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CFRM 462/Econ 424
Summer 2015

Econ 424
Problem Set #7
Introduction to Portfolio Theory and Portfolio Theory with Matrix Algebra
Due: Tuesday 8/11/15 at 8 pm via Canvas

Readings

- My lecture notes and class slides on introduction to portfolio theory
    - portfolioFunctions.pdf
- Ruppert, Chapter 11 (portfolio theory)
- Tutorial on using Excel's solver (on class web page)

Programs and Data

- econ424lab7.r (on homework page)
- introductionToPortfolioTheory.r
- portfolioTheoryMatrix.r
- introPortfolioTheory.xls
- 3firmExample.xls

Instructions

In this lab you will

- Compute portfolios consisting of Boeing and Microsoft, T-bills and Boeing, T-bills and Microsoft and T-bills and combinations of Boeing and Microsoft.
- Use the solver and R functions to compute the global minimum variance portfolio and the tangency portfolio
- Use R and Excel to compute efficient portfolios using matrix algebra

Excel and R Exercises – Introduction to Portfolio Theory

The following questions require Excel and the Solver add-in (see the lecture notes for the calculations using R). Organize your results in an Excel spreadsheet and submit this spreadsheet as your homework. If you don't have the Solver add-in, then you can download a version from the class announcements page. If you can't get the solver to work, then use R to answer the following questions. You will find the Excel spreadsheet `introPortfolioTheory.xls` and `introPortfolioTheory.r` to be helpful for this assignment. Use R to check your answers in Excel.

The ***annual*** estimates of the CER model parameters for Boeing and Microsoft (based on monthly data downloaded from Yahoo! And in the file lab7returns.csv on the class webpage) are given below:

```
> muhat.annual
   rboeing     rmsft
 0.1492271 0.3307664

> sigma2.annual
    rboeing    rmsft
 0.06953347 0.136935

> sigma.annual
  rboeing     rmsft
 0.263692 0.3700472

> covhat.annual
 rboeing,rmsft
 -0.0008089043

> rhohat.vals
 rboeing,rmsft
   -0.00828978
```

1. Create the following portfolios.
    - Combinations of Boeing and Msft (with x_{boeing} = -1, -0.9, …, 2 and $x_{msft} = 1 - x_{boeing}$)
    - Combinations of Boeing and T-bills (with x_{boeing} =0, 0.1, …, 2)
    - Combinations of Msft and T-bills (with x_{msft} = 0, 0.1, …, 2)

   Use an annual risk-free rate of 3% per year for the T-bill. For each portfolio compute $E[R_p]$, $var(R_p)$ and $SD(R_p)$ using the appropriate formulas. For each portfolio plot $E[R_p]$ vs. $SD(R_p)$ and put these values on the same graph. Compute Sharpe's slope for Boeing and Microsoft. Which asset has the highest slope value?

2. Compute the global minimum variance portfolio using the analytical formula presented in class (see my R function `globalMin.portfolio()`), as well as the solver.
    - Make a pie chart showing the weight of Boeing and msft in global minimum variance portfolio.

- Compute $E[R_p]$, $var(R_p)$ and $SD(R_p)$ for the global minimum variance portfolio.
- Compute Sharpe's slope for the global minimum variance portfolio.
- Indicate the location of the global minimum variance portfolio on the graph you created previously in question 1.

3. Using a risk-free rate of 3% per year for the T-bill, compute the tangency portfolio using the analytical formula presented in class (see my R function `tangency.portfolio()`) as well as the solver.
    - Make a pie chart showing the weight of Boeing and msft in the tangency portfolio.
    - Compute $E[R_p]$, $var(R_p)$ and $SD(R_p)$ for the tangency portfolio.
    - Compute Sharpe's slope for the tangency portfolio.
    - Indicate the location of the tangency portfolio on the graph you created previously in question 1.

4. Consider a portfolio that has 10% in the tangency portfolio and 90% in T-bills.
    - In this portfolio, what is the percent invested in Boeing and what is the percent invested in Microsoft? Give a pie chart showing the percent invested in T-bills, Boeing and Microsoft.
    - Compute $E[R_p]$, $var(R_p)$ and $SD(R_p)$ for this portfolio.
    - Compute Sharpe's slope for this portfolio
    - Indicate the location of the tangency portfolio on the graph you created previously in question 1.

5. Find the efficient portfolio (combination of T-bills and tangency portfolio) that has the same risk (SD) as Microsoft (Hint: you do not need the solver for this).
    - In this portfolio, what is the percent invested in Boeing and what is the percent invested in Microsoft? Give a pie chart showing the percent invested in T-bills, Boeing and Microsoft.
    - Compute $E[R_p]$, $var(R_p)$ and $SD(R_p)$ for this portfolio.
    - Compute Sharpe's slope for this portfolio
    - Indicate the location of the tangency portfolio on the graph you created previously in question 1.

Excel and R Exercises –Portfolio Theory with Matrix Algebra

Using the monthly closing price data on the four Northwest stocks (Boeing, Microsoft, Nordstrom and Starbucks) over the period February 1995 - January 2000, you will estimate expected returns, variances and covariances to be used as inputs to the Markowitz algorithm. You will compute efficient portfolios allowing for short-sales. In all cases, you will use matrix algebra formulas to simplify the running of the solver in Excel.

The script file econ424lab7.r walks you through the portfolio theory calculations using the R functions in the package **IntroCompFinR**. The Excel file 3firmExample.xls gives a spreadsheet template for the portfolio calculations. Use this spreadsheet as a template for your analysis in this part of the lab. I want you to do all of the calculations in an Excel spreadsheet. You can check your results using the R code.

Return data on Boeing, Nordstrom, Starbucks and Microsoft stock are in the Excel file **econ424lab7returns.csv** on the class webpage.

In the following exercises you will use matrix algebra formulas to simplify the computation of efficient portfolios when there are more than two risky assets.

6. Using the return data on Boeing, Nordstrom, Starbucks and Microsoft in the matrix ret.mat, estimate the parameters $\mu_i, \sigma_i^2, \sigma_i, \sigma_{ij}$ and ρ_{ij} of the constant expected return (CER) model

$$
\begin{aligned}
&R_{it} = \mu_i + \varepsilon_{it},\ t = 1,\ldots,T \\
&\varepsilon_{it} \sim iid\ N(0, \sigma_i^2) \\
&\text{cov}(\varepsilon_{it}, \varepsilon_{jt}) = \sigma_{ij}
\end{aligned}
$$

where R_{it} denotes the simple return on asset *i* (*i* = Boeing, Nordstrom, Starbucks, and Microsoft). Compute estimated standard errors for the means and volatilities and briefly comment. Arrange these estimates and standard errors nicely in a table.

7. Show the estimated risk-return tradeoff of these assets (i.e., plot the means on the y-axis and the standard deviations on the horizontal axis. Briefly comment.
   a. Assuming a risk free rate of 0.005 (0.5% per month or about 6% per year) compute the Sharpe ratios for each asset. Which asset has the highest Sharpe ratio?
8. Compute the global minimum variance portfolio allowing short-sales. The minimization problem is

$$
\begin{aligned}
&\min_m \sigma_p^2 = m'\Sigma m \text{ subject to} \\
&m'1 = 1
\end{aligned}
$$

   where ***m*** is the vector of portfolio weights and ***Σ*** is the covariance matrix. Briefly comment on the weights. Compute the expected return and standard deviation and add the points to the risk return graph.

9. Of the four stocks, determine the stock with the largest estimated expected return. Use this maximum average return as the target return for the computation of an efficient portfolio allowing for short-sales. That is, find the minimum variance portfolio that has an expected return equal to this target return. The minimization problem is

$$\min_{m} \sigma_p^2 = x'\Sigma x \text{ subject to}$$
$$x'\mu = \mu_0$$
$$x'1 = 1$$

where ***x*** is the vector of portfolio weights, ***μ*** is the vector of expected returns and μ_0 is the target expected return. Are there any negative weights in this portfolio? Compute the expected return, variance and standard deviation of this portfolio. Finally, compute the covariance between the global minimum variance portfolio and the above efficient portfolio using the formula cov($R_{p,m}$, $R_{p,x}$) = **m'Σx**.

10. Using the fact that all efficient portfolios can be written as a convex combination of two efficient portfolios, compute efficient portfolios as convex combinations of the global minimum variance portfolio and the efficient portfolio computed in question 3. That is, compute

$$z = \alpha \bullet m + (1-\alpha) \bullet x$$

for values of α between 1 and -1 (e.g., make a grid for α = 1, 0.9, ..., -1). Compute the expected return, variance and standard deviation of these portfolios.

11. Plot the Markowitz bullet based on the efficient portfolios you computed in the previous questions. On the plot, indicate the location of the minimum variance portfolio and the location of the frontier of efficient portfolio.

12. Compute the tangency portfolio assuming the risk-free rate is 0.005 (r_f = 0.5%) per month. That is, solve

$$\max_{t} \text{slope} = \frac{\mu_p - r_f}{\sigma_p} \text{ subject to}$$
$$\mu_p = t'\mu$$
$$\sigma_p = (t'\Sigma t)^{1/2}$$
$$1 = t'1$$

where ***t*** denotes the portfolio weights in the tangency portfolio. Are there any negative weights in the tangency portfolio? If so, interpret them.

13. On the graph with the Markowitz bullet, plot the efficient portfolios that are combinations of T-bills and the tangency portfolio. Indicate the location of the tangency portfolio on the graph.