Econ 422 Lec 17

Note Title 8/13/2010

1 risky asset (stock)

1 safe asset (T-bill)

x_1 = share of wealth in T-bill

x_2 = " " Stock

$x_1 + x_2 = 1$

r_f = risk free rate (T-bill rate) is

constant ~ Not random

$E[r_f] = r_f$, $Var(r_f) = 0$

R_2 = random variable

$\sim N(\mu_2, \sigma_2^2)$

$Cov(r_f, R_2) = 0$

$R_p = x_1 r_f + x_2 R_2$

$E[R_p] = \mu_p = x_1 r_f + x_2 \mu_2$

$$\begin{aligned} Var(R_p) &= Var(x_1 r_f + x_2 R_2) \\ &= Var(x_2 R_2) = x_2^2 \sigma_2^2 \\ &= \sigma_p^2 \end{aligned}$$

Derive equation for portfolios of T-Bills & Stocks

$x_1 + x_2 = 1 \Rightarrow x_1 = 1 - x_2$

$$\begin{aligned} \mu_p &= x_1 r_f + x_2 \mu_2 \\ &= (1 - x_2) r_f + x_2 \cdot \mu_2 \\ &= r_f - x_2 r_f + x_2 \cdot \mu_2 \\ &= r_f + x_2 (\underbrace{\mu_2 - r_f}_{\text{risk premium on Stock}}) \end{aligned}$$

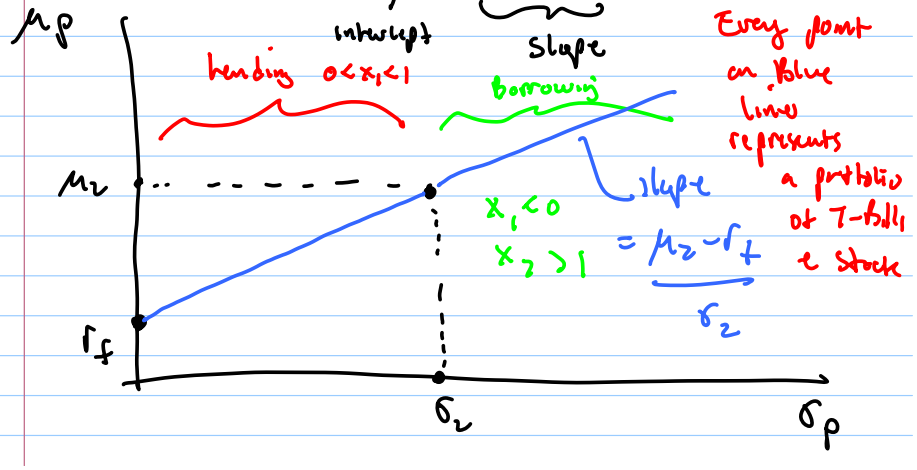
$$\sigma_p^2 = x_2^2 \sigma_2^2 \quad / \quad \Rightarrow \sigma_p = x_2 \cdot \sigma_2$$

$$\Rightarrow \boxed{x_2 = \frac{\sigma_p}{\sigma_2}}$$

$$\mu_p = r_f + \frac{\sigma_p}{\sigma_2}(\mu_2 - r_f)$$

$$= r_f + \sigma_p \left(\frac{\mu_2 - r_f}{\sigma_2} \right)$$

$$\mu_p = r_f + \left(\frac{\mu_2 - r_f}{\sigma_2} \right) \cdot \sigma_p$$



Ex: Look at types of portfolios

(1) T-Bills + Stock 1 (MSFT)

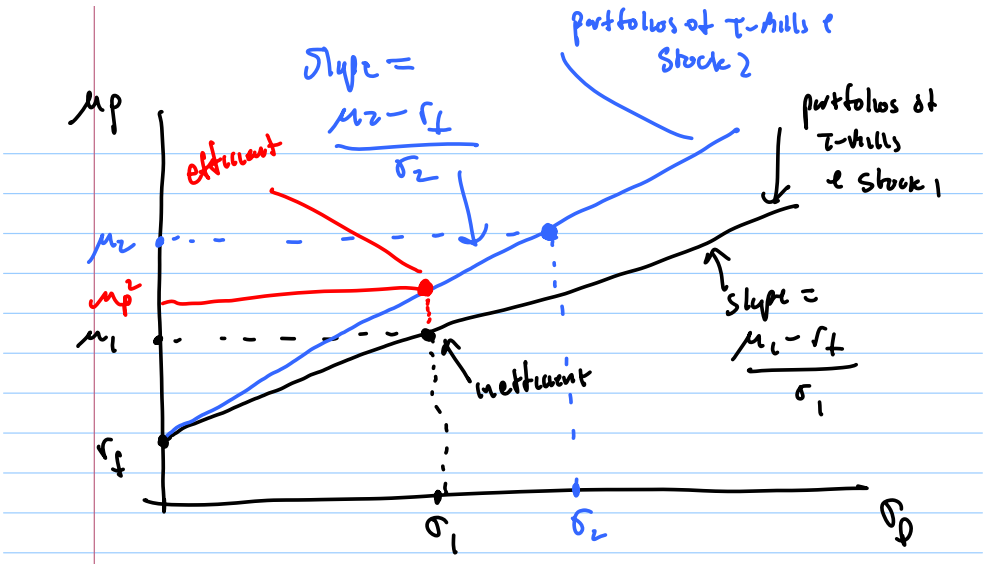
(2) T-Bills + Stock 2 (Store)

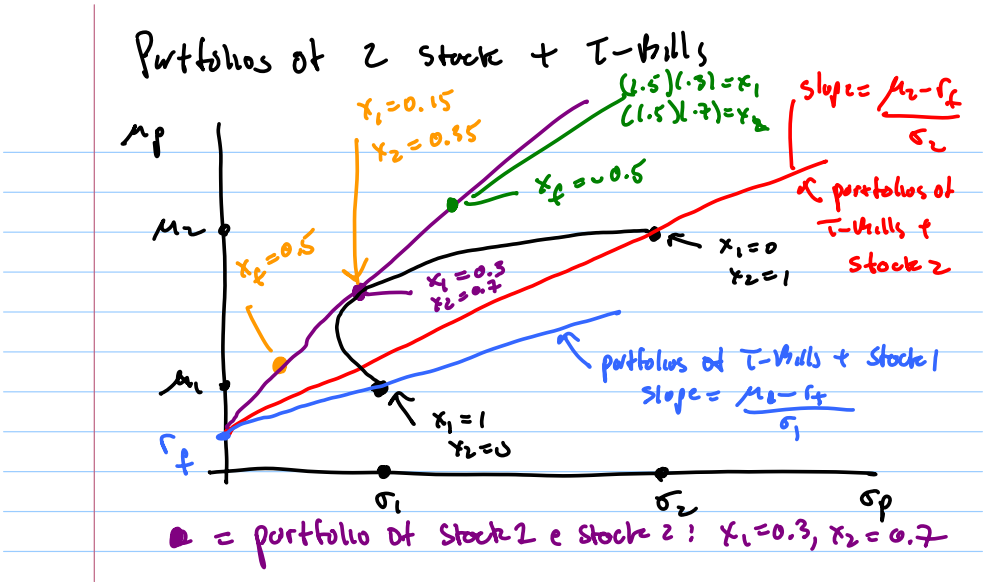
$R_p^1 = x_f^1 \cdot r_f + x_1 \cdot R_1$

$R_p^2 = x_f^2 \cdot r_f + x_2 \cdot R_2$

$\mu_p^1 = r_f + \left(\frac{\mu_1 - r_f}{\sigma_1} \right) \cdot \sigma_p^1$, $\frac{\mu_1 - r_f}{\sigma_1}$ = Sharpe's slope for stock 1

$\mu_p^2 = r_f + \left(\frac{\mu_2 - r_f}{\sigma_2} \right) \cdot \sigma_p^2$, $\frac{\mu_2 - r_f}{\sigma_2}$ = Sharpe's slope for stock 2


portfolios of T-bills & Stock 2
Slope = $\frac{\mu_2 - r_f}{\sigma_2}$
μ_p
efficient
portfolios of T-bills & Stock 1
μ_2
μ_p^2
μ_1
Slope = $\frac{\mu_1 - r_f}{\sigma_1}$
inefficient
r_f
σ_1
σ_2
σ_p


Portfolios of 2 stocks + T-bills
μ_p
$x_1 = 0.15$
$x_2 = 0.85$
$(.5)(.3) = x_1$
$(.5)(.7) = x_2$
slope = $\frac{\mu_2 - r_f}{\sigma_2}$
$x_f = 0.5$
portfolios of T-bills & stock 2
μ_2
$x_f = 0.5$
$x_1 = 0$
$x_2 = 1$
$x_1 = 0.5$
$x_2 = 0.5$
portfolios of T-bills & Stock 1
slope = $\frac{\mu_1 - r_f}{\sigma_1}$
μ_1
$x_1 = 1$
$x_2 = 0$
r_f
σ_1
σ_2
σ_p
● = portfolio of Stock 2 & Stock 2 : $x_1 = 0.3$, $x_2 = 0.7$

First look portfolios of 2 stocks only

$$R_p = x_1 R_1 + x_2 R_2 \quad \text{(stocks)}$$

$$\mu_p = x_1 \mu_1 + x_2 \mu_2$$

$$\sigma_p^2 = x_1^2 \sigma_1^2 + x_2^2 \sigma_2^2 + 2 x_1 x_2 \sigma_{12}$$

$$\rho_{12} = 0.2$$

Second: Look portfolios of Stock 1 & T-Bills

$$\mu_p = r_f + \left(\frac{\mu_1 - r_f}{\sigma_1} \right) \cdot \sigma_p$$

Third: Look at portfolios of Stock 2 & T-Bills

$$\mu_p = r_f + \left(\frac{\mu_2 - r_f}{\sigma_2} \right) \cdot \sigma_p$$

The best combination of T-bills & 2 risky is

1. T-Bills
2. Highest sharpe slope portfolio of the 2 risky assets (tangency portfolio)

Invest in portfolio of assets

$$R_p \sim N(\mu_p, \sigma_p^2)$$

$$R_p = x_1 R_1 + x_2 R_2 + \cdots + x_n R_n$$

$$R_p = \mu_p + \sigma_p * Z \quad , Z \sim N(0,1)$$

$$E[Z] = 0, \quad Var(Z) = 1$$

$$E(f_p) = \mu_p + \sigma_p \cdot \underbrace{E[z]}_{=0} = \mu_p$$

$$Var(f_p) = Var(\mu_p + \sigma_p \cdot z) = \sigma_p^2 \cdot \underbrace{Var(z)}_{=1}$$

$$= \sigma_p^2$$