University of Washington Fall 2007
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Economics 424

Midterm Exam Solutions

This is a closed book and closed note exam. However, you are allowed one page of notes (double-sided). Answer all questions and write all answers in a blue book or on separate sheets of paper. Time limit is 1 hours and 50 minutes. Total points = 110.

I. Return Calculations (25 pts, 5 points each)

1. Consider a one year investment in two Northwest stocks: Amazon and Costco. Suppose you buy Amazon and Costco at the end of September 2006 at $P_{A,t-1} = \$32.12,\ P_{C,t-1} = \$49.19$ and then sell at the end of September 2007 for $P_{A,t} = \$93.15,\ P_{C,t} = \$61.37$. (Note: these are actual closing prices taken from Yahoo! The data for Amazon is not a mistake.)

```
> pa.1 = 32.12
> pa.2 = 93.15
> pc.1 = 49.19
> pc.2 = 61.37
```

a. What are the simple annual returns for the two stocks?

```
> ra = (pa.2 - pa.1)/pa.1
> rc = (pc.2 - pc.1)/pc.1
> ra
[1] 1.9
> rc
[1] 0.2476
```

b. What are the continuously compounded annual returns for the two stocks?

```
> log(1 + ra)
[1] 1.065
> log(1 + rc)
[1] 0.2212
```

c. Costco paid the following per share cash dividends between September 2006 and September 2007: \$0.13 in November, \$0.13 in February, \$0.145 in April, and \$0.145 in July. What is the annual simple total return on Costco? What is the annual dividend yield?

```
> rc.total = (pc.2 + 0.13 + 0.13 + 0.145 + 0.145 -
pc.1)/pc.1
> div.y = (0.13 + 0.13 + 0.145 + 0.145)/pc.1
> rc.total
[1] 0.2588
> div.y
[1] 0.01118
> # total return = cap gain + div yeild
rc + div.y
[1] 0.2588
```

d. The annual inflation rate between September 2006 and September 2007 was about 3%. Using this information, determine the simple and continuously compounded real annual returns on Amazon and Costco. Note: for Costco, do not include the dividend adjustments.

```
> inflat = 0.03
> # simple real returns
ra.real = (1 + ra)/(1 + inflat) - 1
> rc.real = (1 + rc)/(1 + inflat) - 1
> ra.real
[1] 1.816
> rc.real
[1] 0.2113
> # cc real returns
log(1 + ra.real)
[1] 1.035
> log(1 + rc.real)
[1] 0.1917
```

e. At the end of September, 2006, suppose you have $100,000 to invest in Amazon and Costco over the next year. Suppose you sell short $60,000 in Costco and use the proceeds to buy $160,000 in Amazon. Using the results from part a, compute the annual simple return on the portfolio. Assume that both stocks do not pay a dividend.

```
> xc = -60000/100000
> xa = 160000/100000
> xa
[1] 1.6
> xc
[1] -0.6
> rp = xa * ra + xc * rc
> rp
[1] 2.892
```

II. Probability Theory (30 points, 5 points each)

1. Suppose you currently hold $2M (million) in Starbucks stock. That is, your initial wealth at the beginning of the month is $W_0 = \$2M$. Let R_{sbux} denote the monthly *simple* return on Starbucks stock, and assume that $R_{sbux} \sim N(0.03, (0.20)^2)$. Let $W_1 = W_0(1 + R_{SBUX})$ be a random variable representing your wealth at the end of the month.

a) Compute $E[W_1]$, $\mathrm{var}(W_1)$ and $SD(W_1)$

```
> w0 = 2
> e.rsbux = 0.03
> sd.rsbux = 0.2
> e.w = w0 + w0 * e.rsbux
> e.w
[1] 2.06
> var.w = w0 * (w0 * sd.rsbux * sd.rsbux)
> var.w
[1] 0.16
> sd.w = w0 * sd.rsbux
> sd.w
[1] 0.4
```

b) What is the probability distribution of W_1? Sketch the distribution, indicating the location of $E[W_1]$ and $E[W_1] \pm 2 \cdot SD(W_1)$.

Since R is normally distributed and W_1 is a linear function of R, W_1 is also normally distributed with mean $2.06M and SD $0.4M.

c) Briefly explain why the normal distribution may not be appropriate for describing the distribution of simple returns.

The normal distribution is defined from $-\infty$ to ∞ . Simple returns cannot be smaller than -1. Also, multi-period

```
simple returns are multiplicative (geometric average). That
is, the 2 period return is a geometric average
(multiplicative) of two 1 period returns. If the 1 period
returns are normally distributed then the 2 period return
will not be normal.
```

2. Let r_t denote the continuously compounded return on some asset in month t and assume

$$r_t \sim iid\ N(0.02, (0.10)^2)$$

Suppose you currently hold \$2M (million) in the asset. That is, your initial wealth at the beginning of the month is $W_0 = \$2M$.

a. Compute the 1% and 5% value-at-risk (VaR) associated with a one-month investment in the asset. Hint: the 1% and 5% quantiles of the standard normal distribution are -2.326 and -1.645.

```
> mu = 0.02
> sd = 0.1
> q.05 = mu + sd * (-1.645)
> q.01 = mu + sd * (-2.326)
> VaR.05 = (exp(q.05) - 1) * w0
> VaR.05
[1] -0.2691
> VaR.01 = (exp(q.01) - 1) * w0
> VaR.01
[1] -0.383
```

b. Now consider a 12-month (one year) investment. Let $r_t(12)$ denote the 12-month continuously compounded return. What are the mean and standard deviation of the 12-month return?

```
> mu.a = 12 * mu
> sd.a = sqrt(12) * sd
> mu.a
[1] 0.24
> sd.a
[1] 0.3464
```

c. Compute the 1% and 5% value-at-risk on a 12-month investment with initial wealth of \$2M.

```
> q.05 = mu.a + sd.a * (-1.645)
> q.01 = mu.a + sd.a * (-2.326)
```

```
> VaR.05 = (exp(q.05) - 1) * w0
> VaR.05
[1] -0.5619
> VaR.01 = (exp(q.01) - 1) * w0
> VaR.01
[1] -0.8641
```

III. Time Series Concepts (15 points)

1. Let $\{Y_t\}$ represent a stochastic process. Under what conditions is $\{Y_t\}$ covariance stationary? (5 points)

$$E[Y_t] = \mu$$
$$\mathrm{var}(Y_t) = \sigma^2$$
$$\mathrm{cov}(Y_t, Y_{t-j}) = \gamma_j \text{ (depends on } j \text{ and not } t\text{)}$$

2. Consider the random walk model

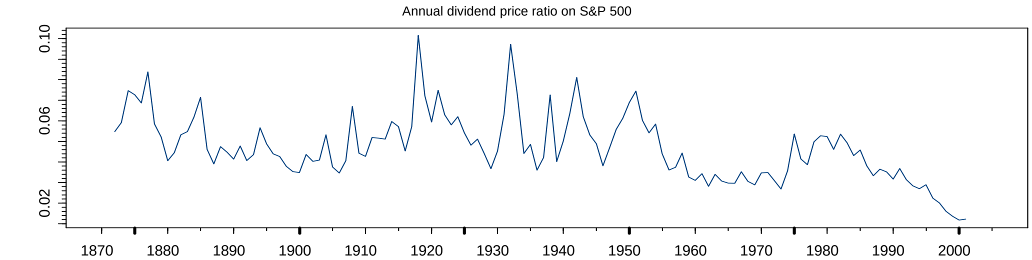
$$Y_t = Y_{t-1} + \varepsilon_t = Y_0 + \sum_{j=1}^{t} \varepsilon_j,\ Y_0 = \text{constant.}$$
$$\varepsilon_t \sim iid\ N(0, \sigma^2)$$

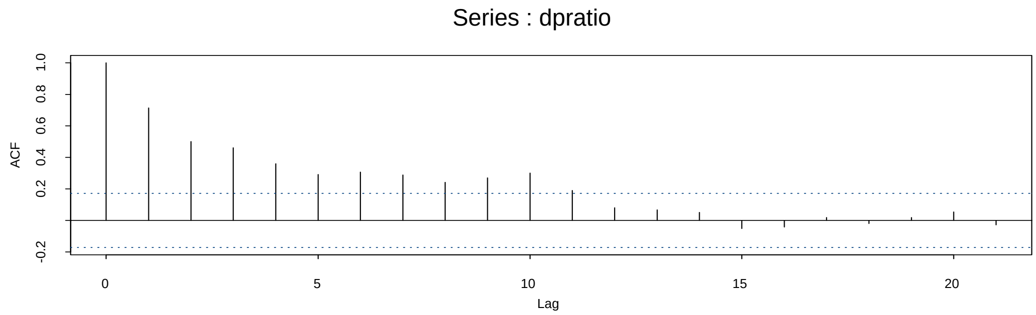
Is $\{Y_t\}$ a covariance stationary stochastic process? Why or why not? (5 points)

```
No. The random walk process is not stationary. The variance
of the random walk process depends on time:
```

$$\mathrm{var}(Y_t) = \sum_{j=1}^{t} \mathrm{var}(\varepsilon_j) = \sum_{j=1}^{t} \sigma^2 = t\sigma^2$$

3. The figure below shows annual observations on the dividend yield of the S&P 500 index over the period 1871 through 2000 along with the sample ACF. (5 points)





a) Assume the dividend yield is covariance stationary. Based on the shape of the sample autocorrelation function, would an MA(1) process or an AR(1) process best describe the data? Briefly justify your answer.

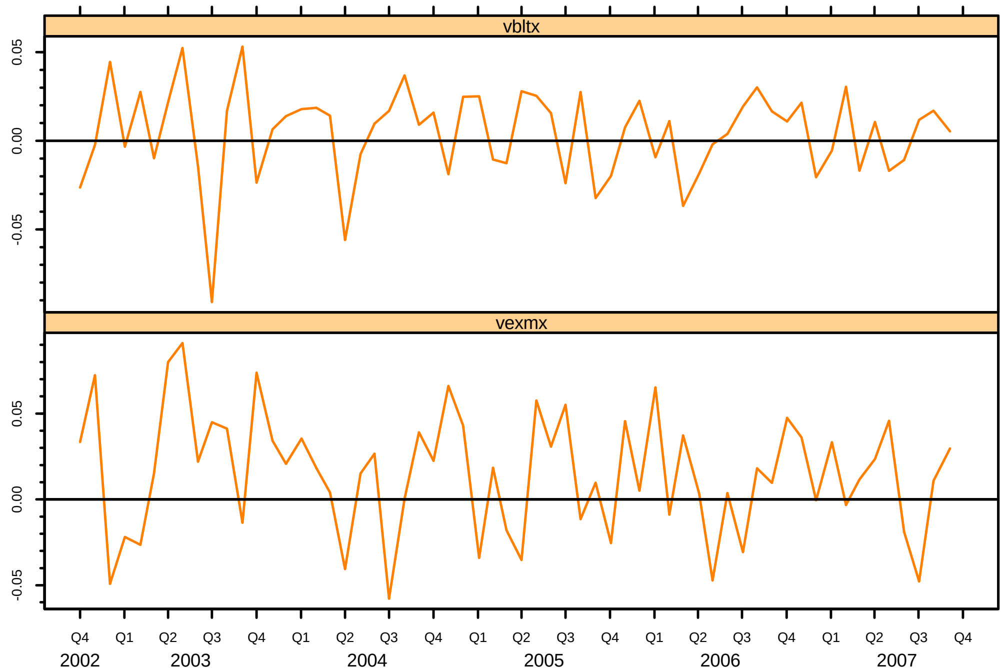
```
The SACF decays toward zero and does not cut off at lag 1.
Therefore, it looks more like an AR(1) process than an
MA(1) process.
```

VI. Constant Expected Return Model (40 points, 5 points each)

Consider the constant expected return model

$$r_{it} = \mu_i + \varepsilon_{it},\ \varepsilon_{it} \sim iid\ N(0, \sigma_i^2)$$
$$\mathrm{cov}(r_{it}, r_{jt}) = \sigma_{ij},\ corr(r_{it}, r_{jt}) = \rho_{ij}$$

for the *monthly* continuously compounded returns on the Vanguard extended market index (vexmx) and the Vanguard long-term bond index (vbltx) (subset of class project data) over the period September 2002 through September 2007. For this period there are *T*=60 monthly observations. The data are shown in the graph below.



a) Do the monthly continuously compounded return data look like they come from the CER model? Why or why not?

The CER model postulates that cc returns are (covariance stationary) iid normal random variables with constant means, variances and covariances (correlations). The above two return series look a bit like computer simulations from the CER model. The returns appear to fluctuate randomly about a constant mean. However, the volatility appears to be slightly higher before 2004 than after 2004 suggesting that the variances of the returns are not constant through time.

b) What are the estimators (formulas used to compute estimates) for μ_i, σ_i^2, and σ_{ij}?

$$\hat{\mu}_i = \frac{1}{T}\sum_{t=1}^{T} r_t,\ \hat{\sigma}_i^2 = \frac{1}{T-1}\sum_{t=1}^{T}(r_t - \hat{\mu})^2$$

$$\hat{\sigma}_{ij} = \frac{1}{T}\sum_{t=1}^{T}(r_{it} - \hat{\mu}_i)(r_{jt} - \hat{\mu}_j)$$

c) The following S-PLUS output gives the estimates of $\mu_i, \sigma_i,$ and ρ_{ij} for vexmx and vbltx from the 5 years of monthly data.

```
       muhat.vals sigmahat.vals
vexmx 0.0151      0.0349
vbltx 0.0042      0.0253

> rhohat.vals
 vexmx,vbltx
  -0.1237
```

Briefly discuss these estimates in light of what we have learned about the CER model so far.

The monthly mean estimates are positive with the stock fund around 1.5% and the bond fund around 0.4% per month. The annualized average returns are about 18% for vexmx and 5% for vbltx. Over this period an investment in stocks did much better than an investment in long-term bonds. The SD estimates are also fairly small, with the SD of vbltx smaller than vexmx. The annualized SD values are about 12% for vexmx and about 9% for bonds. This is expected since bonds are generally thought to be safer than stocks. Since vexmx is a well diversified portfolio, its small SD relative to individual stocks is due to the diversification of risk. Interesting, the correlation between vbltx and vexmx is negative. Typically, stock and bond returns are slightly negatively correlated or uncorrelated. This is one reason to hold both stocks and bonds in your portfolio.

d) Using the above output, compute estimated standard errors for $\hat{\mu}_i,\ \hat{\sigma}_i,\ (i = vexmx, vbltx)$ and $\hat{\rho}_{vexmx,vbltx}$. Briefly comment on the precision of the estimates.

```
> nobs = 60
> se.muhat = sigmahat.vals/sqrt(nobs)
> se.sigma = sigmahat.vals/sqrt(2 * nobs)
> se.rho = (1 - rhohat.vals^2)/sqrt(nobs)
> se.muhat
    vexmx    vbltx
 0.004516 0.003268
> se.sigma
    vexmx    vbltx
 0.003193 0.002311
```

```
> se.rho
 vexmx,vbltx
      0.1271
```

For both vexmx and vbltx the SE values for the mean are fairly small, around 0.45% for vexmx and 0.33% for vbltx. For vexmx the SE value for the mean estimate is about 1/3 the size of the mean estimate whereas for vbltx the SE values is almost as big as the mean estimate. However, the mean estimate for vbltx is closer to zero than the mean estimate for vexmx. The SE value for the estimated correlation is the same size of the estimated correlation. This implies a fairly imprecise estimate of correlation.

e) For vexmx and vbltx, compute 95% confidence intervals for μ. Also, compute a 95% confidence interval for $\rho_{vexmx,vbltx}$. Briefly comment on the precision of the estimates. In particular, note if both positive and negative values are in the respective confidence intervals.

Since T = 60, we can use our rule-of-thumb for calculating a 95% confidence interval:

```
> # 95% ci for mu
upper = muhat.vals + 2 * se.muhat
> lower = muhat.vals - 2 * se.muhat
> cbind(lower, upper)
          lower   upper
vexmx  0.006046 0.02411
vbltx -0.002351 0.01072
```

Notice that the 95% confidence intervals for μ are fairly wide. For vexmx, μ could be as low as 0.6% or as high as 2.4%. Annualized, this range is (7.3%, 29%). For vbltx, μ could be either positive or negative. Hence, μ is not estimated very precisely for both assets.

```
> upper = rhohat.vals + 2 * se.rho
> lower = rhohat.vals - 2 * se.rho
> cbind(lower, upper)
             lower  upper
vexmx,fmagx -0.378 0.1305
```

Here, the 95% confidence interval for ρ contains both positive and negative values. We know that the correlation is not too big but we are not sure about its sign.

f) Test the following hypotheses using a 5% significance level:

$H_0 : \mu_{vexmx} = 0$ vs. $H_1 : \mu_{vexmx} \neq 0$;
$H_0 : \mu_{vbltx} = 0$ vs. $H_1 : \mu_{vbltx} \neq 0$
$H_0 : \rho_{vexmx,vbltx} = 0$ vs. $H_1 : \rho_{vexmx,vbltx} \neq 0$.

Here, we can test hypotheses in two ways: (1) use t-statistics; (2) use 95% confidence intervals. Using t-statistics we have

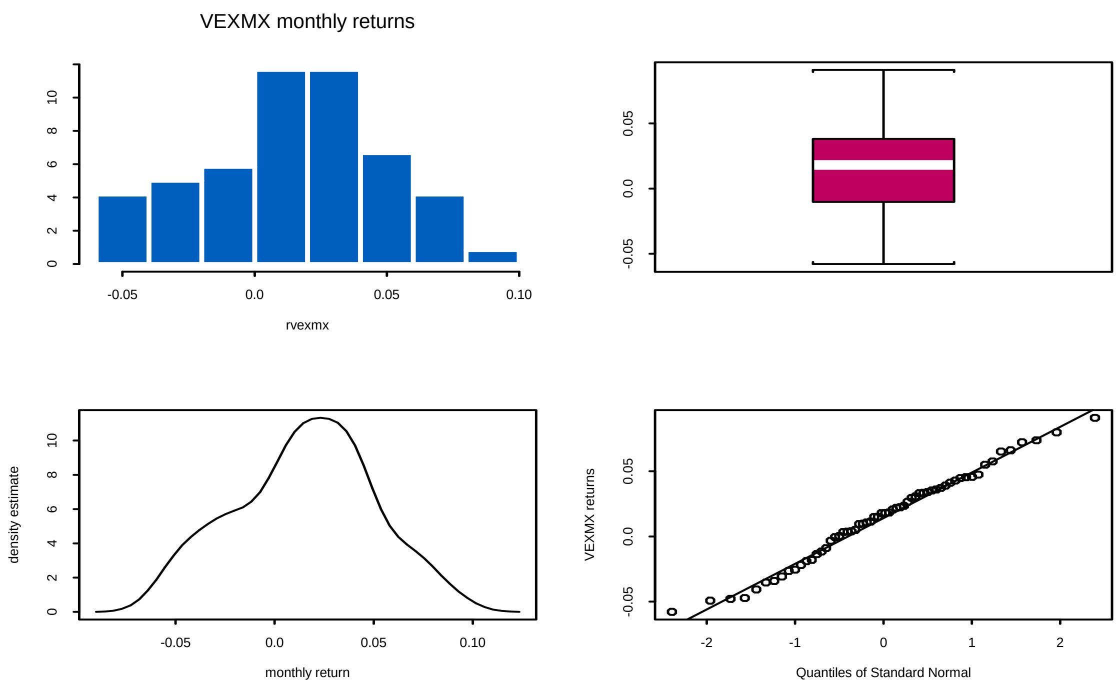
```
> t.stat.mu0 = muhat.vals/se.muhat
> t.stat.rho0 = rhohat.vals/se.rho
> abs(t.stat.mu0)
 vexmx vbltx
 3.339 1.281
> abs(t.stat.rho0)
 vexmx,vbltx
      0.9736
```

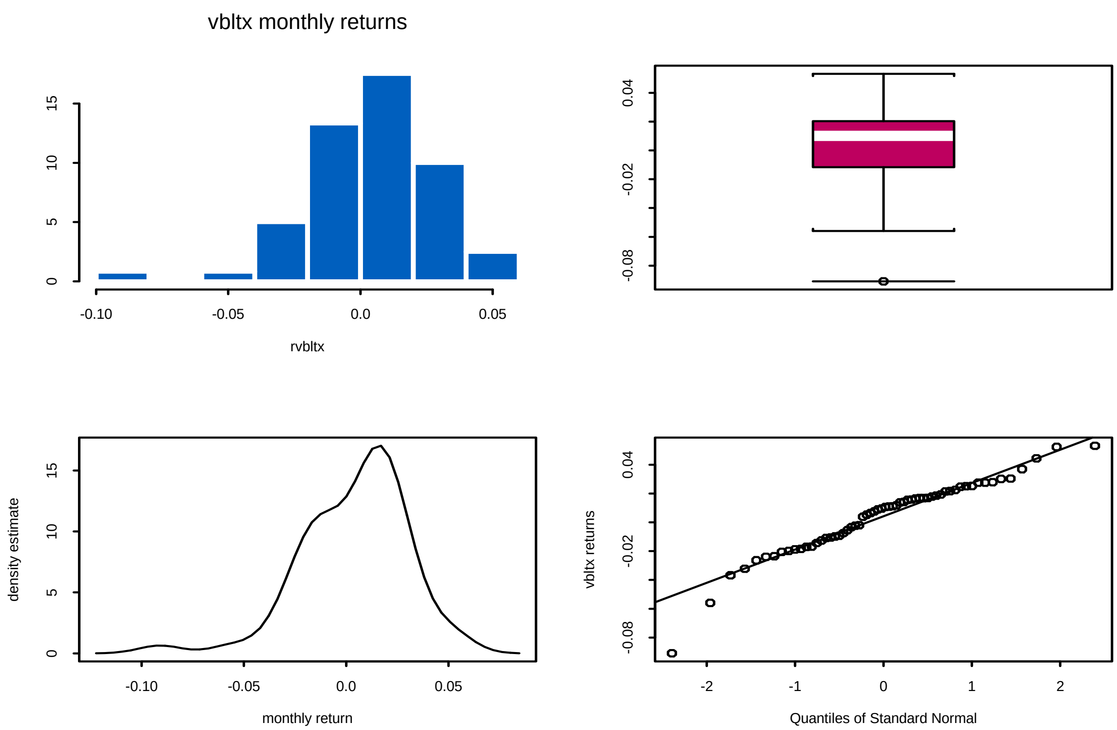
Since T=60, our rule-of-thumb decision rule is: reject the null that the true value is zero at the 5% level if the absolute value of the t-statistic is greater than 2. We reject the null only for vexmx.

Using 95% confidence intervals, our decision rule is: reject the null that the true value is zero at the 5% level if zero is not in the 95% confidence interval. From the previous question, we see that zero is not in the 95% confidence interval only for vexmx.

g) The figures below gives some graphical diagnostics of the return distributions for vexmx and vbltx. Also, estimated values of the skewness and excess kurtosis for vexmx and vbltx are

	vexmx	vbltx
skewness	-0.1111852	-0.9670961
excess kurtosis	-0.4891066	2.4082389





Based on this information, do you think the monthly cc returns on vexmx and vbltx are normally distributed? Briefly justify your answer.

For vexmx, the graphical diagnostics are consistent with a normal distribution: the histogram is bell shaped, the box plot is symmetric with no outliers and the qq-plot against the normal distribution is linear. Additionally, the sample skewness and excess kurtosis values are close to zero. If we compute the JB statistics we get

```
> JB = (nobs * (vexmx.skew^2 + 0.25 * vexmx.ekurt^2))/6
> JB
[1] 0.7217
```

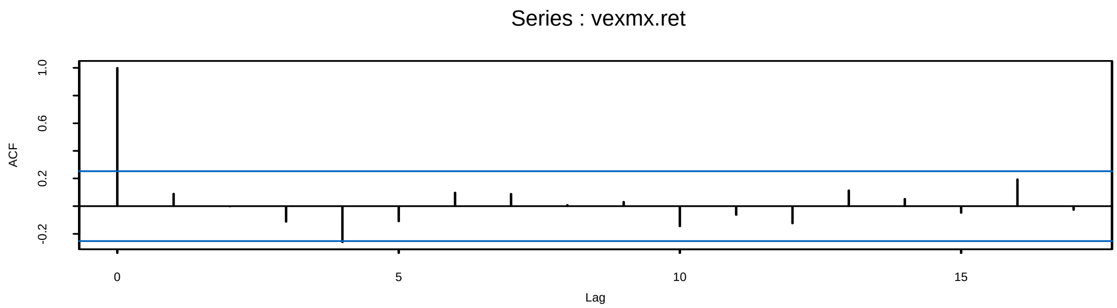
Since JB < 6, we do not reject the null hypothesis that the returns on vexmx are normally distributed.

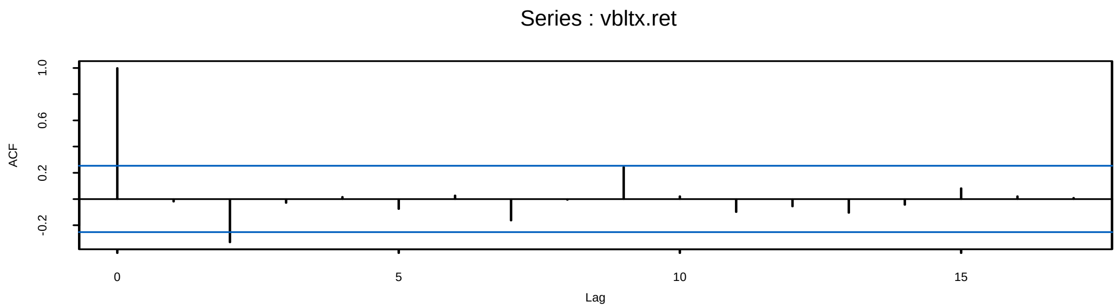
The story is different for vbltx. Here the graphical diagnostics indicate some departures from the normal distribution. This histogram is negatively skewed (long left tail), the boxplot shows one moderate outlier, and the qq-plot deviates from linearity in the lower tail. The sample skewness is fairly negative and the excess kurtosis is quite large. The JB statistic is

```
> JB = (nobs * (vbltx.skew^2 + 0.25 * vbltx.ekurt^2))/6
> JB
[1] 23.85
```

which is greater than 6 so we reject the null hypothesis at the 5% level that the returns are normally distributed.

i) Below are the sample autocorrelation functions for vexmx and vbltx. Using the information in these graphs, would you say that the CER model assumption that returns are uncorrelated over time is appropriate? Briefly justify your answer.





The SACF shows estimates of ρ_j together with error bands equal to $\pm 2/\sqrt{T}$. If the estimate of ρ_j is outside of the error band then we can reject the null hypothesis that the true ρ_j is zero at the 5% level. For both vexmx and vbltx all but one value of ρ_j are below $\pm 2/\sqrt{T}$. Also these values are fairly small. Hence, it appears that the returns are essentially uncorrelated over time.