Econ 422 Lec 11

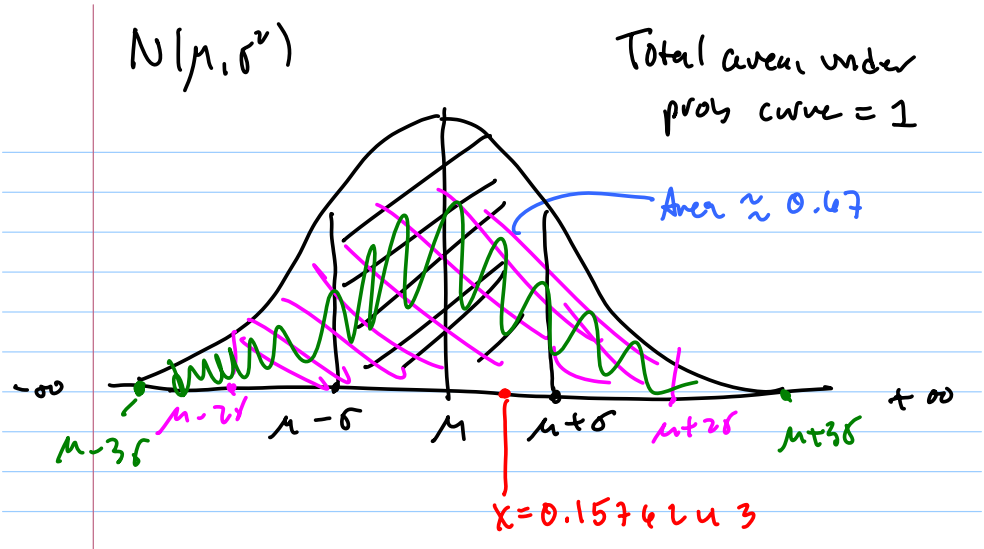
Note Title 8/5/2010

Probability with continuous random variables

X = monthly total rate of return

Assume $X \sim N(\mu, \sigma^2)$

X is distributed as normal with mean μ and variance σ^2



Q: What is $Pr(X = 0.1576243) = 0$

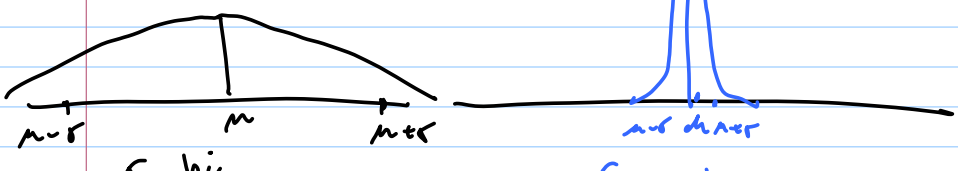
But probability over intervals are well defined

$$Pr(\mu - \sigma \le X \le \mu + \sigma) = 0.67$$

$$Pr(\mu - 2\sigma \le X \le \mu + 2\sigma) = 0.95$$

$$Pr(\mu - 3\sigma \le X \le \mu + 3\sigma) \approx 0.99$$

μ = Center ; σ = spread about center



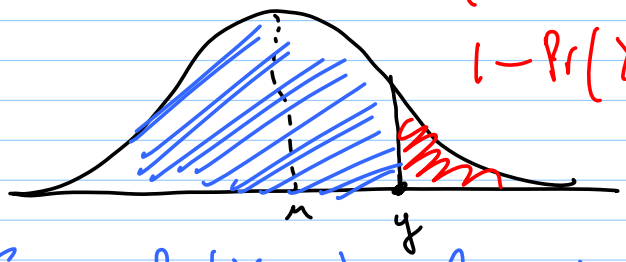
σ big

σ small

CDF for Normal

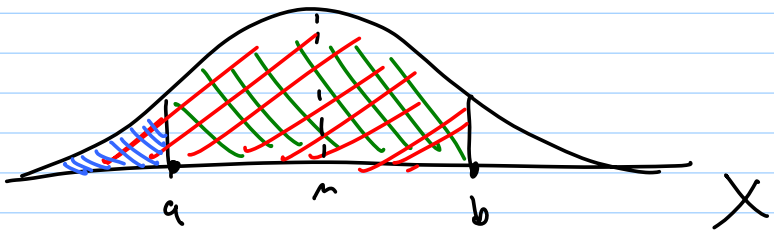
$Pr(X \geq y) =$

$1 - Pr(X \leq y)$



$F_X(y) = Pr(X \leq y)$ = Area under curve to the left of y

Finding Areas (probabilities) over intervals



$Pr(a \leq X \leq b) = F_X(b) - F_X(a)$

Normal distn

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right\}$$

pdf

$$F_X(y) = \int_{-\infty}^{x} f(t)\,dx = \Phi\left(\frac{y-\mu}{\sigma}\right)$$

doesn't have analytic expression

must be evaluated numerically

Excel function NORMDIST( )

Computes both $F_X(y)$ and $f(x)$

Expected Value for Discrete RV

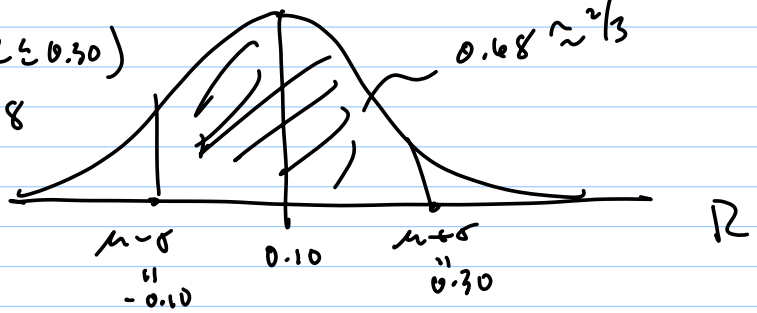
$$E[X] = \sum_{x \in \Omega} x \cdot Pr(X = x)$$

$$= 40(0.25) + 45(0.30) + 50(0.4) + 60(0.05) = 46.5$$

Ex: $R \sim N(0.10, (0.20)^2)$ $V[R] = (0.20)^2$

$\mu = 0.10$, $\sigma = 0.20$, $E[R] = \mu = 0.10$

$Pr(-0.10 \leq R \leq 0.30) = 0.68$



Invest in mutt over next year

$$W_1 = W_0(1+R)$$

W_1: FV, W_0: PV, R: annual rate of return.

$$= 10{,}000(1+R)$$

$$W_1 = 10{,}000 + 10{,}000 \times R$$

$$E[W_1] = E[10{,}000 + 10{,}000 \times R]$$

$$= 10{,}000 + 10{,}000 \cdot E[R]$$

$$= 10{,}000 + 10{,}000 \times (0.10)$$

$$= 11{,}000$$

Find $SD(W_1) = \sqrt{Var(W_1)}$

$$W_1 = 10{,}000 + 10{,}000 * R$$

$$V(W_1) = V\left(10{,}000 + 10{,}000 * R\right)$$

$$= (10{,}000)^2 * V(R)$$

$$= (10{,}000)^2 * (0.20)^2$$

$$SD(W_1) = \sqrt{V(W_1)} = 10{,}000\,(0.20) = 2{,}000$$

Q: What is the distribution of W_1?

A: $W_1 \sim N\left(E[W_1], V(W_1)\right)$

b/c $W_1 = 10{,}000 + 10{,}000 * R$

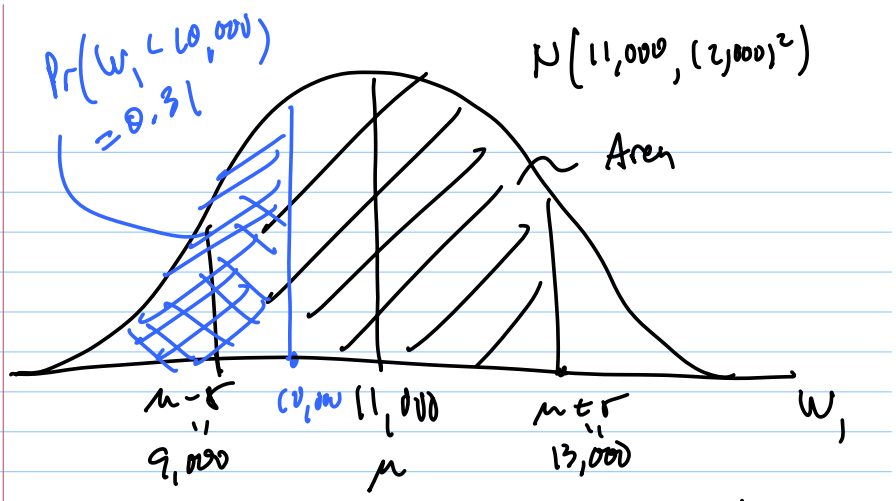
$R \sim N\left(0.10, (0.20)^2\right)$

$$E[W_1] = 11{,}000$$

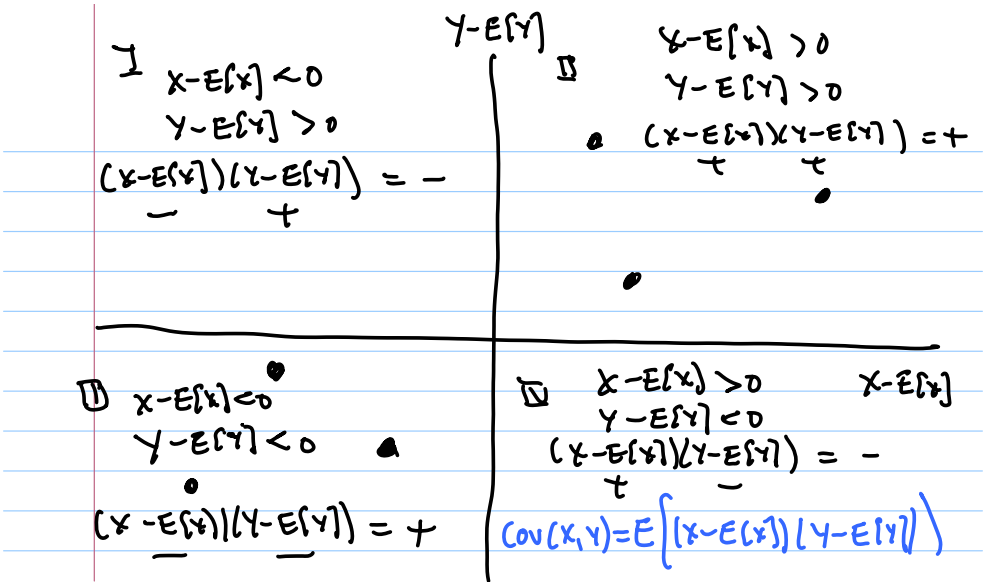
$$V(W_1) = (10{,}000)^2(0.20)^2 = (2000)^2$$

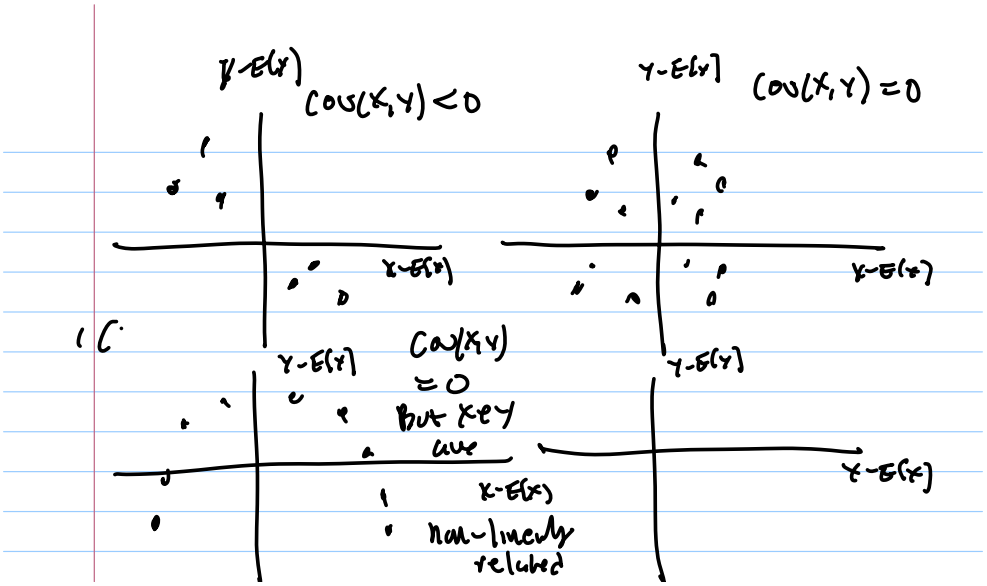
$$SD(W_1) = 10{,}000(0.20) = 2{,}000$$

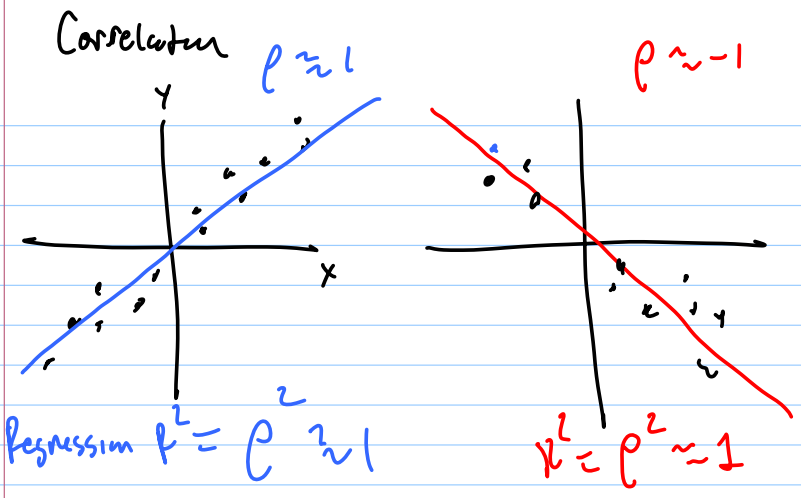
$$W_1 \sim N\left(11{,}000, (2000)^2\right)$$

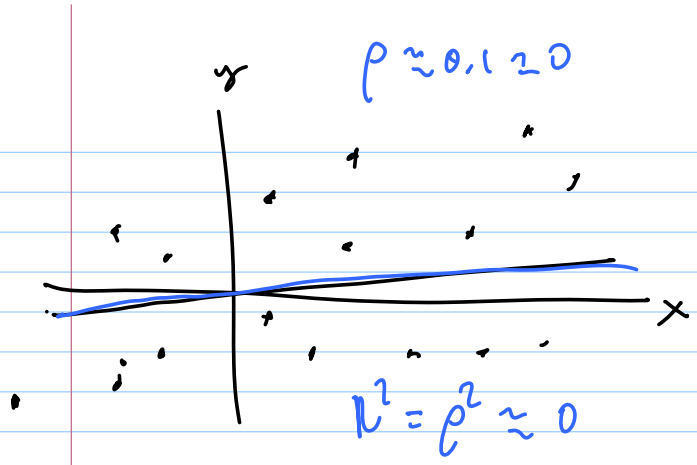


$$Pr(9{,}000 \leq W_1 \leq 13{,}000) \approx 2/3$$


Y-E[Y]
I
X-E[X] < 0
Y-E[Y] > 0
(X-E[X])(Y-E[Y]) = −
II
X-E[X] > 0
Y-E[Y] > 0
(X-E[X])(Y-E[Y]) = +
III
X-E[X] < 0
Y-E[Y] < 0
(X-E[X])(Y-E[Y]) = +
IV
X-E[X] > 0
Y-E[Y] < 0
(X-E[X])(Y-E[Y]) = −
X-E[X]
Cov(X,Y) = E[(X-E[X])(Y-E[Y])]


Y-E[Y]
Cov(X,Y) < 0
X-E[X]
Y-E[Y]
Cov(X,Y) = 0
X-E[X]
Cov(X,Y) = 0 but X ≠ Y are non-linearly related
Y-E[Y]
X-E[X]


Correlation
$\rho \simeq 1$
$\rho \simeq -1$
Y
X
Regression $R^2 = \rho^2 \simeq 1$
$R^2 = \rho^2 \simeq 1$


y
$\rho \simeq 0,1 \simeq 0$
X
$R^2 = \rho^2 \simeq 0$