Answer Key for Review Problems on Random Variables

1. A random variable X has the following distribution:

Row 1	Possible values of X:	0	1	2	3	4
Row 2	Probabilities	.2	.1	.2	.3	.2
Row 3	Values of 3X-1:					
Row 4	Values of X^2					

a. What two sets of information are required to specify the distribution of the random variable X?
Set of possible values, row 1 and corresponding probabilities, row 2.

b. Compute E(X). $= \sum p_i x_i = 2.2$

c. Compute V(X). $= \sum p_i [(x_i - E(X))^2]$ or better $= E(x^2) - [E(X)]^2 = 1.96$

d. What is the probability distribution of the new random variable Y=3X-1. [Hint: see part a.]
Rows 2 and 3 (after you fill it in).

e. Compute E(Y) =5.6 and V(Y)=17.64.

f. Find the probability distribution of the random variable X^2 and compute $E(X^2)$=6.8.
Rows 2 and 4 (filled in)

g. What is the value of $E(9X^2$-6X+1). [Hint: the easy way is to use the results above and the simple theorems.] =49.0

h. What is the value of $E[(X-E(X))^2]$? What is another name for this expression? What does it measure? =1.96 This is V(X). It measures the dispersion of the X distribution.

2. A producer has the following cost function:

Output X	500	700	1000	1500
Average Cost AC	2	3.5	6	9

Price is a random variable with the probability distribution:

Price P	10	12	15	20
Probabilities	.5	.3	.1	.1

The producer wants to maximize expected profits. What output should he/she choose?

$$\pi = \tilde{p}x - AC(x)$$

$$E(\pi) = E(p)x - AC(x)$$

$$E(p) = 12.10$$

Expected profits are maximized at x=1000, expected profit is 6100.

3. An investor can choose between assets A1 and A2. The probability distributions of the rates of return for these assets are given below.

R1 (%)	10	15	17	20
Probabilities	.4	.3	.2	.1

and

R2 (%)	10	15	17	20
Probabilities	.3	.3	.2	.2

a. Compute the expected return for both assets. E(r1)=13.9 E(r2)=14.9)
b. Compute the variance for both assets. V(r1)=12.09 V(r2)=13.29
c. Based on the expected return vs. variance criterion, which asset would you prefer and why? Asset 2 has higher expected return but also has higher variance. There is no clear choice based on mean – variance criterion.
Do you see any other basis for preferring one asset to the other? Asset 2 would be preferred by anyone. You have the same chance of getting 15 or 17, a lower chance of getting 10, and a higher chance of getting 20. A2's distribution lies to the right of A1's. These ideas are formalized with the concept of stochastic dominance.
d. Compute the expected return and variance of a portfolio that has portfolio weights 0.4 and 0.6 and if the covariance between the two returns is zero.

$$E(r_p) = .4E(r1) + .6E(r2) = 14.5$$

$$V(r_p) = .4^2(12.09) + .6^2(13.29) + 2(.4)(.6)(0) = 6.7188$$

Note that the portfolio variance is lower than that of either asset.

4. Random variables X and Y have the following joint distribution:

X	Y	Prob(X,Y)
0	1	.4
0	-1	.3
1	0	.2
1	1	.1

a. First convert this information into a 2-way table showing the joint X,Y probability distribution.

		Y	values		
		-1	0	1	Marginal prob
X	0	.3	0	.4	.7
values	1	0	.2	.1	.3
	Marginal prob	.3	.2	.5	1.0

b. Compute E(X), E(Y), V(X), V(Y),E(XY), Cov(X,Y), and the correlation coefficient.
.3 .2 .21 .76 .1 .04 .1

5. For two jointly distributed random variables X and Y (not necessarily those of problem 4),
a. If V(X)=0 <u>or</u> V(Y)=0, what can you say about Cov(X,Y)? Explain. Covariance would be zero.
b. Show, using the definition, that Cov(aX,bY)=abCov(X,Y).
c. Show, using the definition, that Cov(X+Y,Z)= Cov(X,Z)+Cov(Y,Z).

6. To test your understanding of expected utility concepts, try the following problem.

Will be done in class

A consumer has the utility function u=U(W)=ln(W) where u represents the level of utility, U(W) represents the general form of a utility function, W is the consumer's wealth, and ln(W) represents a specific utility function that specifies that the value of the utility index associated with a given level of wealth, W, is given by the natural log (base e) of W.

The consumer's wealth is not fixed, but is a random variable given by the probability distribution:

Possible value of W:	100,000	150,000	200,000	250,000
Probability of W:	0.2	.5	0.2	0.1
Value of U(W)	11.5129255	11.9183906	12.2060727	12.4292162

a. Compute the consumer's expected wealth i.e. E(W).
E(W)=.2(100000)+...+.1(250000)=160000
b. Extend the table to show the possible values of U(W).
See
c. Compute the expected utility of wealth, i.e. .E[U(W)]
E[U(W)]=.2(ln(100000))+...+.1(ln(250000))=11.9459165
d. Prove that $E[U(W)] \neq U(E(W))$. result in part c is not equal to ln(160000)=11.9829291
e. Find the certainty equivalent wealth, that is the wealth, which if held for certain, would be regarded as equivalent in the utility sense to the random wealth distribution given above. i.e. find W_c such that $U(W_c)=E[U(W)]$.

If you had W_c for sure it would be worth $\ln(W_c)$. We showed above that the risky wealth has an expected utility value of 11.9459165. For what value of W_c does $\ln(W_c)$ have the same value. $\ln(W_c)=11.9459165$. Take the exponential of both sides to get

$$W_c=\exp\{11.9459165\}=154{,}186.24.$$