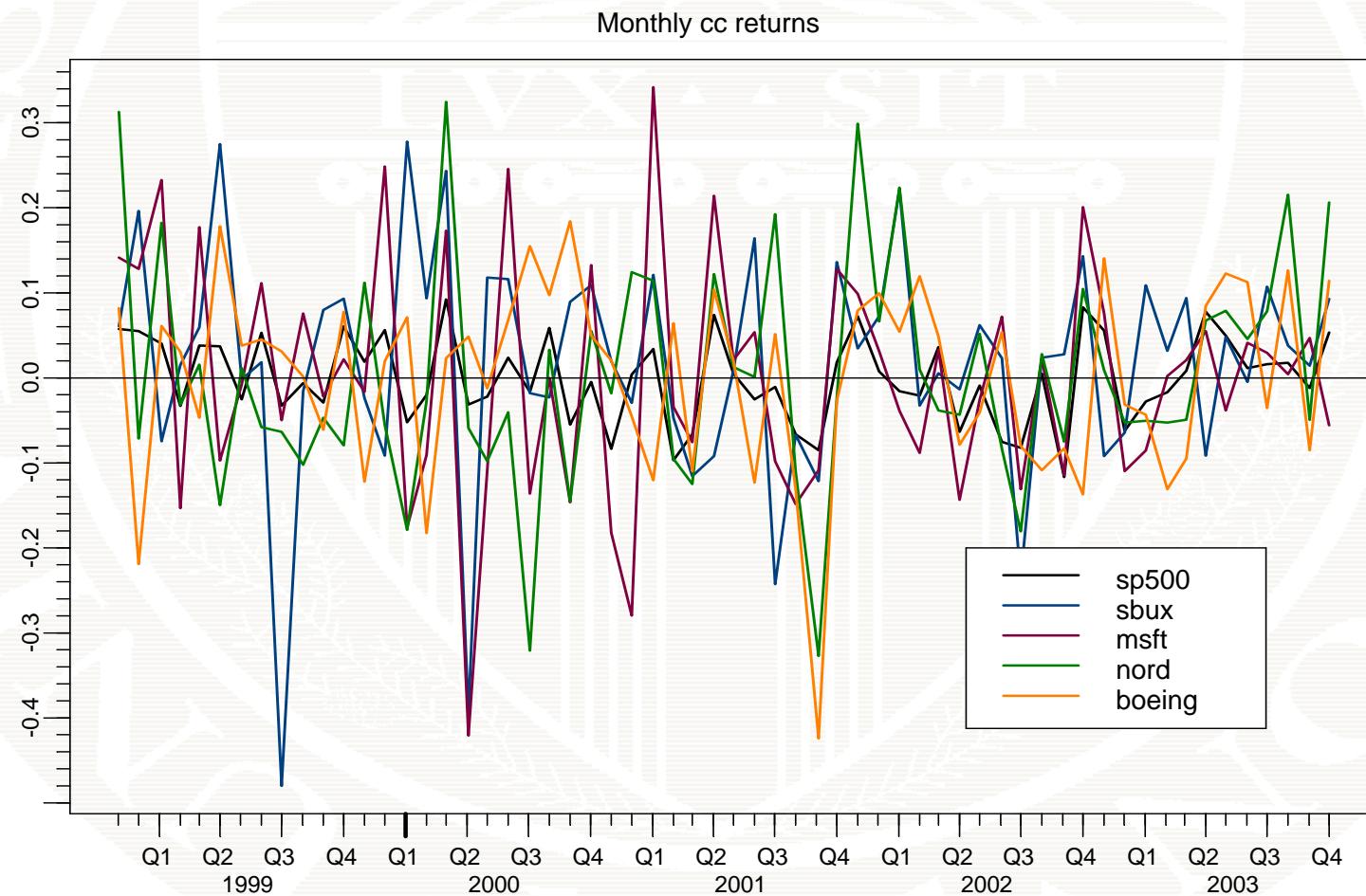
Econ 424/Amath 462

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Relationship between $E[R]$ and β



Tangency Portfolio (computed in R) using $r_f = 0.01/12$

Portfolio expected return: 0.023903

Portfolio standard deviation: 0.17149

Portfolio Sharpe Ratio: 0.13452

Portfolio weights:

sbux	msft	nord	boeing
1.3079	-0.5003	0.1604	0.0319

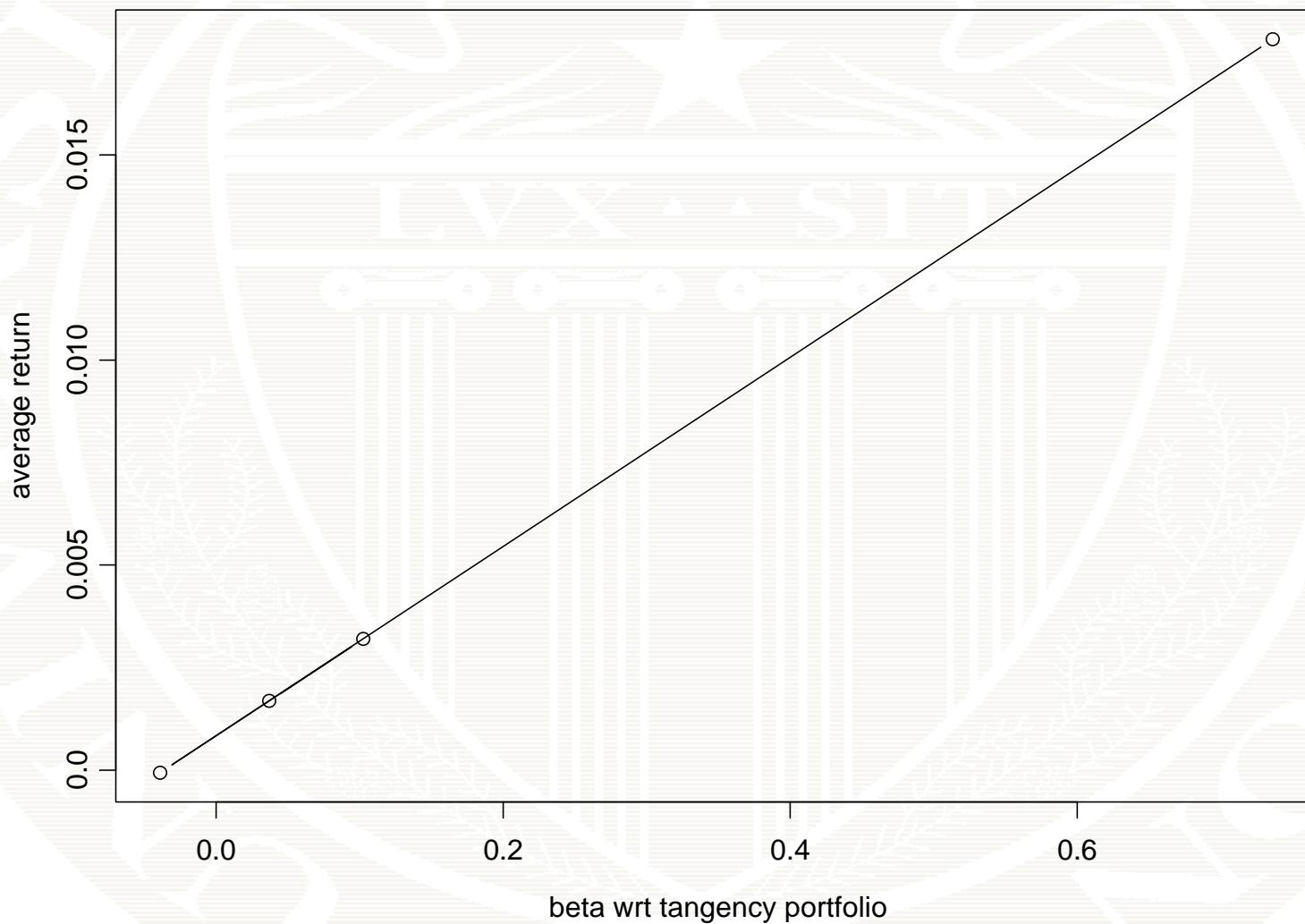
Beta values computed from SI regression $R_i = \alpha_i + \beta_i * R_{tan} + e_i$

sbux	msft	nord	boeing
0.73618	-0.038882	0.10269	0.037027

Average returns on four assets

sbux	msft	nord	boeing
0.017817	-0.000063638	0.0032024	0.0016875

Linear relationship between average return and beta



Regression of average returns on beta

$$\hat{\mu}_i = \gamma_0 + \gamma_1 \hat{\beta}_{i,\text{tan}} + \eta_{it}$$

Coefficients:

	Value	Std. Error	t value	Pr(> t)
(Intercept)	1.0000e-003	0.0000e+000	7.4306e+014	0.0000e+000
beta.vec	2.3000e-002	0.0000e+000	7.6649e+015	0.0000e+000

Residual standard error: 1.86e-018 on 2 degrees of freedom

Multiple R-Squared: 1

Note: Estimated coefficients are

(Intercept)	beta.vec
0.00083333	0.023069

Risk-free rate and risk premium on tangency portfolio

```
> 0.01/12
[1] 0.00083333
> tan.port$er - 0.01/12
[1] 0.023069
```

TESTING THE CAPM

Monthly Returns: Jan 1978 – Dec 1987

```
> colnames(berndt.dat)
[1] "CITCRP"  "CONED"   "CONTIL"   "DATGEN"   "DEC"      "DELTA"
[7] "GENMIL"   "GERBER"  "IBM"      "MARKET"   "MOBIL"    "PANAM"
[13] "PSNH"     "TANDY"   "TEXACO"   "WEYER"    "RKFREE"

# create excess returns by subtracting off risk free rate

> returns.mat = as.matrix(coredata(berndt.dat))
> excessReturns.mat = returns.mat -
+                      returns.mat[, "RKFREE"]
> excessReturns.df = as.data.frame(excessReturns.mat)
```

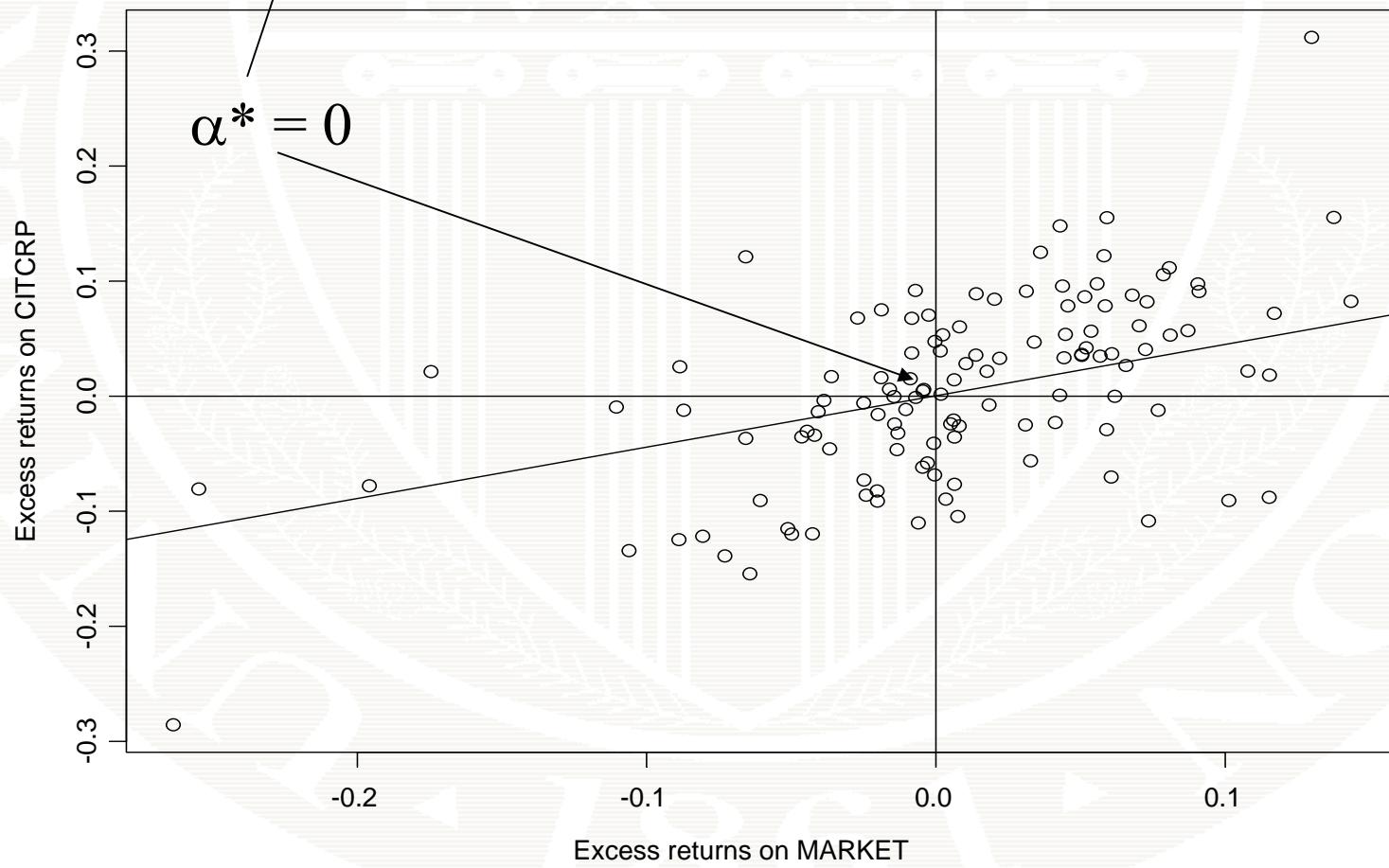
Coefficients:

	Value	Std. Error	t value	Pr(> t)
(Intercept)	0.001	0.009	0.065	0.948
MARKET	0.447	0.119	3.746	0.000

Residual standard error: 0.0703 on 58 degrees of freedom

Multiple R-Squared: 0.195

CAPM regression for CITCRP



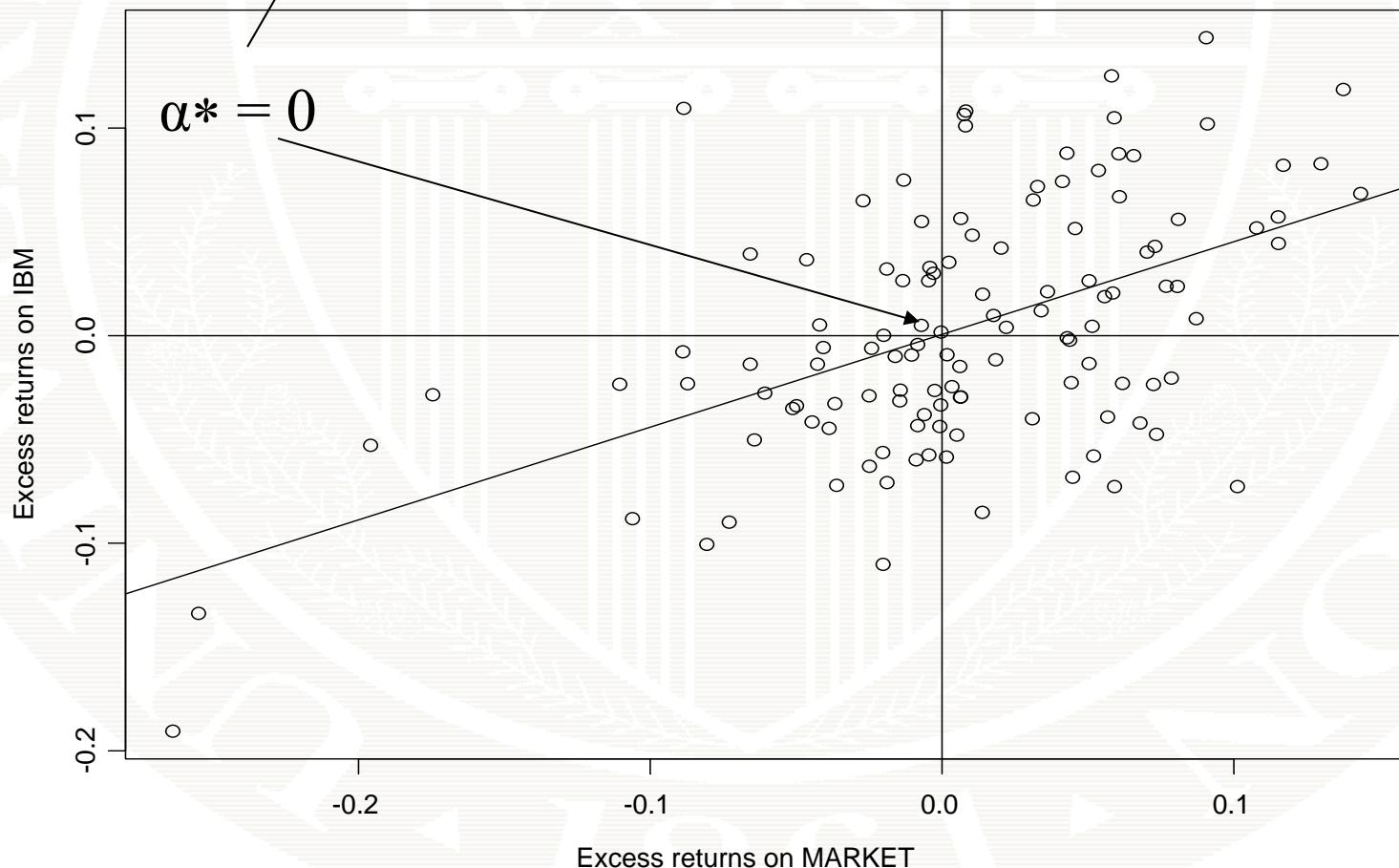
Coefficients:

	Value	Std. Error	t value	Pr(> t)
(Intercept)	0.000	0.007	-0.036	0.971
MARKET	0.339	0.089	3.818	0.000

Residual standard error: 0.0524 on 58 degrees of freedom

Multiple R-Squared: 0.201

CAPM regression for IBM



Test CAPM: $H_0: \alpha^*_i = 0$

$$t = \frac{\hat{\alpha}_i}{SE(\alpha_i)}$$

Decision: Reject H_0 at 5% level if $|t| > 2$

```
> tstats
```

CITCRP	CONED	CONTIL	DATGEN	DEC	DELTA	GENMIL	GERBER
0.06502	1.2141	-0.6703	-1.043	0.030922	0.62137	0.5411	-0.068848
IBM	MOBIL	PANAM	PSNH	TANDY	TEXACO	WEYER	
-0.036301	0.085956	-0.89396	-0.27455	1.997	-0.40445	-0.52	

Conclusion: Do not reject CAPM for any of the assets!

Relationship Between Average Return and β

```
> print(rbind(mu.hat,betas), digits=3)
    CITCRP CONED CONTIL DATGEN DEC DELTA GENMIL GERBER
mu.hat 0.0056 0.0098 -0.00319 -0.00372 0.00818 0.012 0.00535 0.00453
betas  0.4466 0.1405  0.38855  1.00562 0.70677 0.392 0.09874 0.46316

    IBM MOBIL PANAM PSNH TANDY TEXACO WEYER
mu.hat 0.00355 0.00838 -0.0053 0.000831 0.0427 0.00371 0.00325
betas  0.33901 0.67978 0.7466 0.218017 1.0308 0.64326 0.70789
```

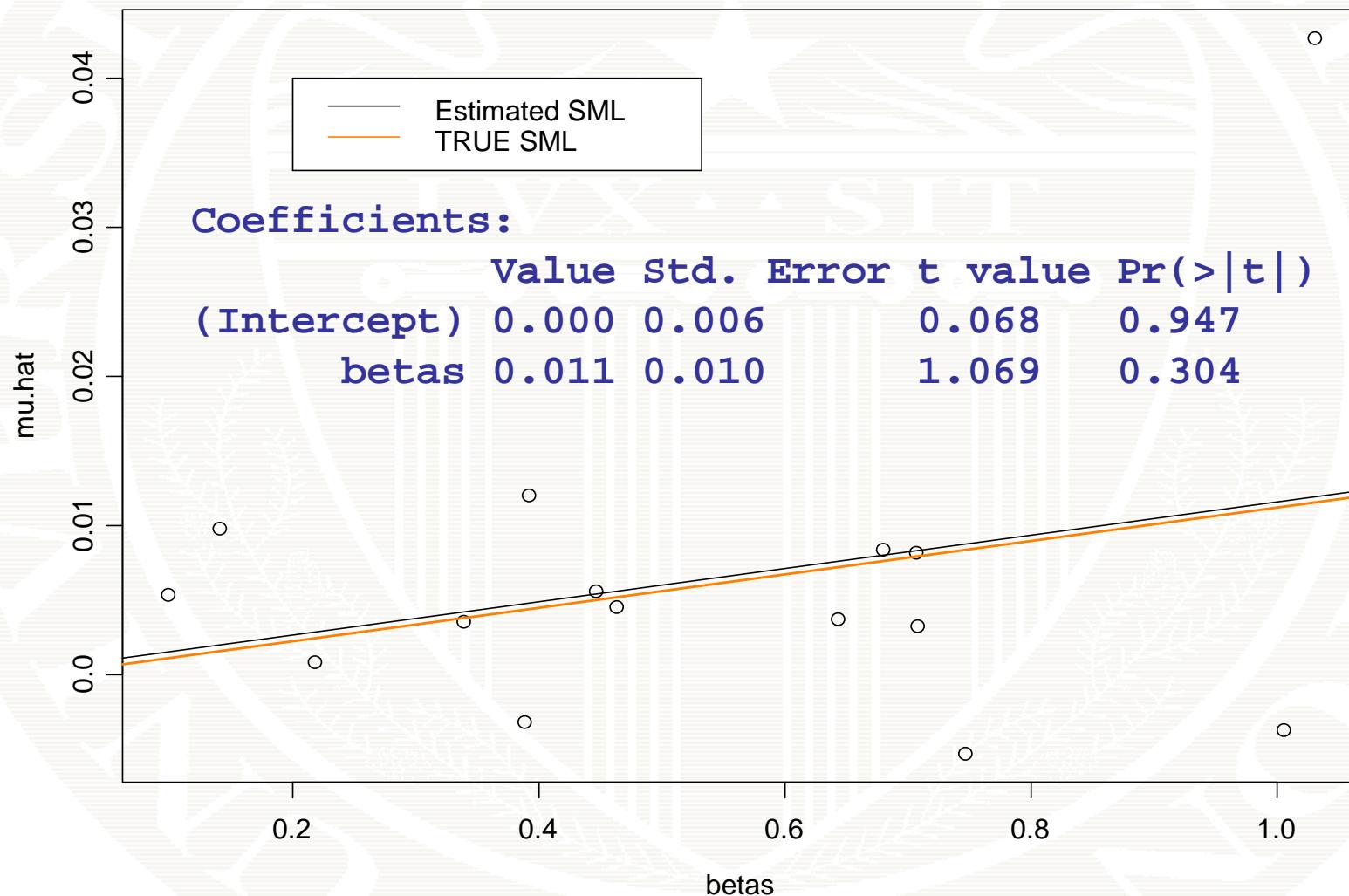
Estimated Security Market Line (SML)

$$\hat{\mu}_i = \gamma_0 + \gamma_1 \hat{\beta}_i + error_i$$

$$CAPM \Rightarrow \gamma_0 = 0, \gamma_1 = \hat{\mu}_M - r_f = 0.011$$

Jan 1978 – Dec 1982

TRUE and Estimated SML



$$t_{\gamma_0=0} = 0.068, \quad t_{\gamma_1 = \mu_M - r_t} = \frac{0.011 - 0.011}{0.010} = 0$$

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Jan 1983 – Dec 1987

TRUE and Estimated SML

