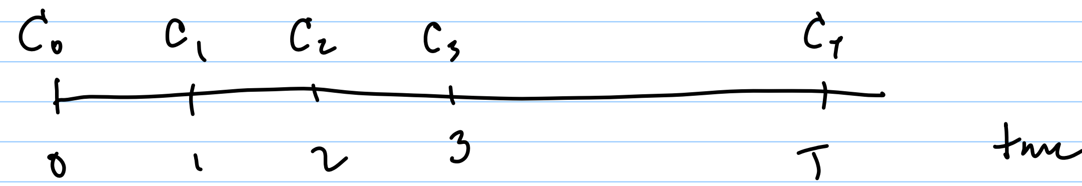
Econ 422 Lec 4

Note Title 7/27/2010

PV calculations



r = annual risk free interest rate

C_i = risk free cash flows — they occur with certainty

$$PV = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \cdots + \frac{C_T}{(1+r)^T}$$

$$= C_0 \cdot 1 + C_1 \cdot \left(\frac{1}{1+r}\right) + C_2 \cdot \left(\frac{1}{1+r}\right)^2 + \cdots + C_T\left(\frac{1}{1+r}\right)^T$$

discount factors

$\left(\frac{1}{1+r}\right)^t$ = price today of \$1 to be received in year t

ir

Review of Geometric Series

$$\sum_{t=0}^{\infty} a^t = 1 + a + a^2 + a^3 + \cdots\cdots$$

Need $|a| < 1$ for this sum to converge.

Result: If $|a| < 1$ then

$$\sum_{t=0}^{\infty} a^t = \frac{1}{1-a}$$

Pf:
$$A = \sum_{t=0}^{\infty} a^t = 1 + a + a^2 + a^3 + \cdots$$

$$a \cdot A = a + a^2 + a^3 + \cdots\cdots$$

$$A - aA = 1 + \cancel{a} + \cancel{a^2} + \cancel{a^3} \cdots\cdots - (\cancel{a} + \cancel{a^2} + \cancel{a^3} + \cdots\cdots)$$

$$= 1$$

$$A(1-a) = 1 \Rightarrow A = \frac{1}{1-a}$$

$$\Rightarrow \sum_{t=0}^{\infty} a^t = \frac{1}{1-a}$$

Perpetuity

$$PV = \frac{c}{1+r} * \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t$$

$$q = \frac{1}{1+r}, \quad \sum_{t=0}^{\infty} q^t = \frac{1}{1-q}$$

$$= \frac{1}{1 - \frac{1}{1+r}}$$

$$= \frac{1}{\frac{1+r-1}{1+r}}$$

$$= \frac{1}{\frac{r}{1+r}}$$

$$= \frac{1+r}{r}$$

$$PV = \frac{c}{1+r} \cdot \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t$$

$$= \frac{c}{\cancel{1+r}} * \frac{\cancel{1+r}}{r} = \frac{c}{r}$$

Growing Perpetuity.

$$PV = \frac{C}{1+r} \cdot \sum_{t=0}^{\infty} a^t , \quad a = \frac{1+g}{1+r}$$

If $g < r$ then $|a| < 1$ and so

$$\sum_{t=0}^{\infty} a^t = \frac{1}{1-a} = \frac{1}{1 - \frac{1+g}{1+r}}$$

$$= \frac{1}{\frac{1+r-1-g}{1+r}} = \frac{1}{\frac{r-g}{1+r}}$$

$$= \frac{1+r}{r-g}.$$

$$PV = \frac{C}{1+r} \times \frac{1+r}{r-g} = \frac{C}{r-g}.$$

PV of finite annuity

(1)

0	C	C	...	C	0	0	...
0	1	2	...	T	T+1	T+2	...

(2)

0	C	C	...	C	C	C	...
0	1	2	...	T	T+1	T+2	...

Perpetuity, with 1st payment in period 1

(3)

0	0	0	...	0	C	C	C	...
0	1	2	...	T	T+1	T+2	T+3	...

$PV_T = C/r$

Perpetuity with 1st payment in period T+1

(2) $PV = \frac{C}{r}$

(3) In period T, $PV = \frac{C}{r}$

In period 0, $PV = \dfrac{\frac{C}{r}}{(1+r)^T}$

Black cash flows = Blue Cash Flows − Purple cash flows

PV of Black cash flows (finite annuity)

= PV of blue cash flows

− PV of Purple cash flows

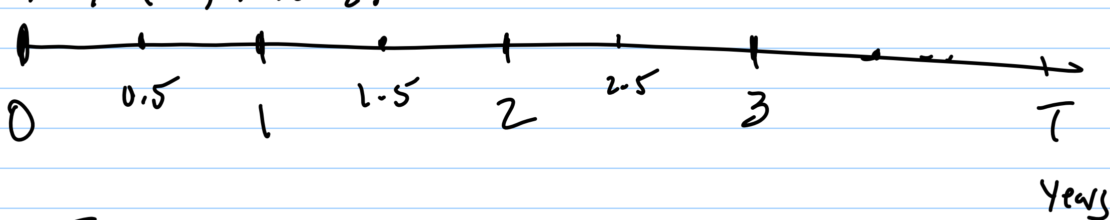
PV of finite annuity =

$$\frac{c}{r} - \frac{\frac{c}{r}}{(1+r)^T} = \frac{c}{r}\left[1 - \left(\frac{1}{1+r}\right)^T\right]$$

FV compounding n times per year

Ex $n = 2$ $\quad r$ = annual interest rate

$V_0 \rightarrow V_0(1+\frac{r}{2}) \rightarrow V_0(1+\frac{r}{2})^2$



$\frac{r}{2}$ = semi-annual interest rate

$$FV = V_0 * \left(1 + \underset{\substack{\uparrow \\ \text{6 mo interval}}}{\frac{r}{2}}\right)^{\underbrace{2 \cdot T}_{\text{\# of 6 month periods in T years}}}$$

Compounding n times per year

$$FV = V_0 * \left(1 + \frac{r}{n}\right)^{n \cdot T}$$

$\frac{r}{n}$ = periodic interest rate

$n \cdot T$ = # of compounding periods in T years.