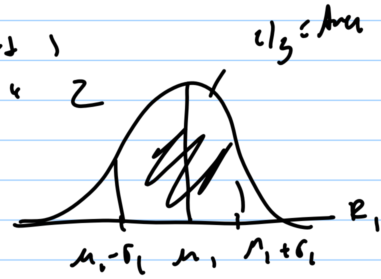
Econ 422 Lec 16

Note Title 8/12/2010

Portfolio theory

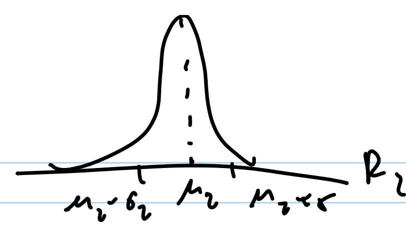
R_1 = annual return on asset 1

R_2 = " " " 2



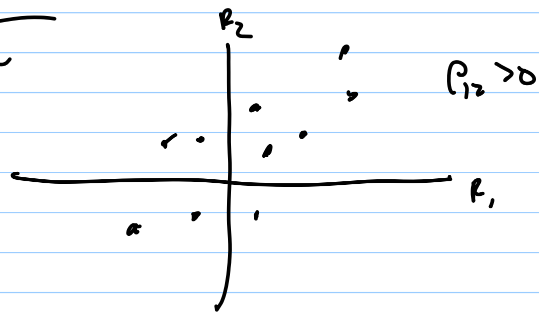
$R_1 \sim N(\mu_1, \sigma_1^2)$

$R_2 \sim N(\mu_2, \sigma_2^2)$



$\mathrm{cov}(R_1, R_2) = \sigma_{12}$

$$\rho_{12} = \frac{\sigma_{12}}{\sigma_1 \cdot \sigma_2}$$



Example portfolio

$$R_p = \underset{x_1}{\underset{\shortparallel}{0.25}} \cdot R_1 + \underset{x_2}{\underset{\shortparallel}{0.75}} \cdot R_2$$

$$E[R_p] = 0.25\, \underset{0.12}{\underset{\shortparallel}{\mu_1}} + 0.75 \cdot \underset{0.17}{\underset{\shortparallel}{\mu_2}}$$

$$= 0.16$$

$$Var(R_p) = (0.25)^2 \cdot \sigma_1^2 + (0.75)^2 \sigma_2^2$$
$$+ 2(0.25)(0.75) \cdot \sigma_{12}$$

$$\sigma_{12} = \rho_{12} \cdot \sigma_1 \cdot \sigma_2$$

$$= (0.4)(0.2)(0.3) = 0.024 > 0$$

$$Var(R_p) = (0.25)^2(0.2)^2 + (0.75)^2(0.3)^2 + 2(0.25)(0.75)(0.024)$$

$$= 0.06215$$

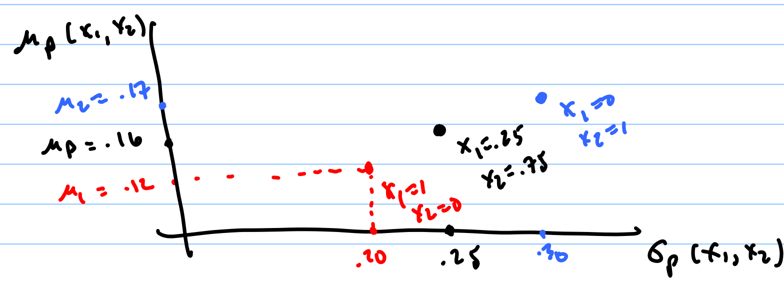
$$SD(R_p) = \sigma_p = \sqrt{Var(R_p)}$$

$$= \sqrt{\sigma_p^2}$$

$$= \sqrt{0.0625} = 0.25$$

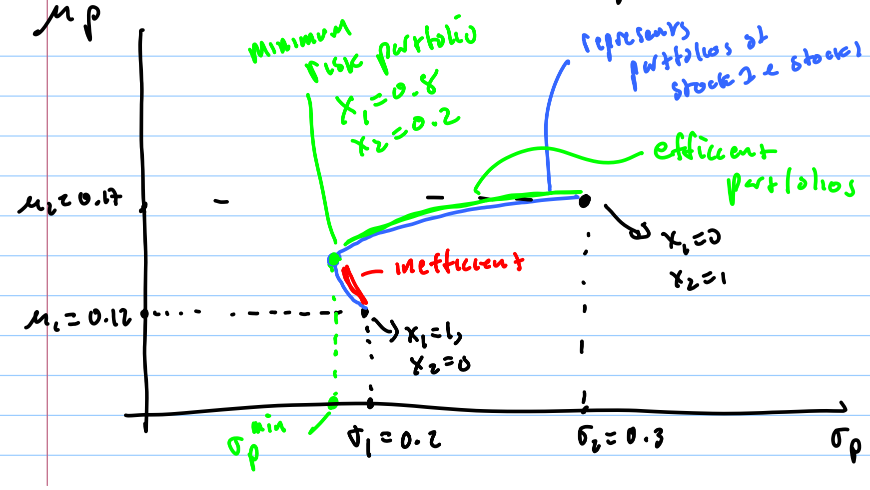
$$E\{R_p\} = \mu_p = 0.16$$

$$SD(R_p) = \sigma_p = 0.25$$



$\mu_p(x_1, x_2)$

Risk - Return characteristics of 2 asset portfolios



$\sigma_p(x_1, x_2)$

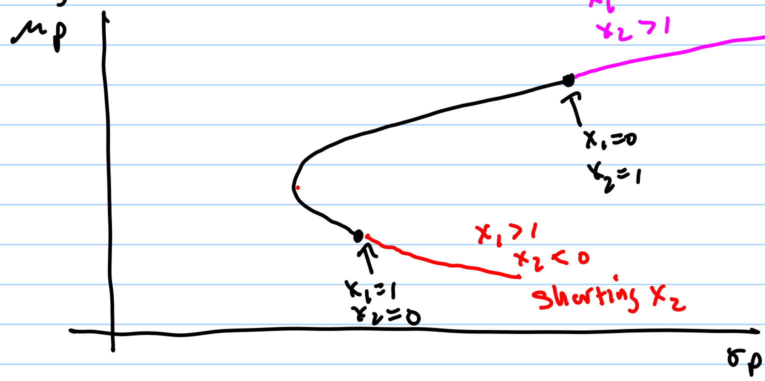
Any portfolios below the min risk portfolio are call "inefficient" portfolios

— there will be some other portfolio that has the risk but higher expected return!

An efficient portfolio has the highest expected return for any given level

of risk where risk is measured by
σ_p (portfolio SD)

- Adding short sales



n asset portfolios (n is big, e.g. n = 100)

$X_1, \ldots, X_n$: portfolio weights

$X_1 + X_2 + \ldots + X_n = 1$

special case: equally weighted portfolio

$X_i = \frac{1}{n}$ (e.g. $n = 100$
$X_i = \frac{1}{100} = 0.01$)

$R_1, \ldots, R_n$ = annual return on n assets

Assume $R_i \sim N(\mu_i, \sigma_i^2)$

$\sigma_{ij} = \text{cov}(R_i, R_j) \quad i,j = 1, \ldots, n$

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \cdot \sigma_j} \quad i,j = 1, \ldots, n$$

$R_p = x_1 R_1 + x_2 R_2 + \cdots + x_n R_n$

Derive $E[R_p] = \mu_p$ and $V(R_p) = \sigma_p^2$

$E[R_p] = x_1\mu_1 + x_2\mu_2 + \cdots + x_n\mu_n$

How to compute $\sigma_p^2 = V(x_1R_1 + x_2R_2 + \cdots + x_nR_n)$

	x_1	x_2	$\cdots$	x_n
x_1	σ_{11}	σ_{12}		σ_{1n}
x_2	σ_{12}	σ_{22}		σ_{2n}
$\vdots$	$\vdots$		$\ddots$	
x_n	σ_{1n}	σ_{2n}	$\cdots$	σ_{nn}

covariance matrix

n^2 total elements in cov matrix

There n variances (σ_{ii} $i = 1, \ldots, n$)
i.e n diagonal elements in cov matrix

n^2 total elements

n diag elements

$n^2 - n = n(n-1)$ off diagonal elements

= covariance terms

Ex: $n = 100$

$n^2 = (100)(100) = 10{,}000$ total elements

varns + covs

$n = 100$ variances

$n(n-1) = 100(99) = 9900$

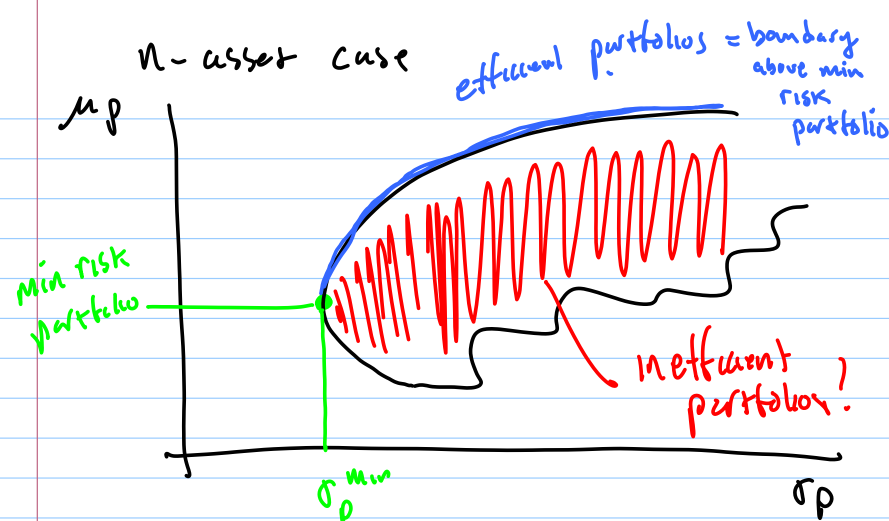
covariances !!!

$$V(R_p) = x_1^2\sigma_{11} + x_1x_2\sigma_{12} + \cdots x_1x_n\sigma_{1n}$$
$$+ x_2x_1\sigma_{12} + x_2^2\sigma_{22} \cdots - - -$$
$$\cdots - - - \cdots x_n^2\sigma_{nn}$$
$$= \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}\cdot x_i\cdot x_j$$

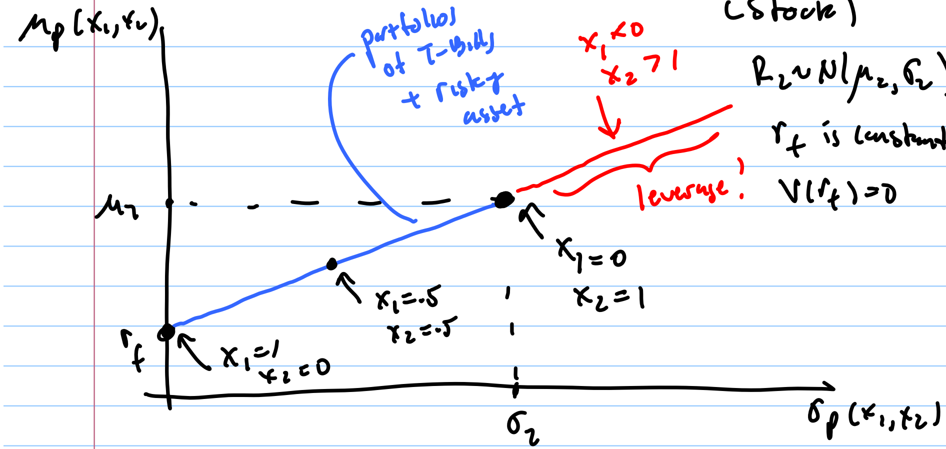
Note: in the sum above

- n variance terms
- $n(n-1)$ covariance terms

n-asset case



Portfolios of T-Bills & Risky asset (stock)



$R_2 \sim N(\mu_2, \sigma_2)$

r_f is constant

$V(r_f) = 0$