Econ 422 Lec 21

Note Title 8/19/2010

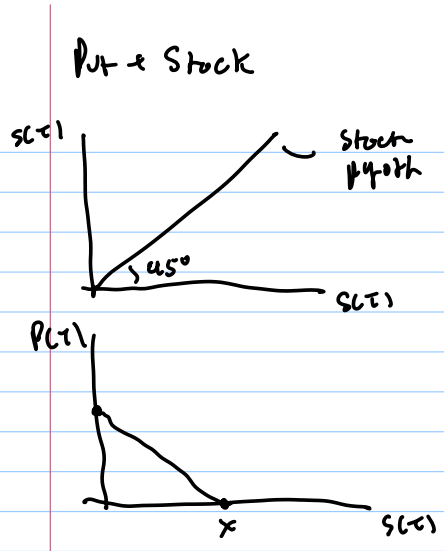
Put Call Parity

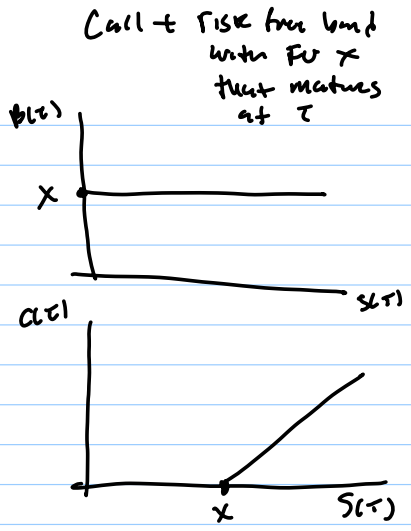
Put + Stock = Call + PV(K)

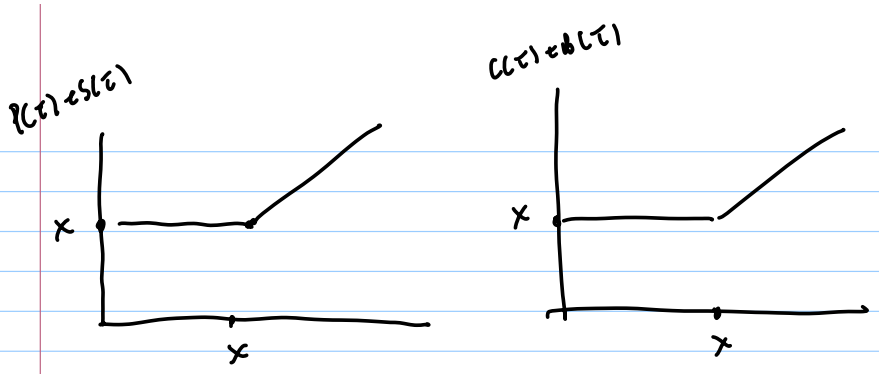
K = exercise price on both put & call

Put & Call have same maturity date

Stocks at any time prior maturity.

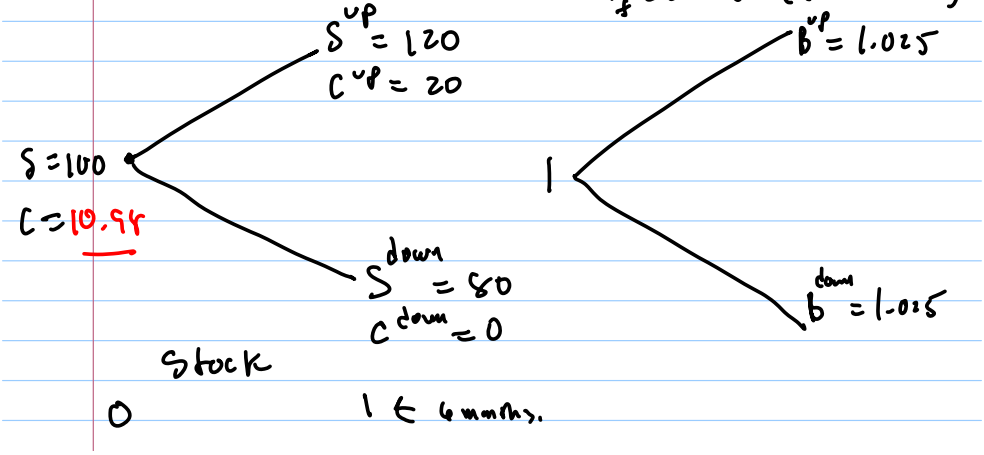






Replicating portfolio of Stock & risk-free bond

$r_f = 0.025$ (6 month rate)



Δ = shares of stock in replicating portfolio

B = $ of bonds in "

up state: $\Delta \cdot S^{up} + B \cdot B^{up} = C^{up}$

$\Delta \cdot 120 + B \cdot 1.025 = 20$

down state: $\Delta \cdot S^{down} + B \cdot B^{down} = C^{down}$

$\Delta \cdot 80 + B \cdot 1.025 = 0$

2 linear equations in 2 unknowns Δ & B

$\Delta \cdot 120 + B \cdot 1.025 = 20$

$-\ \Delta \cdot 80 + B \cdot 1.025 = 0$

$\Delta \underbrace{(120 - 80)}_{S^{up} - S^{down}} = \underbrace{20 - 0}_{C^{up} - C^{down}}$

$\Delta = \frac{20}{40} = 0.5 \Rightarrow$ 1/2 share of stock in replicating ptf.

Substitute $\Delta = 0.5$ into

$$\Delta \cdot 80 + B \cdot (1.025) = 0$$

$$\Rightarrow (0.5)(80) + B(1.025) = 0$$

Solve for B

$$B(1.025) = -40$$

$$\Rightarrow B = \frac{-40}{1.025} = -39.024$$

Replicating portfolio:

$$\Delta = 0.5 \quad , B = -39.024$$

By arbitrage arguments it must be the case that the current value of the call, C, must be equal to the current value of the replicating portfolio

$$C = \Delta \cdot 100 + B \cdot 1$$

$$= (0.5)(100) - 39.025$$

100 = Stock at 0

1 = Bond value at time 0

$= 10.96$!

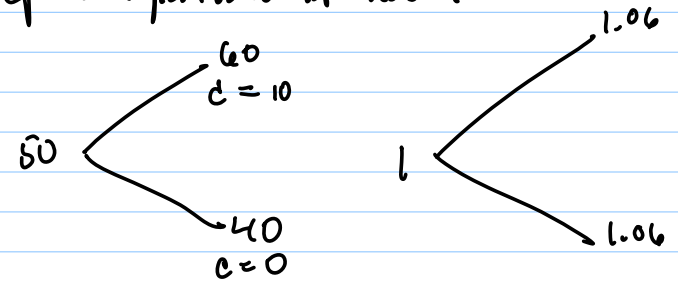
Option Delta

$$\Delta = \frac{C^{up} - C^{down}}{S^{up} - S^{down}} = \frac{\text{Change in Call}}{\text{Change in Stock}}$$

$\sim$ derivative of Call value with respect to change in Stock

$r_f = 0.06$ per period

Replicating portfolio in model



$\Delta \cdot 60 + B(1.06) = 10$

$\Delta \cdot 40 + B(1.04) = 0$

$\Delta =$

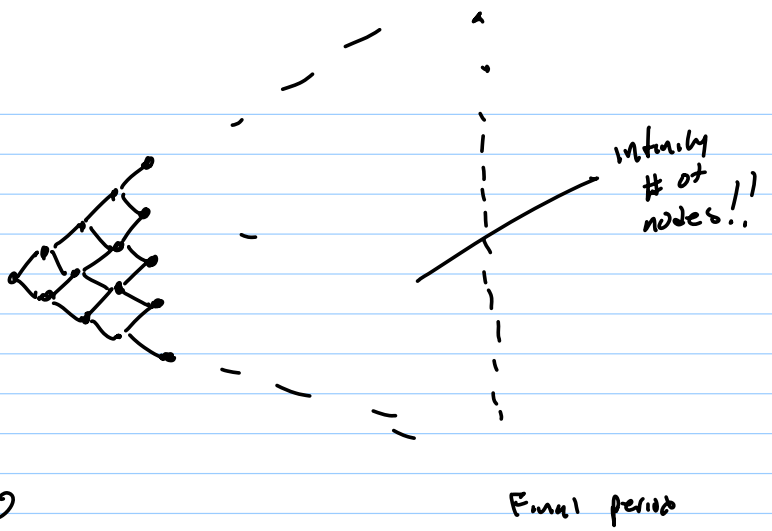
$\Delta(60-40) = 10 \Rightarrow \Delta = \frac{10}{20} = 0.5$

$\Rightarrow (0.5)40 + B(1.04) = 0$

$\Rightarrow B = \frac{-20}{1.04} = -18.86$

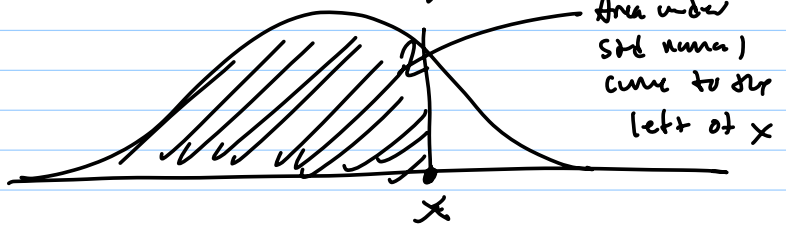
$C = \Delta \cdot (50) + B$

$= (0.5)(50) - 18.86 = 6.13$



$= \Phi(x)$

$N(x) = Pr(Z \leq x)$, $Z \sim N(0,1)$



Excel: NORMSDIST( )