University of Washington
Department of Economics

Spring 2006

Eric Zivot

Economics 424

Final Exam

This is a closed book and closed note exam. However, you are allowed one page of handwritten notes. Answer all questions and write all answers in a blue book. Total points = 100.

I. Matrix Algebra and Portfolio Math (25 points)

Let R_i denote the continuously compounded return on asset i (i = 1, …, N) with $E[R_i] = \mu_i$, var(R_i) = σ_i^2 and cov(R_i, R_j) = σ_{ij}. Define the ($N \times 1$) vectors $\boldsymbol{R} = \left(R_1, \ldots, R_N\right)'$, $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_N)'$, $\boldsymbol{m} = \left(m_1, \ldots, m_N\right)'$, $\boldsymbol{x} = \left(x_1, \ldots, x_N\right)'$, $\boldsymbol{y} = \left(y_1, \ldots, y_N\right)'$, $\boldsymbol{t} = \left(t_1, \ldots, t_N\right)'$, $\mathbf{1} = \left(1, \ldots, 1\right)'$ and the ($N \times N$) covariance matrix

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \sigma_2^2 & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{1N} & \sigma_{2N} & \cdots & \sigma_N^2 \end{pmatrix}.$$

The vectors $\boldsymbol{m}$, $\boldsymbol{x}$, and $\boldsymbol{y}$ contain portfolio weights that sum to one. Using simple matrix algebra, answer the following questions.

a. For the portfolio defined by the vector $\boldsymbol{x}$, give the expression for the portfolio return ($R_{p,x}$), the portfolio expected return ($\mu_{p,x}$), and the portfolio variance ($\sigma_{p,x}^2$).

b. Write down the optimization problem used to determine the global minimum variance portfolio assuming short sales are allowed. Let $\boldsymbol{m}$ denote the vector of portfolio weights in the global minimum variance portfolio.

c. Write down the optimization problem used to determine an efficient portfolio with target return equal to μ_0 assuming short sales are allowed. Let $\boldsymbol{x}$ denote the vector of portfolio weights in the efficient portfolio.

d. Using the results from parts b. and c., briefly describe how you can compute the efficient frontier containing only risky assets (Markowitz bullet) using just two efficient portfolios. Give the expressions for the expected return and variance for portfolios on the efficient frontier.

e. Write down the optimization problem used to determine the tangency portfolio, assuming short sales are allowed and the risk free rate is give by r_f. Let $\boldsymbol{t}$ denote the vector of portfolio weights in the tangency portfolio.

II. Efficient Portfolios (40 points)

The graph below shows the efficient frontier computed from three Vanguard mutual funds: S&P 500 index (vfinx), European stock index (veurx), and long-term bond index (vbltx).

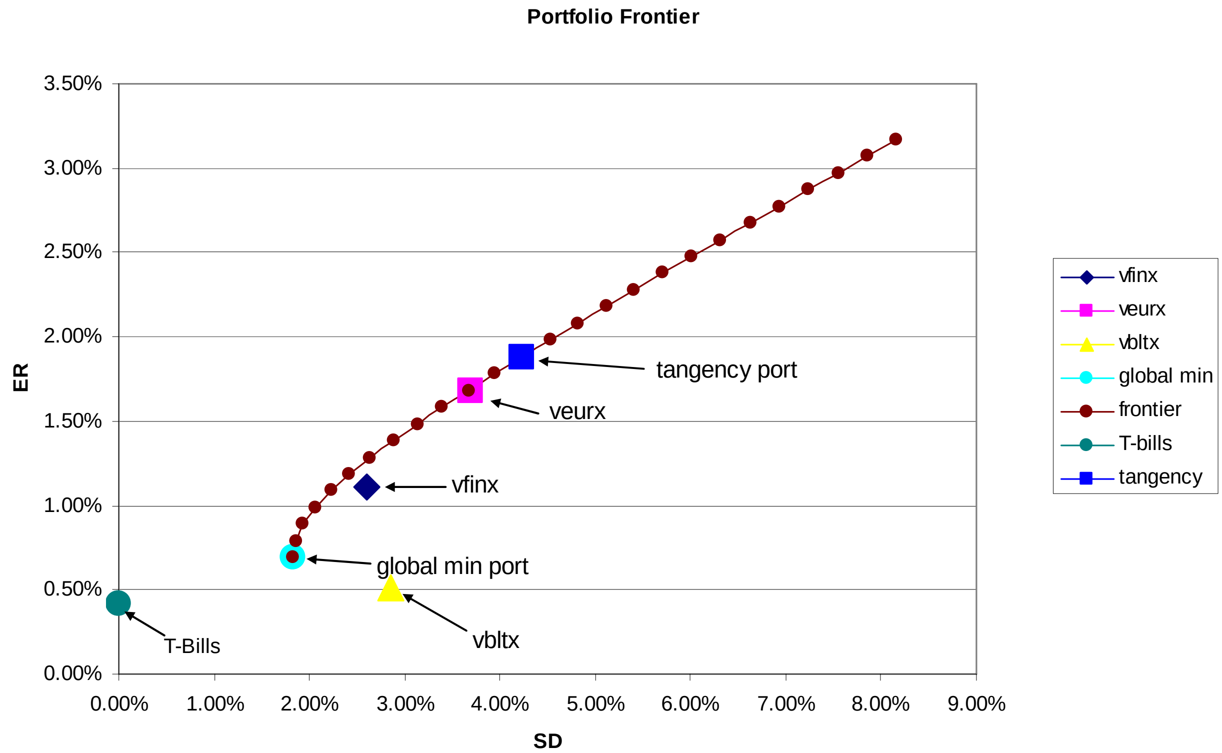


Figure 1 Markowitz Bullet

Expected return and standard deviation estimates for specific assets are summarized in the table below. These estimates are based on monthly continuously compounded return data over the three year period January 2003 – January 2006.

Table 1 Portfolio Statistics

Asset	Mean (E[R])	Standard deviation (SD(R))	Weight in Global Min Portfolio	Weight in Tangency portfolio
VFINX	1.11%	2.60%	83%	-44%
VEURX	1.68%	3.70%	-27%	139%
VBLTX	0.51%	2.85%	44%	5%
T-Bills	0.42%	0%		
Global Min Portfolio	0.69%	1.83%		
Tangency Portfolio	1.88%	4.23%		

The pair-wise scatter plots of the returns and sample correlation matrix are given below:

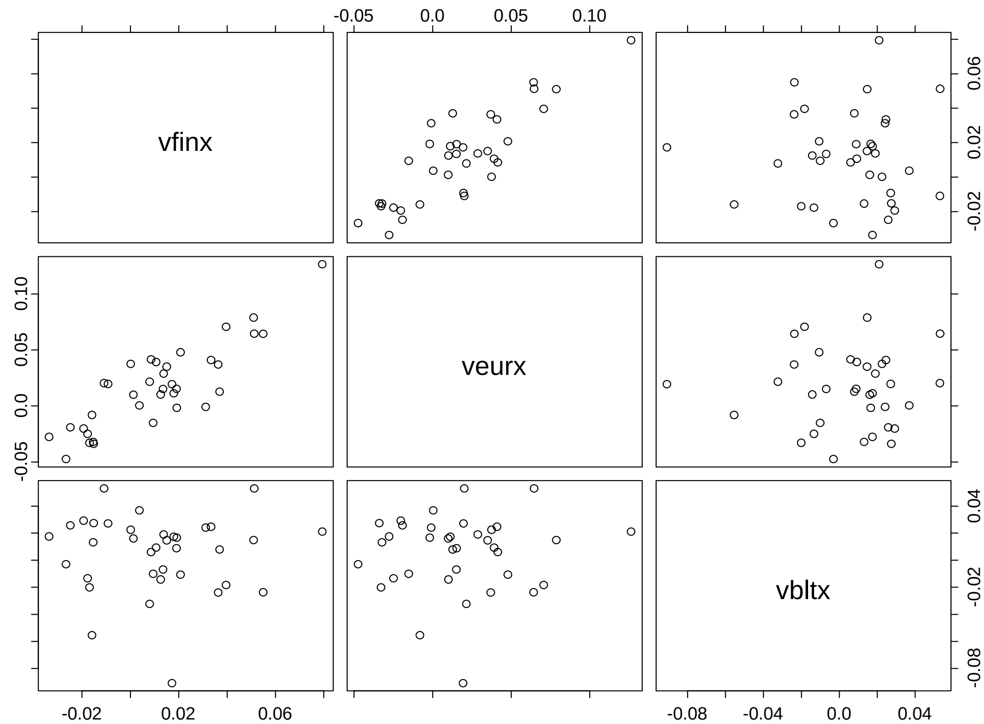


Figure 2 Return Scatter Plots

```
> print(cor.mat, digits=4)
         vfinx   veurx    vbltx
vfinx  1.00000 0.85441 -0.01635
veurx  0.85441 1.00000  0.04798
vbltx -0.01635 0.04798  1.00000
```

Using the above information, please answer the following questions.

a. Based on the pair-wise scatterplots and sample correlations, do you think there will be diversification benefits from forming portfolios of the three mutual funds? If yes, which asset should provide the largest diversification benefit? Briefly justify your answer.

b. Find the efficient portfolio of risky assets only (e.g. a portfolio on the Markowitz bullet) that has an expected monthly return equal to 1.5%. In this portfolio, how much is invested in vfinx, veurx, and vbltx? (Hint: recall, any portfolio on the Markowitz bullet can be expressed as a convex combination of any other two portfolios on the Markowitz bullet)

c. Transfer the graph of the Markowitz bullet to your blue book. On this graph indicate the efficient combinations of T-bills and risky assets. Also, indicate a portfolio that would be preferred by a very risk averse investor and a portfolio that would be preferred by a risk tolerant investor.

d. How much should be invested in T-bills and the tangency portfolio to create an efficient portfolio with the same expected return as vfinx? What is the standard deviation of this efficient portfolio? Transfer the graph of the Markowitz bullet to your blue book and indicate the location of this efficient portfolio on the graph.

e. In the efficient portfolio you found in part d, what are the shares of wealth invested in vfinx, veurx, and vbltx?

f. Assume that the continuously compounded returns for all assets are normally distributed. For an initial investment of $100,000, compute the 1% value-at-risk (VaR) on vfinx and the efficient portfolio that has the same expected return as vfinx. (Hint: the 1% quantile of a standard normal random variable is -2.33). Which portfolio has the smallest 1% VaR?

g. How much should be invested in T-bills and the tangency portfolio to create an efficient portfolio with the same standard deviation as vfinx? What is the expected return on this portfolio? Transfer the above graph to your blue book and indicate the location of this efficient portfolio on the graph.

h. The global minimum variance portfolio has a negative weight on veurx. What is the interpretation of a negative weight? Is the negative weight problematic? Why or why not?

III. Empirical Analysis of the single index model (35 points)

The single index model for asset returns has the form

$$
\begin{aligned}
&R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it} \\
&i = 1,\ldots,n \text{ assets} \\
&t = 1,\ldots,T \text{ time periods}
\end{aligned}
$$

a. What are the assumptions of the SI model?

b. What is the interpretation of ε_{it} in the SI model?

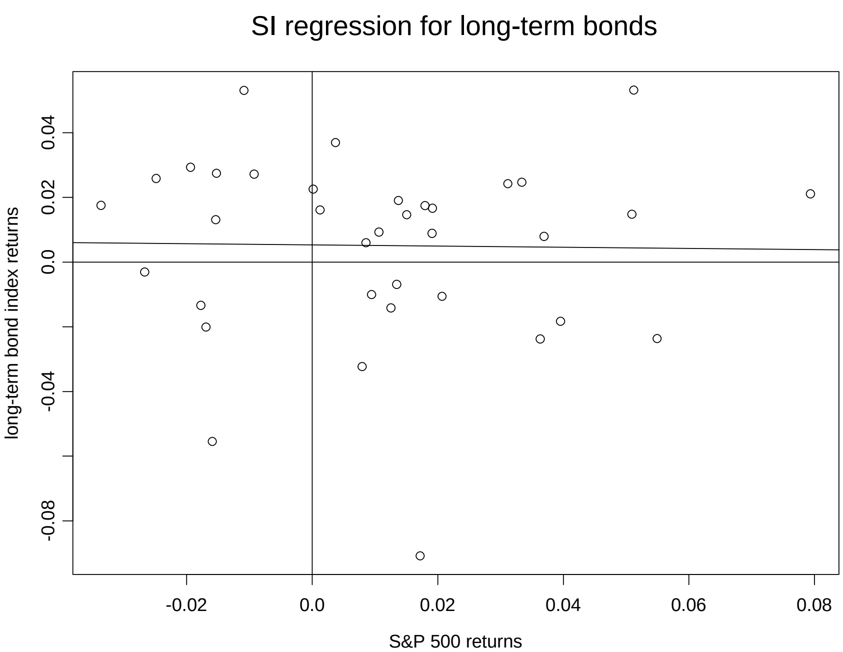
The following represents S-PLUS linear regression output from estimating the single index model for the Vanguard long-term bond index (vbltx) and the Vanguard European Equity index (veurx) using monthly continuously compounded return data over the 3 year period January 2003 –January 2006. In the regressions, the market index is the Vanguard S&P 500 index (vfinx).

```
> summary(vbltx.fit,cor=F)
```

```
Call: lm(formula = vbltx ~ vfinx, data = final.ts)
Residuals:
      Min      1Q   Median      3Q     Max
 -0.09582 -0.0164 0.008568 0.01776 0.04875

Coefficients:
              Value Std. Error t value Pr(>|t|)
(Intercept)  0.0053  0.0053     1.0080  0.3206
      vfinx -0.0179  0.1880    -0.0953  0.9246

Residual standard error: 0.02894 on 34 degrees of freedom
Multiple R-Squared: 0.0002673
F-statistic: 0.009091 on 1 and 34 degrees of freedom, the p-value
is 0.9246
```

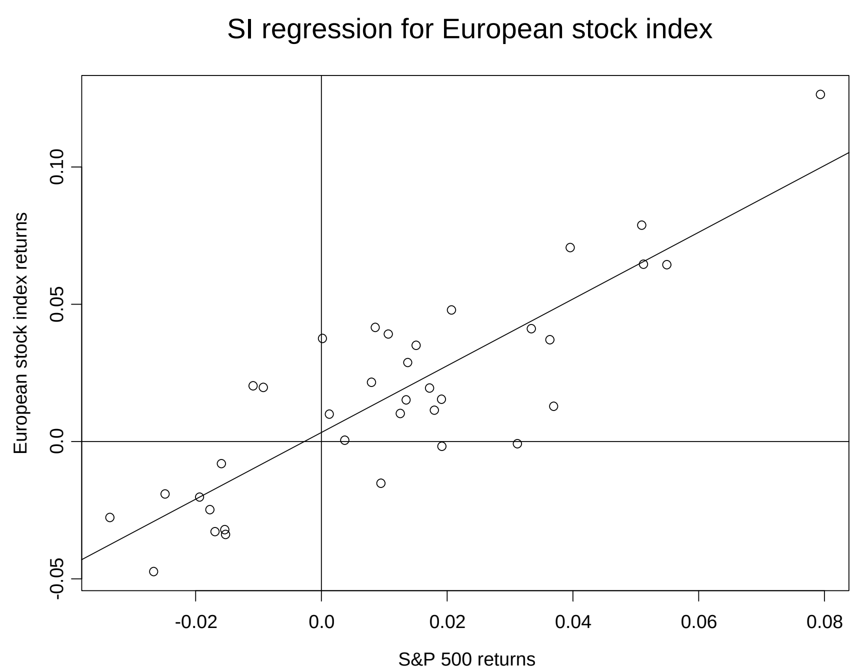


```
> summary(veurx.fit,cor=F)

Call: lm(formula = veurx ~ vfinx, data = final.ts)
Residuals:
      Min       1Q    Median     3Q     Max
 -0.04203 -0.01184 -0.001937 0.0134 0.03397

Coefficients:
             Value Std. Error t value Pr(>|t|)
(Intercept) 0.0033 0.0035     0.9411  0.3533
      vfinx 1.2155 0.1268     9.5881  0.0000

Residual standard error: 0.01952 on 34 degrees of freedom
Multiple R-Squared: 0.73
F-statistic: 91.93 on 1 and 34 degrees of freedom, the p-value is
3.387e-011
```



c. For the long-term bond index and the European equity index, what are the estimated values of α and β and what are the estimated standard errors for these estimates? Is β for the European equity index estimated more precisely than β for the long-term bond index? Why or why not?

d. Based on the estimated values of β, what can you say about the risk characteristics of the long-term bond index and the European equity index relative to the market index?

e. For each regression, what is the proportion of market or systematic risk and what is the proportion of firm specific or unsystematic risk?

f. Why should the European equity index have a greater proportion of market risk than the long-term bond index?

g. The following graphs show the 24-month rolling estimates of β for the SI models for the long-term bond index and the European equity index. Using these graphs, what can you say about the stability of β over time?