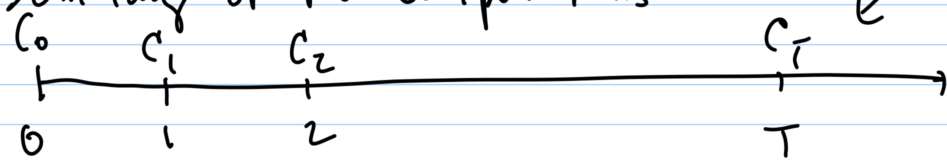
Econ 422 Lec 5

Note Title 7/28/2010

Summary of PV computations



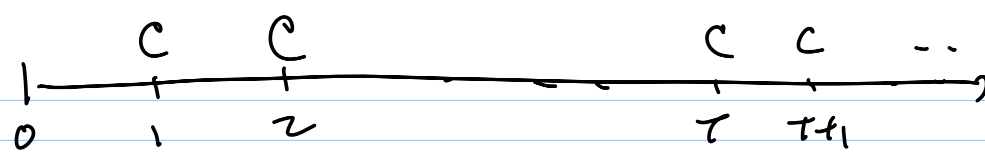
← risk free

r = annual risk free rate

$$PV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_T}{(1+r)^T}$$

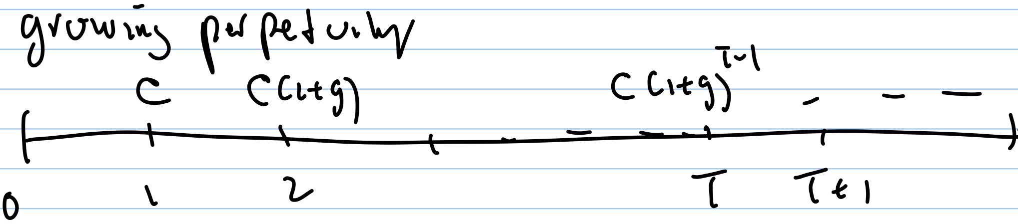
Special cases

(1) Perpetuity



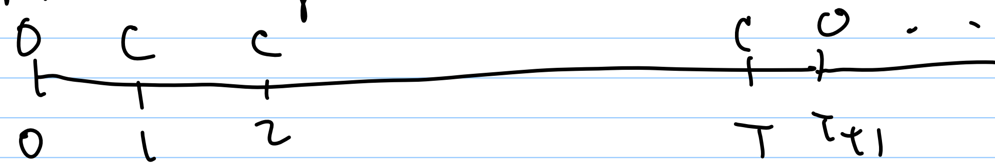
$$PV = \frac{C}{r}$$

(2) growing perpetuity



$$PV = \frac{C}{r-g}, \quad r > g$$

(3) Finite annuity

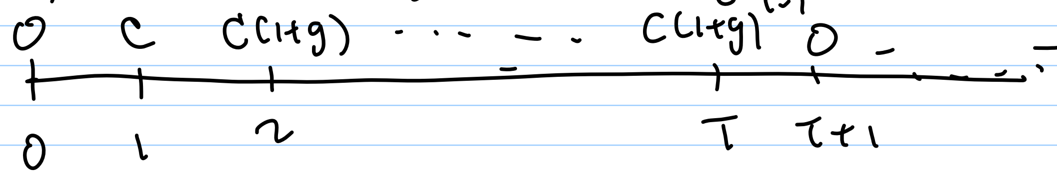


$$PV = C * PVA(r, T)$$

$$PVA(r, T) = \frac{1}{r}\left\{1 - \left(\frac{1}{1+r}\right)^T\right]$$

= PV of finite annuity that pays \$1 starting in period 1 until period T

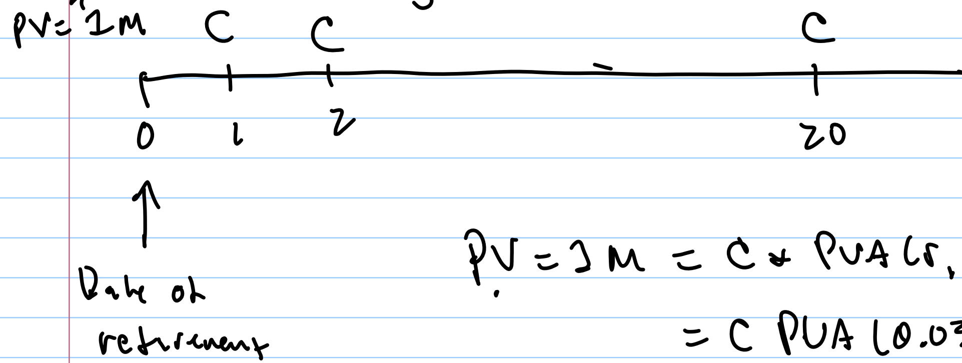
(4) PV of finite growing annuity



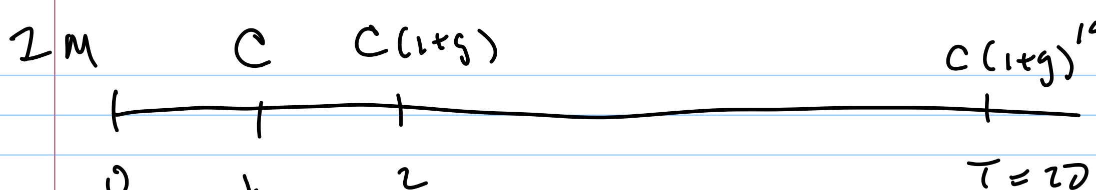
$$PV = C * PVGA(r, g, T)$$

$$PVGA(r, g, T) = \frac{1}{r-g}\left[1 - \left(\frac{1+r}{1+g}\right)^T\right]$$

Ex: Annuitizing retirement



$$PV = 1M = C * PVA(r, T)$$
$$= C\, PVA(0.03, 20)$$
$$\Rightarrow C = \frac{1M}{PVA(0.03, 20)}$$



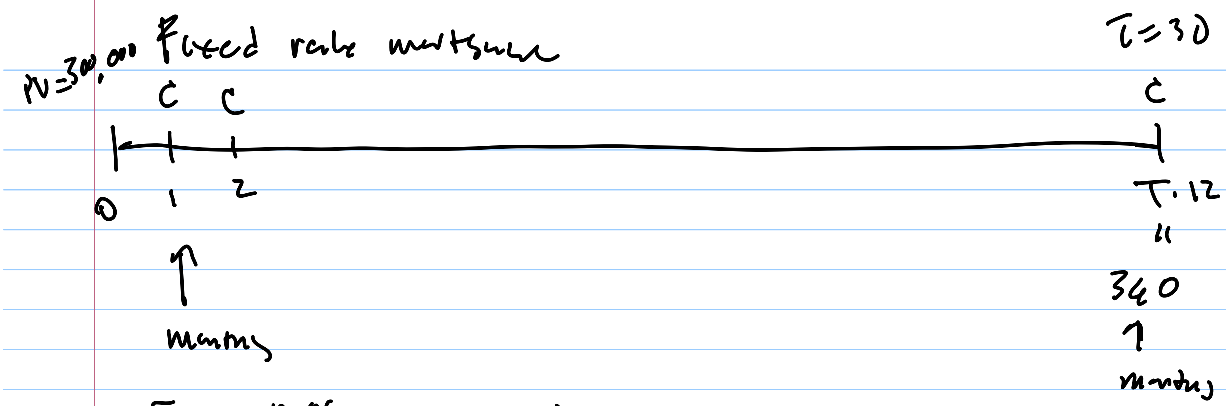
$PV = 1M$

$r = 0.03$

$g = 0.02$

$T = 20$

$$PV = 1M = C * PVGA(r, g, T)$$
$$= C * PVGA(0.03, 0.02, 20)$$
$$\Rightarrow C = \frac{1M}{PVGA(0.03, 0.02, 20)}$$



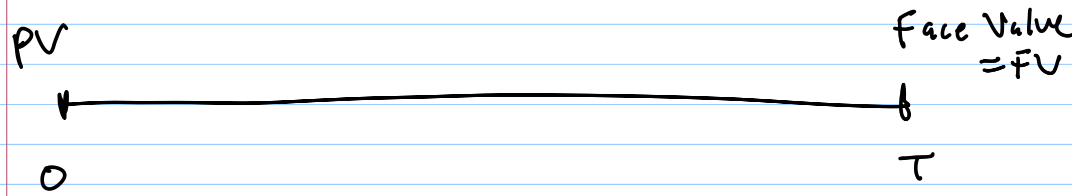
$$\frac{r}{12} = \frac{0.06}{12} = 0.0067$$

Solve: $300,000 = C \times PVA\left(\frac{r}{12}, T \times 12\right)$

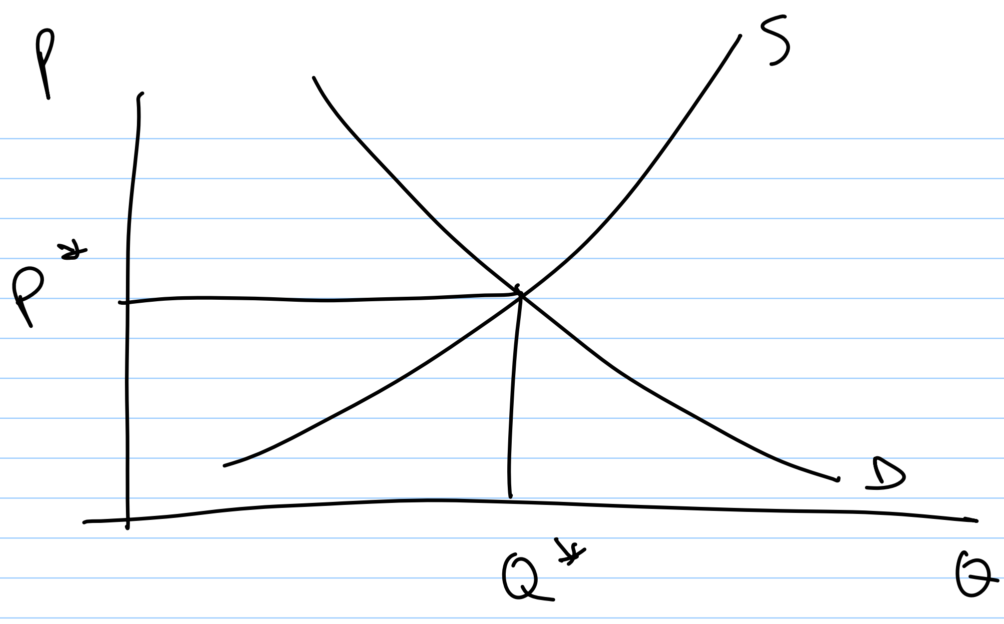
$= C \times PVA(0.0067, 360)$

Ex: A simple explanation of the credit crisis

Valuation of zero coupon bonds



$$PV = \frac{FV}{(1+r)^T}$$, r = annual risk free interest rate.

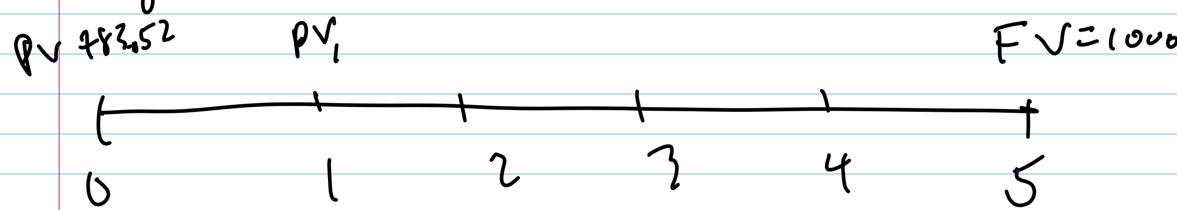


Introduction to Bond Investing

Suppose you buy a 5 yr zero coupon bond (STRIP) from US gov't.

$$PV = \frac{FV}{(1+r)^5} \quad , \; r = \text{interest rate.} = 0.05, \; FV = 1000 \Rightarrow PV = 783.52$$

If I hold the bond for 5-yrs I am guaranteed an annual interest rate of $r = 0.05$



$783.52 = PV$

$FV = 1000$

Annual rate of return $= \left(\frac{1000}{783.52}\right)^{\frac{1}{5}} - 1$

$= 0.05$

Assume $r = 0.05$ in yr 1

$$PV_1 = \frac{FV}{(1+r)^4} = \frac{1000}{(1.05)^4} = 822.7025$$

$$\text{Annual rate of return} = \frac{822.7025 - 783.52}{783.52}$$

$$= 5\% \ !$$

If $r = 0.06$ 1 year from now then

$$PV_1 = \frac{1000}{(1.06)^4} = 792.0937$$

$$< 822.7025$$

$$\text{Annual rate of return} = \frac{792.0937 - 783.52}{783.52} = 0.011$$