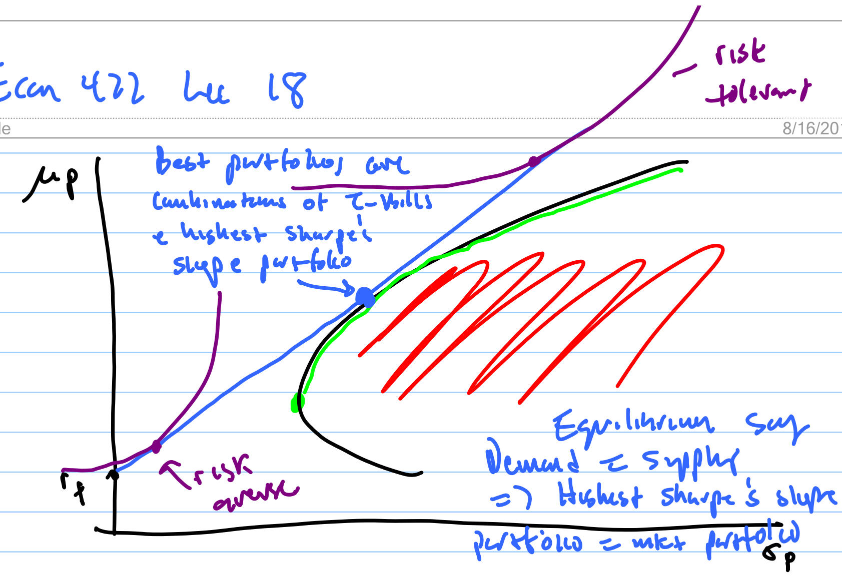
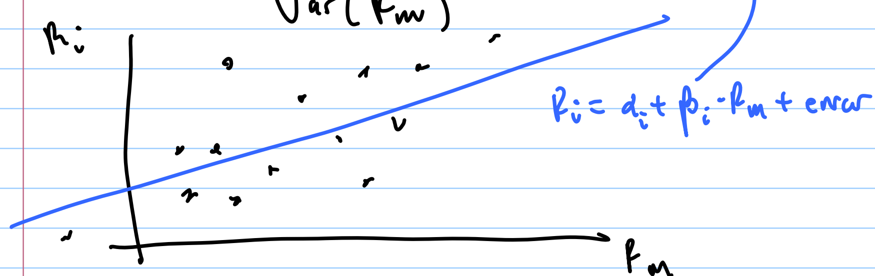
Econ 422 Lec 18

Note Title 8/16/2010



Beta

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)}$$



Ex: Interpretation of β

R_{99} = return on 99 asset portfolio with equal weights $x_i^{99} = \frac{1}{99}$

$\sigma_{99}^2 = Var(R_{99})$

R_{IBM} = return on IBM

$\sigma_{IBM}^2 = Var(R_{IBM})$, $\sigma_{IBM,99} = Cov(R_{IBM}, R_{99})$

$\Rightarrow x_i^{100} = \frac{1}{100}$

$$R_{100} = (0.01) * R_{IBM} + (0.99) * R_{99}$$

$$\begin{aligned} Var(R_{100}) &= (0.01)^2 Var(R_{IBM}) + (0.99)^2 \cdot Var(R_{99}) \\ &\quad + 2(0.01)(0.99) \cdot Cov(R_{IBM}, R_{99}) \\ &\approx (0.0001)\sigma_{IBM}^2 + (0.98) \cdot \sigma_{99}^2 \\ &\quad + 0.02 * \sigma_{IBM,99} \\ &\approx 0.98 \cdot \sigma_{99}^2 + 0.02 \cdot \sigma_{IBM,99} \end{aligned}$$

$$\sigma^2_{100} \approx 0.98 \cdot \sigma^2_{99} + 0.02 \cdot \sigma_{IBM,99}$$

Q: Under condition is $\sigma^2_{100} = \sigma^2_{99}$?

$$\sigma^2_{100} = 0.98 \cdot \sigma^2_{99} + 0.02 \cdot \sigma_{IBM,99} = \sigma^2_{99}$$

$$\Rightarrow \cancel{0.02}\, \sigma_{IBM,99} = \cancel{0.02}\, \sigma^2_{99}$$

$$\Rightarrow \frac{\sigma_{IBM,99}}{\sigma^2_{99}} = 1$$

$$\Rightarrow \frac{\mathrm{cov}(R_{IBM}, R_{99})}{\mathrm{Var}(R_{99})} = 1$$

$$\Rightarrow \beta_{IBM,99} = 1$$

$\Rightarrow$ IBM has the same "common" risk as the 99 asset portfolio!

Using the same algebra we can show

(1) If $\sigma_{100}^2 > \sigma_{99}^2$ then

$$\beta_{IBM,99} > 1$$

$\Rightarrow$ IBM has "more common risk" than the portfolio - IBM is riskier than 99 asset portfolio

(2) If $\sigma_{100}^2 < \sigma_{99}^2$ then

$$\beta_{IBM,99} < 1$$

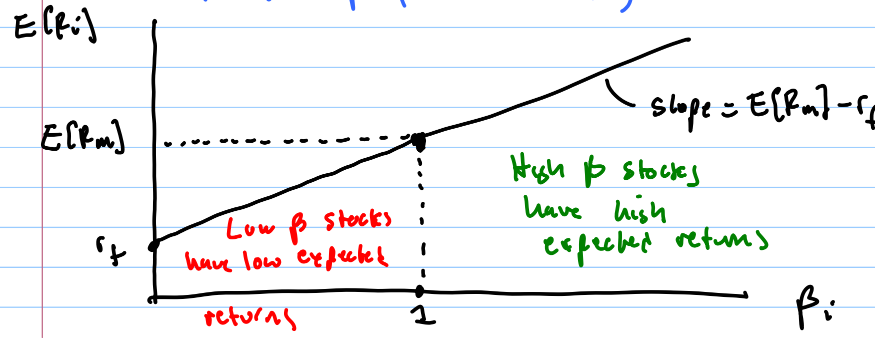
$\Rightarrow$ IBM has smaller common risk than 99 asset portfolio

$\Rightarrow$ IBM is "safer" than 99 asset portfolio

Security Market Line (SML)

=> Graphical Representation of CAPM equation

$$E[R_i] = r_f + \beta_i \left(E[R_m] - r_f \right)$$



Q: How is CAPM related to the price of stocks??

A: Remember simple Dividend discount model for stock prices

$$P = \frac{D_1}{1+R} + \frac{D_2}{(1+R)^2} + \frac{D_3}{(1+R)^3} + \cdots$$

Here Dividends and R are random variables

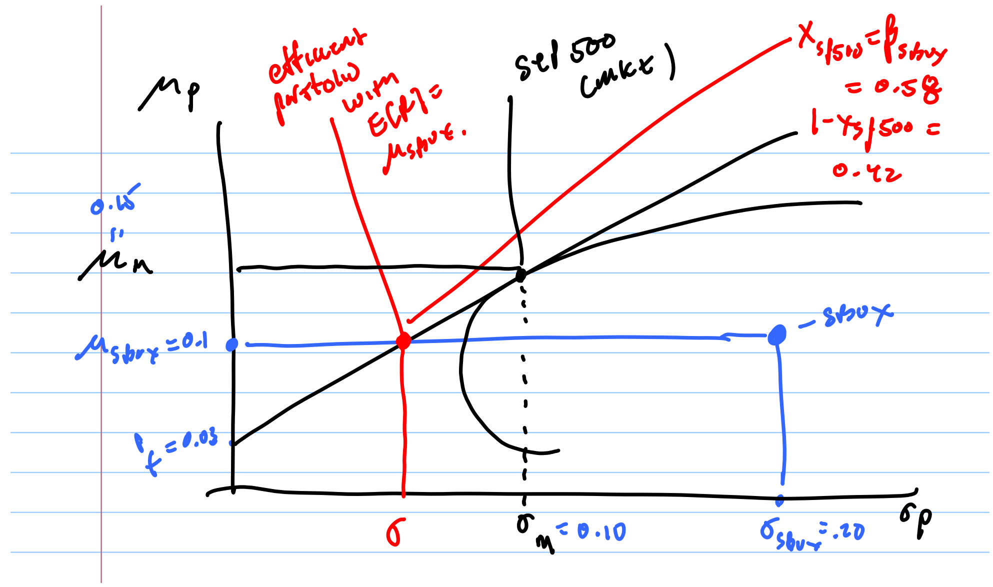
$$E[P] = \frac{E[P_1]}{1+E[R]} + \frac{E[P_2]}{(1+E[R])^2} + \frac{E[P_3]}{(1+E[R])^3} + \cdots$$

CAPM: $E[R] = r_f + \beta\left(E[R_m] - r_f\right)$

$$\beta = \frac{E[R] - r_f}{E[R_m] - r_f}$$

$$= \frac{0.10 - 0.03}{0.15 - 0.03} = \frac{0.07}{0.12}$$

$$= \underline{0.58}$$



Recall, All portfolios of T-Bills & S&P500 satisfy

$$\mu_p^e = r_f + x_{S\&P500}\cdot(\mu_{S\&P500} - r_f)$$

↑ share of wealth in S&P500

CAPM

$$E[R_{sbux}] = r_f + \beta_{sbux}(E(R_m) - r_f)$$

Invest β_{SBUX} in S&P 500

$1-\beta_{SBUX}$ in T-Bills

$\beta_{SBUX} = 0.58$ = invested in S&P 500

$1-\beta_{SBUX} = 1-0.58 = 0.42$ in T-Bills