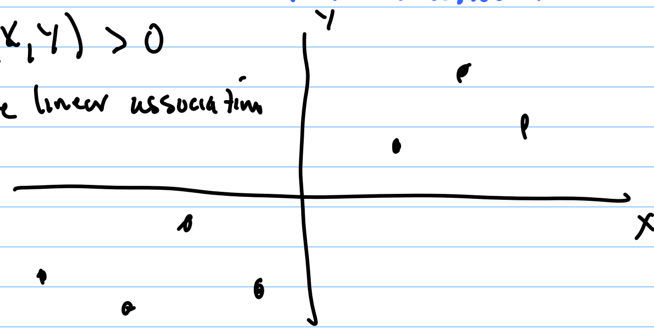
Note Title 8/6/2010

Econ 422 Lec 12

Covariance – Measure of the direction of linear association

$Cov(X,Y) > 0$

positive linear association



Computing Covariance – Discrete case

$$Cov(X,Y) = E\Big[(X-E[X])(Y-E[Y])\Big] \quad ①$$

$$= E[XY] - E[X]\cdot E[Y] \quad ②$$

① $$Cov[X,Y] = \sum_{x}\sum_{y} (x-E[X])(y-E[Y])\cdot Pr(X=x, Y=y)$$

(i) $E\{X \cdot Y\} = \sum_x \sum_y x \cdot y \cdot \Pr(X=x, Y=y)$

$E\{X\} = \sum_x x \Pr(X=x)$

$E\{Y\} = \sum_y y \Pr(Y=y)$

$Z = X + Y$

$\mathrm{Var}(Z) = E\{(Z - E\{Z\})^2\}$

$= E\{(X + Y - E\{X+Y\})^2\}$

$= E\{(X + Y - E\{X\} - E\{Y\})^2\}$

$$= E\left\{\left[(X - E[X]) + (Y - E[Y])\right]^2\right\}$$

$$E\left\{(X-E[X])^2 + 2(X-E[X])(Y-E[Y]) + (Y-E[Y])^2\right\}$$

$$E\left\{(X-E[X])^2\right\} + E\left\{(Y-E[Y])^2\right\} + 2E\left\{(X-E[X])(Y-E[Y])\right\}$$

$$V(Z) = V(X+Y) =$$

$$V(X) + V(Y) + 2\,\mathrm{Cov}(X,Y)$$

Creating a portfolio of 2 assets

Asset 1, Asset 2

Portfolio = collection of assets

w_1 = share of wealth in asset 1

w_2 = " " " 2

Ex: $w_1 = 0.5 \Rightarrow 50\%$ of wealth in asset 1

$w_2 = 0.5 \Rightarrow 50\%$ of wealth in asset 2

$\Rightarrow$ Equally weighted portfolio

Note: $w_1 + w_2 = 1$ (or 100%)

r_p = portfolio rate of return

$= w_1 r_1 + w_2 r_2$

Expected return on portfolio:

$$E\{r_p\} = E\{w_1 r_1 + w_2 r_2\}$$
$$= w_1 E\{r_1\} + w_2 E\{r_2\}$$

In our example,

$$w_1 = w_2 = 0.5$$
$$E\{r_1\} = E\{r_2\} = 0.10$$

$$E\{r_p\} = (0.5)(0.1) + (0.5)(0.1) = \underline{\underline{0.1}}$$

$$V(r_p) = V(w_1 r_1 + w_2 r_2)$$
$$= w_1^2 V(r_1) + w_2^2 V(r_2)$$
$$+ 2 \cdot w_1 \cdot w_2 \operatorname{cov}(r_1, r_2)$$
$$= (0.5)^2 (0.00145) + (0.5)^2 (0.00145)$$
$$+ 2(0.5)(0.5) * (-0.00145) = 0 \, !$$

$$\mathrm{Cov}(r_1, r_2) = E[r_1 \cdot r_2] - \underset{\substack{\| \\ 0.1}}{E[r_1]} \cdot \underset{\substack{\| \\ 0.1}}{E[r_2]}$$

$$E[r_1 \cdot r_2] = \sum_{r_1} \sum_{r_2} r_1 \cdot r_2 \cdot \Pr(r_1 = r_1, r_2 = r_2)$$

Result:

Let X be any random variable and let $g(\cdot)$ be any function.

Ex $g(x) = x^2$, $g(x) = \ln(x)$

Then $E[g(x)] = \sum_x g(x) \cdot \Pr(X = x)$

If $g(x) = x^2$ then

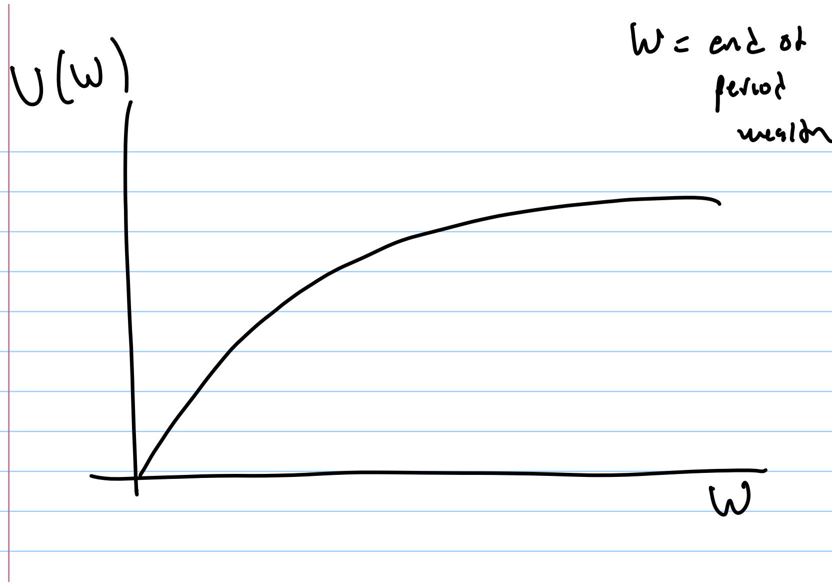
$$E\{x^2\} = \sum_x x^2 \cdot \Pr(X = x)$$

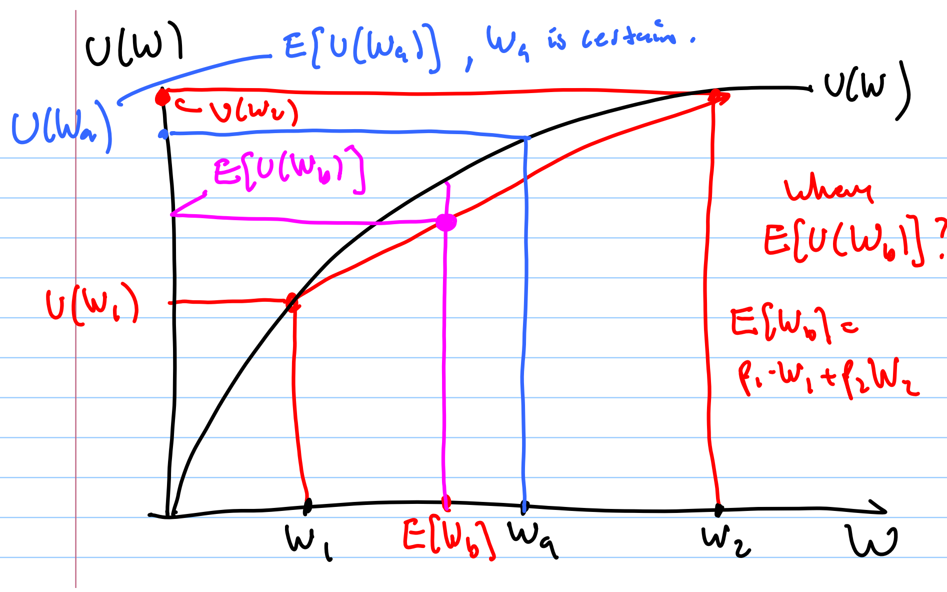
Expected Utility Hypothesis

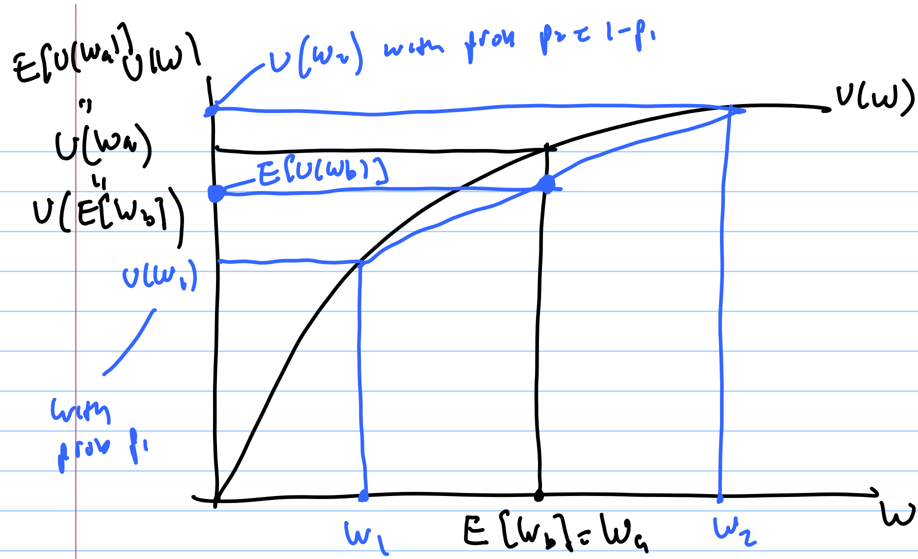
W = random end of period wealth

$U(W)$, U = utility function

Assume individuals maximize $E\{U(W)\}$


U(W)
W = end of period wealth
W


U(W)
E[U(Wa)] , Wa is certain.
U(W)
U(Wa)
U(W2)
E[U(Wb)]
where E[U(Wb)]?
U(W1)
E[Wb] = p1·W1 + p2W2
W1
E[Wb]
Wa
W2
W


E[U(Wa)] U(W)
=
U(Wa)
≥
U(E[Wb])
U(W2) with prob p2 = 1-p1
E[U(Wb)]
U(W1)
with prob p1
U(W)
W1
E[Wb] = Wa
W2
W