Econ 422 Lec 15 !

Note Title 8/11/2010

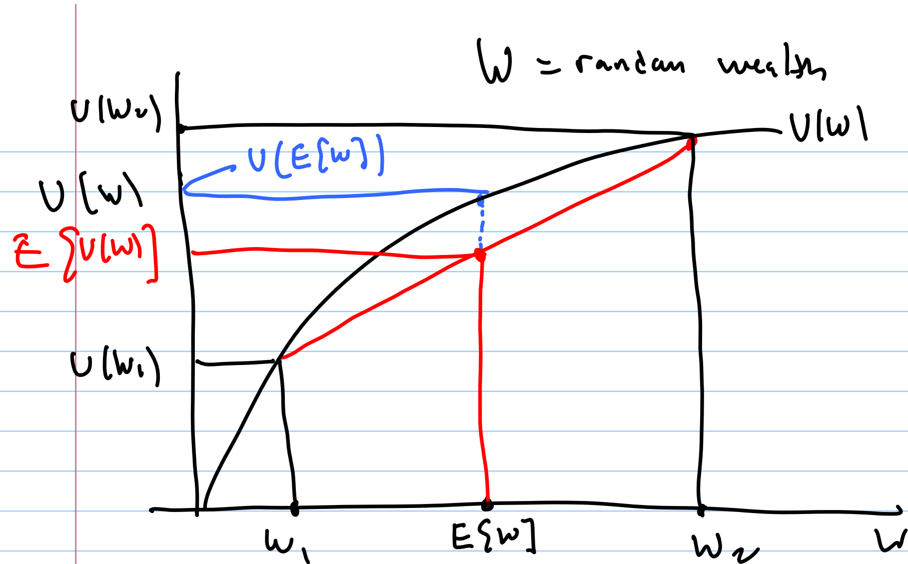
Expected Utility

Rational individuals make choices under uncertainty to maximize expected utility

$$E\{U(W)] = \sum_{w} U(w) \cdot Pr(W=w)$$

s.t. budget constraints

Here, W = random variable = end of period wealth



Probability distn of W:

$$W = \begin{cases} W_1 & \text{with prob } p \\ W_2 & \text{with prob } 1-p \end{cases}$$

= random wealth

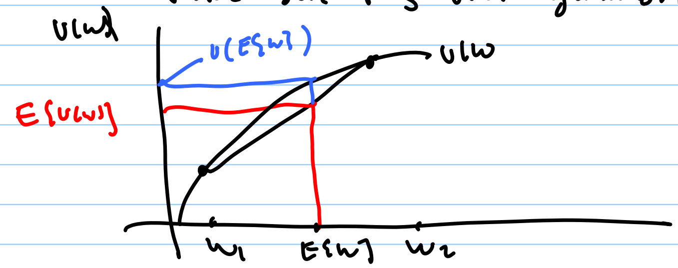
$W_2 > W_1$

$$E[W] = p \cdot W_1 + (1-p) \cdot W_2$$

(1) For risk averse individuals

$$E[U(w)] < U(E[w])$$

— take sure thing over gamble

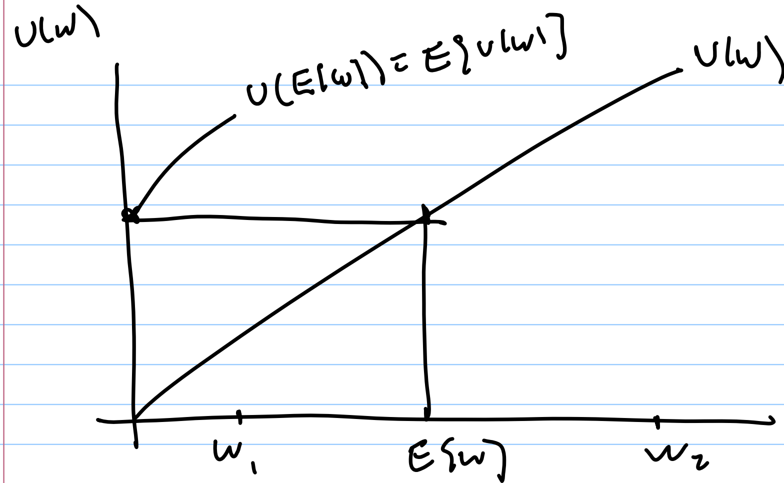


(2) For risk neutral individuals

$$E[U(W)] = U(E[W])$$

Person is indifferent b/w sure thing and gamble where Expected payoff is the same as the sure thing'

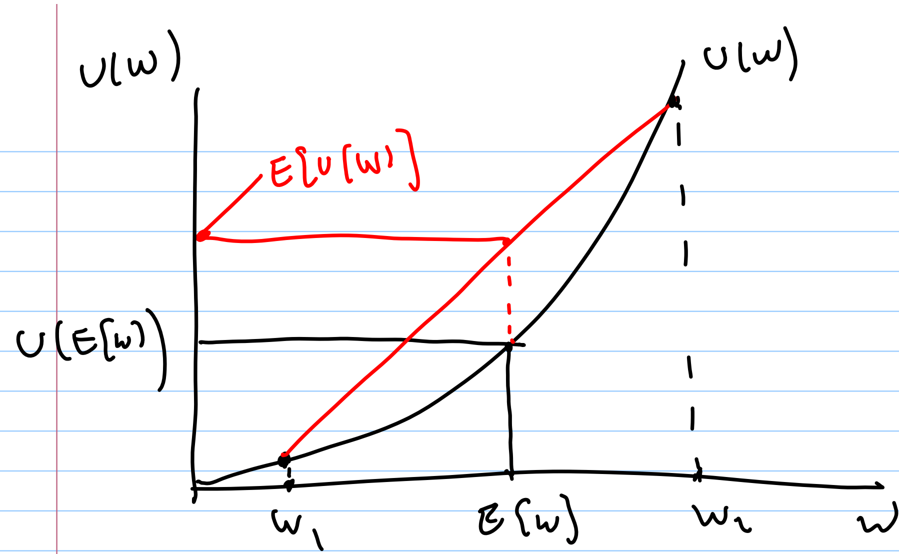
Risk neutral



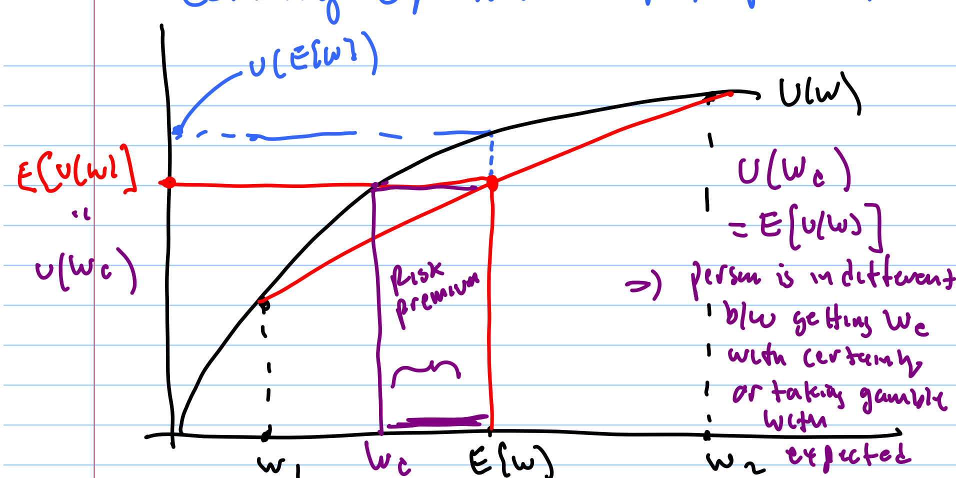
(3) Risk lover

$$E[U(W)] > U(E[W])$$

Prefers gamble over sure thing where expected payoff = sure thing



Certainty Equivalent & Risk Premium



W_c = certainty equivalent wealth.

payoff $E[W]$

Risk Premium = $E[W] - W_c > 0$

Risk versus Return

R = random rate of return on some risky asset

e.g. R = annual rate of return on some stock (e.g. MSFT)

Not a random variable! It is constant
eg. $r_f = 0.005$

r_f = rate of return on a safe asset — usually a risk free US gov't bond (with maturity date the same as the holding period of the risky asset)

Excess Return = rate of return on risky asset
— rate of return on safe asset

$= R - r_f$

= random variable b/c R is a random variable

Risk premium = expected excess return

$= E[R] - r_f$

= "average" excess return
by investing in risky
asset vs. safe asset

For risk averse individuals it must the case that the risk premium is positive:

$$E\{R\} - r_f > 0$$

$$\Rightarrow E\{R\} > r_f$$

Risk neutral: risk premium = 0

$$E\{R\} = r_f \quad !$$

Measuring systematic using "Beta"

$$\underbrace{E[R] - r_f}_{\text{risk premium on any risky asset}} = \beta * \underbrace{\left(E[R_p] - r_f\right)}_{\text{risk premium on a well diversified portfolio}}$$

asset beta

β = sensitivity of asset to common risk captured by diversified portfolio

that only has systematic risk

Solving for β gives

$$\beta = \frac{E[R] - r_f}{E[R_p] - r_f}$$

CAPM

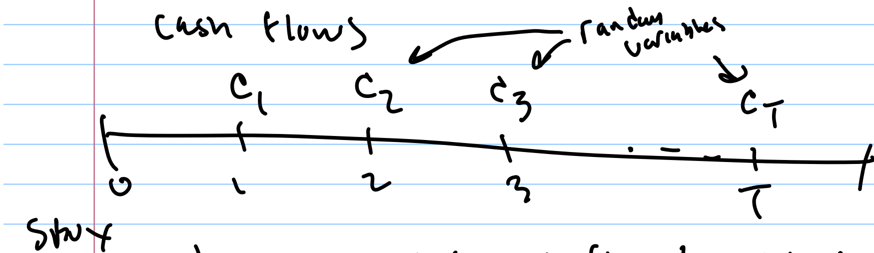
$$\underbrace{E[R] - r_f}_{\text{risk premium on any asset}} = \beta_{mkt} \underbrace{\left(E(R_m) - r_f\right)}_{\substack{\text{risk premium on mkt portfolio} \\ \approx \text{ risk premium on S\&P 500}}}$$

beta with respect to mkt portfolio

$$\Rightarrow E[R] = r_f + \beta_{mkt}\left(E[R_m] - r_f\right)$$

risk adjusted expected return.

So for PV calculations with risky cash flows



$\tilde{c}_i$ = uncertain cash flow for SBUX with "risk" associated with typical SBUX business

$$PV = \frac{E[C_1]}{1+E[R]} + \frac{E[C_2]}{(1+E[R])^2} + \cdots + \frac{E[C_T]}{(1+E[R])^T}$$

$$E[R] = r_f + \beta_{SBUX} \cdot (E[R_m] - r_f)$$

= risk adjusted "cost of capital"

Portfolio theory with 2 risky assets

R_1, R_2 rates of return (annual) on
2 risky assets (SBUX, MSFT)

$\mu_1 = E[R_1]$, $\mu_2 = E[R_2]$: $R_1 \sim N(\mu_1, \sigma_1^2)$

$\sigma_1^2 = V(R_1)$, $\sigma_2^2 = V(R_2)$ $R_2 \sim N(\mu_2, \sigma_2^2)$

$\sigma_{12} = cov(R_1, R_2)$

risk-return characteristics