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0. Executive Summary

0.1. Data Set

This project looks at monthly closing prices from September 2005 through end of September 2010.

0.2. Mutual fund descriptions

S&P 500 index: vfinx

The mutual fund holds stocks in the same proportion as in the SP500 index, which is known as a benchmark of US's stock performance. The SP500 contains a high proportion of stocks of many large US companies.

European stock index: veurx

The mutual fund invests in all of the stocks in the MSCI Europe index, which has the stocks of companies located in Europe.

Emerging markets fund: veiex

The mutual fund invests in all of the stocks in the MSCI Emerging Markets index, which has the stocks of companies located in emerging markets, or nations with rapid growth and industrialization. These emerging markets are all around the world, with China and India being the two largest.

Long-term bond fund: vbltx

The mutual fund tracks the performance of a market-weighted bond index called the Barclays Capital U.S. Long Government/Credit Float Adjusted Index, which includes government and high-quality corporate bonds that have an average maturity of 15-30 years.

Short-term bond fund: vbisx

The mutual fund tracks the performance of a market-weighted bond index called the Barclays Capital U.S. 1–5 Year Government/Credit Float Adjusted Index, which includes government and high-quality corporate bonds that have an average maturity of 1-5 years.

Pacific stock index: vpacx

The mutual fund attempts to replicate the performance of MSCI Pacific index, which has the stocks of companies located in Japan, Australia, Hong Kong, Singapore, and New Zealand.

0.3. Main Findings

- The mutual funds that index groups of countries all had prices and returns that dropped dramatically around mid-2008 to 2009. The two bond funds had prices and returns drop only slightly during this period.

- None of the six mutual funds have returns that are normally distributed and rolling analysis suggest that expected returns and standard deviations are not stationary.

- The emerging market fund (veiex) has the highest risk and highest return. The short term bond (vbisx) has the lowest risk, and the S&P 500 (vfinx) has the lowest return. Expected returns are not measured as precisely as standard deviations.

- There is usually a tradeoff between return and risk, but country stock indexes SP500 (vfinx), European (veurx), and Pacific (vpacx) funds performed very poorly—they have a *lower* return and *higher* risk than the two bond funds.

- The Sharpe Ratio measures excess return per unit of risk. The short term bond fund (vbisx) has the highest Sharpe Ratio, and the S&P 500 mutual fund (vfinx) has the lowest Sharpe Ratio.

- Country stock index funds (vfinx, veurx, veiex, vpacx) have strong, precisely measured positive linear relationships with each other. Country stock index funds and bond funds have slight, positive, and imprecisely measured correlations with each other. The two bond funds have a moderate positive linear relationship.

- None of these correlations between the Vanguard assets are negative, making diversification difficult since mutual funds cannot be shorted.

- Value at risk over a one month investment horizon is largest (in absolute value) for SP500 (vfinx) and smallest for short term bonds (vbisx). Over a one year investment horizon, European fund (veurx) and emerging markets fund (veiex) have largest value at risk, and short term bonds (vbisx) have the smallest value at risk.

- The global minimum portfolio without short sale has a higher standard deviation and higher expected return than the global minimum portfolio with short sales. The potential loss (value at risk) is greater with a global minimum variance portfolio without short sale.

- The tangency portfolio with no shorting has a lower expected return, higher standard deviation, and lower Sharpe Ratio than the tangency with shorting.

- When we introduce the risk free asset, we can achieve the target portfolio return with a lower variance. As such, the value at risk is lower in absolute value when holding a riskless asset.

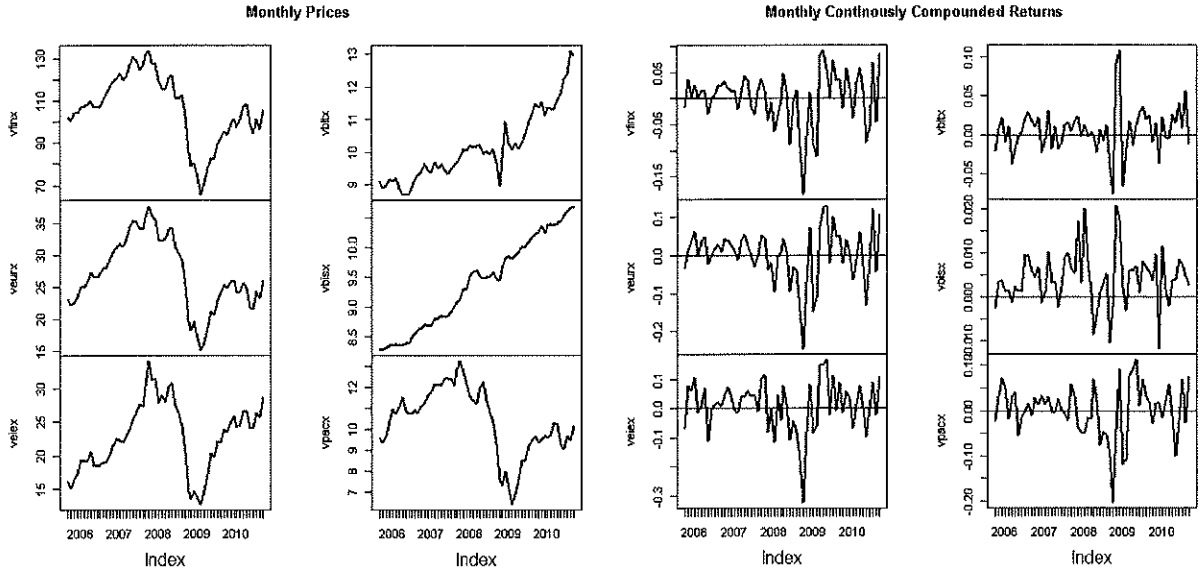
- Mutual funds that peg market returns in different parts of the world have high non-diversifiable risk (R^2) and the mutual funds that peg bonds have very low non-diversifiable risk.

- Mutual funds that peg market returns in different parts of the world have higher β (risk with respect to the market) than the mutual funds that peg bonds. The β values for the global minimum and tangency portfolios are very low.

- Asset statistics do not support CAPM.

1. Return Calculation and Sample Statistics

1.1.Time Trends of Mutual Fund Prices and Returns



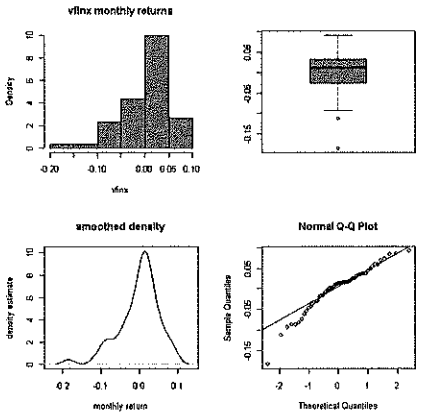
The mutual funds that replicate indexes of different countries like the S&P 500 index (vfinx), European stock index (veurx), emerging markets fund (veiex), and Pacific stock index (vpacx) have prices and continuously compounded returns that seem to move together. For example, prices rose and peaked at mid-2007. The prices then dipped until 2008, then rose briefly, then plummeted until 2009. After 2009, these country stock indexes rose again. The continuously compounded return for these four funds fluctuated around their respective means until mid-2008, when the returns dropped dramatically. After early to mid-2009, the returns fluctuated around their respective means again.

The two bond funds, long-term bond fund (vbltx) and short-term bond fund (vbisx), move together. Their prices trended up steadily, but they too had a drop in price around mid-2008. Compared to the country stock indexes, the movements of the bond indexes are smoother, with the short term bond index having the smoothest upward trending in price.

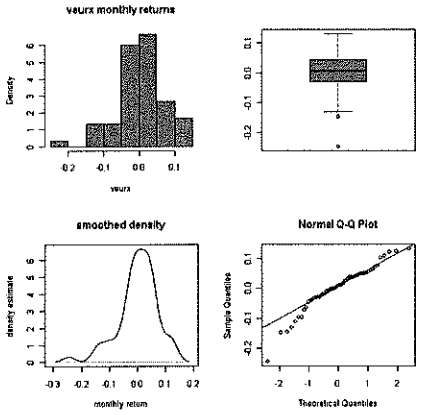
1.2. Normality of Monthly Continuously Compounded Returns

In this section, there are four panel diagnostic plots for each asset containing histograms, smoothed density plots, boxplots and qq-plots, which are used to determine if these returns are normally distributed.

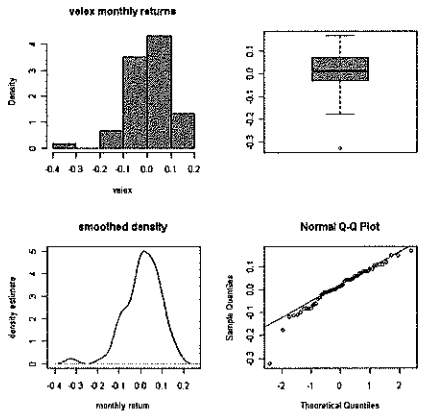
S&P 500 index: vfinx



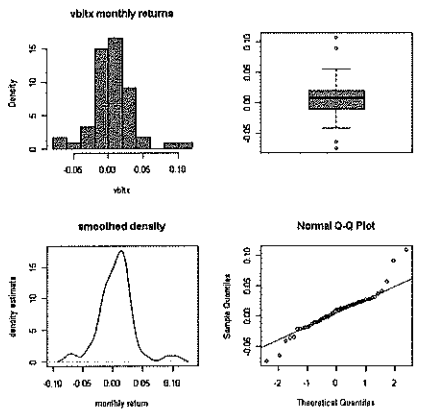
European stock index: veurx



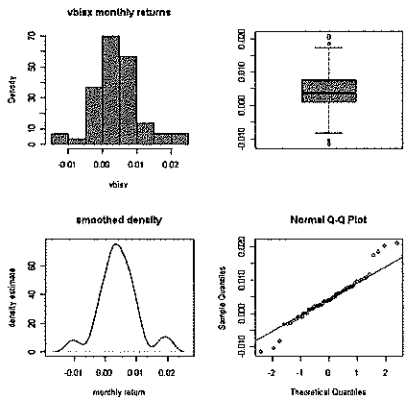
Emerging markets fund: veiex



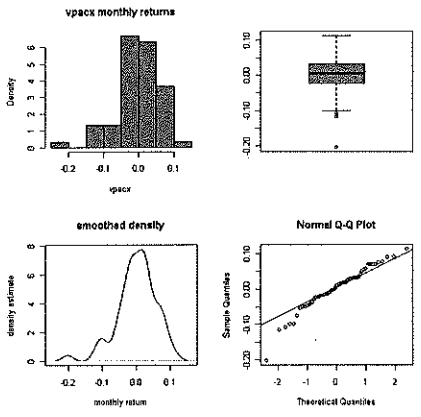
Long-term bond fund: vbltx



Short-term bond fund: vbisx



Pacific stock index: vpacx

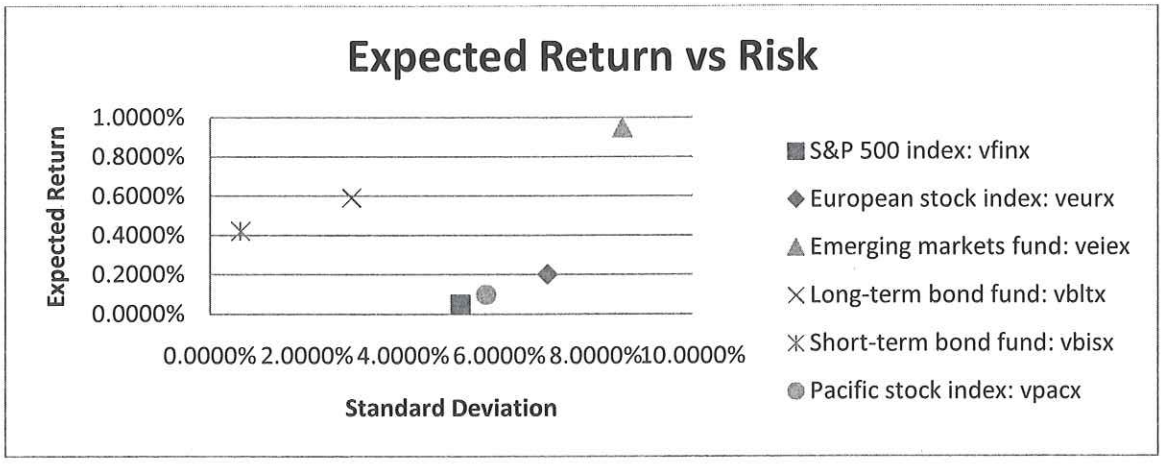


1.3. Constant Expected Return (CER) Model Parameters

CER Parameter Values							
Asset	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\sigma}^2$	Skew	Excess Kurtosis	1% Quantile	5% Quantile
S&P 500 index: vfinx	0.0470%	5.1952%	0.0027	-0.9468	1.3551	-0.1418	-0.0885
European stock index: veurx	0.1998%	6.9973%	0.0049	-0.8900	1.5167	-0.1878	-0.1313
Emerging markets fund: veiex	0.9486%	8.5491%	0.0073	-1.0045	2.2700	-0.2373	-0.1138
Long-term bond fund: vbltx	0.5899%	2.9326%	0.0009	0.4372	2.6176	-0.0690	-0.0363
Short-term bond fund: vbisx	0.4218%	0.6293%	0.0000	0.2600	0.9788	-0.0110	-0.0036
Pacific stock index: vpacx	0.0980%	5.7243%	0.0033	-0.8035	1.3844	-0.1521	-0.1005

These are the estimates of the parameters of the constant expected return (CER) model. As seen above, SP500 (vfinx), European (veurx), emerging market (veiex), and Pacific (vpacx) stock index funds have negative skewness, and the two bond funds have positive skewness. All six mutual funds have fat tails.

The emerging market fund (veiex) has the highest risk and highest return. The two bond funds have a lower risk and return than the emerging market fund (veiex), where the long term bond (vbltx) has a higher risk and return than the short term bond (vbisx), but notice that the three other country stock indexes SP500 (vfinx), European (veurx), and Pacific (vpacx) funds perform very poorly—they have a *lower* return and *higher* risk than the two bond funds.



2. Value-at-Risk (VaR) Calculations

2.1. Value at Risk Over Different Investment Horizons

VaR of $100,000 Over a One Month Investment Horizon		
Asset	VaR 1%	VaR 5%
S&P 500 index: vfinx	-$11,342.42	-$8,147.27
European stock index: veurx	-$14,852.40	-$10,693.66
Emerging markets fund: veiex	-$17,254.09	-$12,289.97
Long-term bond fund: vbltx	-$6,042.14	-$4,145.45
Short-term bond fund: vbisx	-$1,036.80	-$611.47
Pacific stock index: vpacx	-$12,382.33	-$8,896.75

VaR of $100,000 Over a One Year Investment Horizon		
Asset	VaR 1%	VaR 5%
S&P 500 index: vfinx	-$33,835.65	-$25,202.22
European stock index: veurx	-$41,720.53	-$31,252.55
Emerging markets fund: veiex	-$43,735.79	-$31,153.27
Long-term bond fund: vbltx	-$15,256.86	-$9,182.08
Short-term bond fund: vbisx	-$10.28	$1,486.29
Pacific stock index: vpacx	-$36,208.38	-$26,978.12

The short term bond (vbisx) has the lowest value at risk, while the emerging markets fund (veiex) has the highest value at risk.

2.2. Bootstrapped Standard Errors and 95% Confidence Intervals for 5% Value at Risk

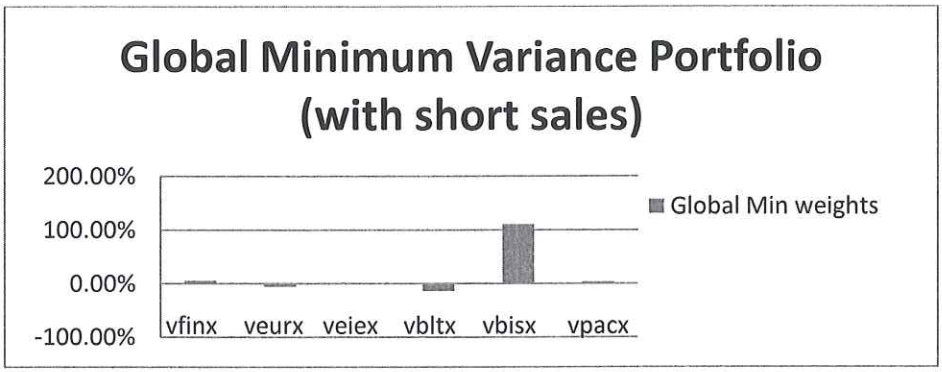
5% VaR of $100,000 Over a One Month Investment Horizon		
Asset	Bootstrapped SE	95% Confidence Interval
S&P 500 index: vfinx	$1,343.00	(-$10912, -$5645)
European stock index: veurx	$1,748.00	(-$14268, -$7416)
Emerging markets fund: veiex	$2,322.00	(-$17074, -$7970)
Long-term bond fund: vbltx	$691.90	(-$5615, -$2903)
Short-term bond fund: vbisx	$132.60	(-$880.1, -$361.2)
Pacific stock index: vpacx	$1,421.00	(-$11859, -$6288)

Since the estimated 5% VaR is within the confidence interval for all six assets, the 5% VaR for all assets are statistically significant and are reliable measures.

4. Portfolio Theory

4.1. Global Minimum Variance Portfolio with Short Sale

Global Minimum Variance Portfolio (with short sale)	
Asset	Weights
S&P 500 index: vfinx	5.73%
European stock index: veurx	-5.80%
Emerging markets fund: veiex	-0.19%
Long-term bond fund: vbltx	-14.25%
Short-term bond fund: vbisx	110.39%
Pacific stock index: vpacx	4.13%

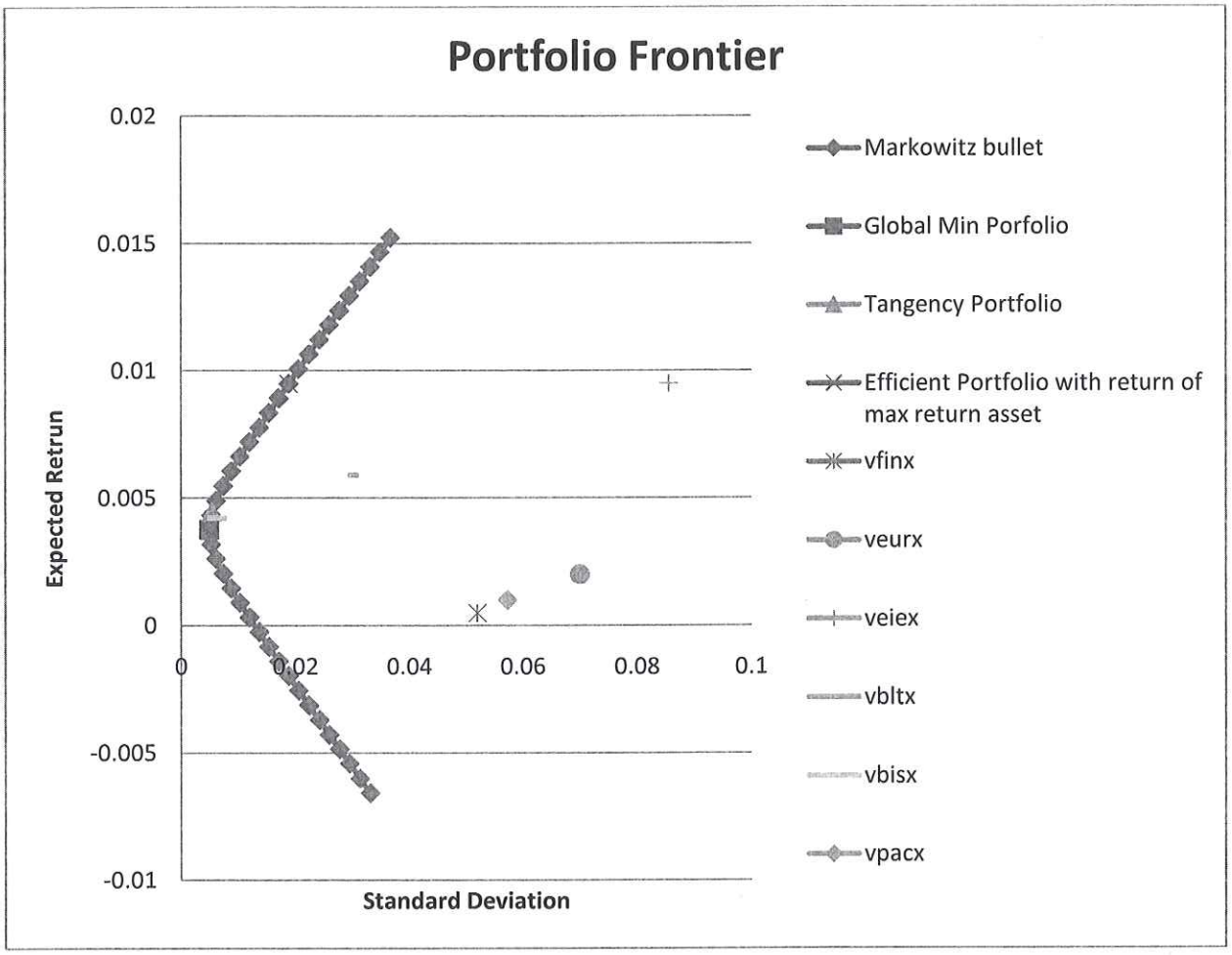


Global Minimum Variance Portfolio (with short sale)		
	Monthly Returns	Annualized Returns
E[Rp]	0.3748%	4.4980%
Var[Rp]	0.0000	0.0003
SD[Rp]	0.4876%	1.6891%
Sharpe[Rp]	0.6833	2.6383
VaR 5%	-$426.29	$1,734.55
VaR 1%	-$756.62	$570.18

Since there are negative weights in the portfolio, it means this global minimum variance portfolio was constructed by shorting mutual funds. Since mutual funds cannot be shorted, this global min portfolio that allows for short sales is not replicable.

Comparing the annualized return statistics with those of the six assets, the annual mean for this global minimum variance portfolio is within the range of expected returns of the six assets, but the annual standard deviation of this global minimum variance portfolio is lower than any standard deviation of any individual assets. The VaR is much smaller (in absolute value) for the global minimum variance portfolio than any individual asset, implying this portfolio has "safer" worse-case-scenario than holding any one asset.

4.3. Markowitz Bullet



This Markowitz Bullet is constructed with linear combinations of the global minimum variance portfolio and an efficient portfolio that minimizes variance subject to having the expected return of the asset with the highest expected return. Both efficient portfolios used to construct this Markowitz Bullet used short sales.

Efficient portfolio (min variance subject to return of max return asset)				
Asset	**Weights**		**Monthly Returns**	**Annualized Returns**
S&P 500 index: vfinx	-19.54%	**E[Rp]**	0.9486%	11.3835%
European stock index: veurx	-5.86%	**Var[Rp]**	0.0004	0.0042
Emerging markets fund: veiex	49.57%	**SD[Rp]**	1.8793%	6.5100%
Long-term bond fund: vbltx	7.02%	**Sharpe[Rp]**	0.4826	1.7422
Short-term bond fund: vbisx	120.59%	**VaR 5%**	-$2,119.69	$677.89
Pacific stock index: vpacx	-51.78%	**VaR 1%**	-$3,365.26	-$3,691.05

5. Asset Allocation

The efficient portfolio below achieves a target expected return of 8% per year (which corresponds 0.67% per month) using only the six risky assets with no short sales.

Efficient portfolio (min variance subject to 8% annual return)	
Asset	Weights
S&P 500 index: vfinx	0.00%
European stock index: veurx	23.85%
Emerging markets fund: veiex	70.92%
Long-term bond fund: vbltx	5.22%
Short-term bond fund: vbisx	0.00%
Pacific stock index: vpacx	0.00%

Efficient portfolio (min variance subject to 8% annual return)		
	Monthly Returns	**Annualized Returns**
E[Rp]	0.6667%	8.0000%
Var[Rp]	0.0011	0.0129
SD[Rp]	3.2840%	11.3760%
Sharpe[Rp]	0.1903	0.6996
VaR 5%	-$4,624.64	-$10,158.13
VaR 1%	-$6,735.45	-$16,860.16

The efficient portfolio below achieves a target expected return of 8% per year (0.67% per month) using a combination of T-Bills and the no-short-sale tangency portfolio.

Efficient portfolio (min variance subject to 8% annual return)	
Asset	
T-bills: riskless asset	-62.84%
S&P 500 index: vfinx	0.00%
European stock index: veurx	0.00%
Emerging markets fund: veiex	1.15%
Long-term bond fund: vbltx	0.00%
Short-term bond fund: vbisx	161.68%
Pacific stock index: vpacx	0.00%

Efficient portfolio (min variance subject to 8% annual return)		
	Monthly Returns	**Annualized Returns**
E[Rp]	0.6667%	8.0000%
Var[Rp]	0.0001	0.0013
SD[Rp]	1.0207%	3.5359%
Sharpe[Rp]	0.6123	2.2507
VaR 5%	-$1,007.18	$2,207.95
VaR 1%	-$1,693.41	-$225.53

When we introduce the risk free asset, we can achieve the target portfolio return with a lower variance. As such, the value at risk is lower in absolute value when holding a riskless asset.