Econ 422
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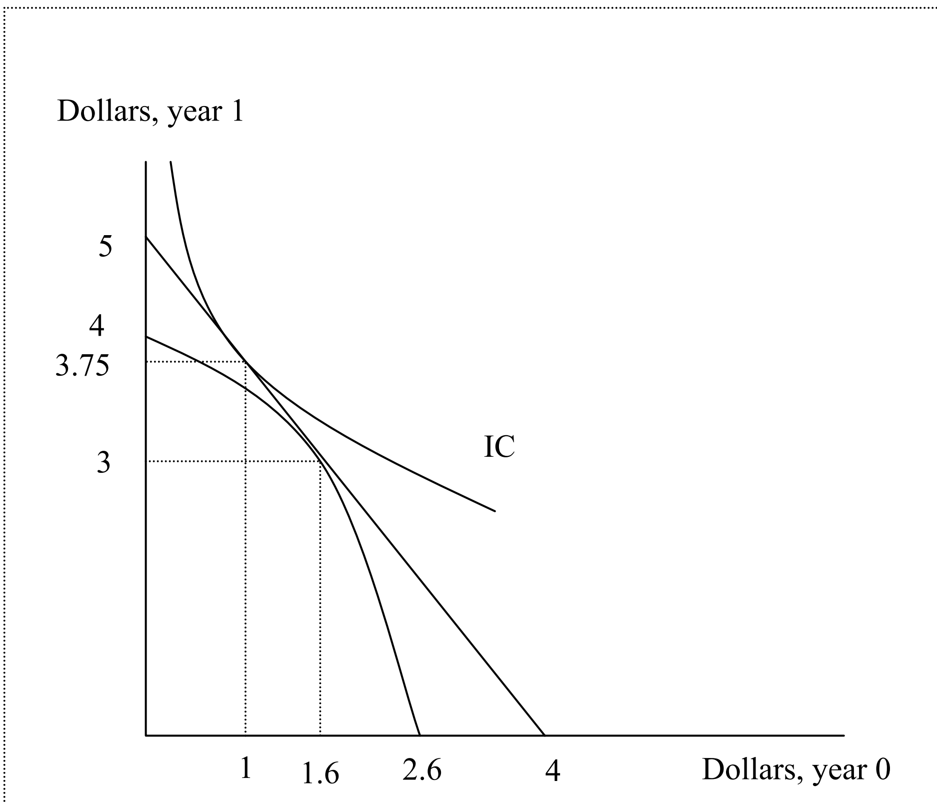
Midterm Exam Solutions

This is a closed book exam. However, you are allowed one page of notes (double-sided). Answer all questions. For the numerical problems, if you make a computational error you may still receive full credit if you provide the correct formula for the problem. There are 25 questions, and each question is worth 4 points. Total points = 100. You have 1 hour and 50 minutes to complete the exam. Good luck.

I. Intertemporal Consumption and Investment Decisions (25 points, 5 points each)

In the figure below, the downward sloping straight line connecting the points (0,5) and (4,0) represents the opportunities for investment in the capital market (money market), and the downward sloping curved line connecting the points (0,4) and (2.6,0) represents the opportunities for investment in physical capital (e.g. plant and machinery). The only asset at time 0 is \$2.6 million in cash (initial endowment). There is no additional endowment present at time 1.



Please answer the following questions related to the figure:

a. What is the interest rate, r , and what is the slope of the capital market line?

To determine the interest rate, r, we use the money market line and solve

$$4(1 + r) = 5 \Rightarrow 1 + r = 5/4 = 1.25 \Rightarrow r = 0.25 \text{ or } 25\%$$

The slope of the capital market line (intertemporal budget constraint) is –(1+r) = -1.25.

b. How much should be invested in physical capital (plant and equipment), and how much will this investment be worth next year?

Optimal investment occurs where the slope of production function (MRT) = slope of budget constraint = -(1 + r). This tangency point occurs when 2.6 – 1.6 = 1 M is invested. The investment of 1 M returns 3 M next year.

c. What is the present value (PV) and net present value (NPV) of this investment?

The PV and NPV of the investment project is

$$PV = 3/1.25 = 2.4 \text{ M}$$
$$NPV = 2.4 - 1 = 1.4 \text{ M}$$

d. What is the optimal consumption at times 0 and 1?

The optimal allocation of consumption at times 0 and 1 is determined by the tangency point of the IC with the budget constraint. From the figure, we see that optimal consumption at time 0 is 1 M and optimal consumption at time 1 is 3.75 M.

e. How much is borrowed or lent at time 0?

The owner lends 1.6 – 1 = 0.6 M (at r = 25%) today and receives 0.6(1.25) = 0.75 M next period. Consumption next period is then consists of the 3M investment return plus the 0.75 M money market return (= 3.75 M).*

II. Present Value Computations (25 points, 5 points each)

1. You have $1,000 to invest today in a money market account that pays an annual interest rate of r.

a. If you invest the $1000 for T years, what is the future value of the investment as a function of r?

The FV of the investment is $FV = \$1,000(1+r)^T$

b. Using your answer from part a above, determine how many years it will take for your investment to double as a function of r. How many years will it take for your investment to double if $r = 2\%$, 5% and 10%.

To find out how many years it will take for the investment of $1000 to double, we solve the equation

$$FV = \$2,000 = \$1,000(1+r)^T$$

for T. Dividing both sides by $1,000, taking logs of both sides and solving for T gives

$$\ln(2) = T\ln(1+r) \Rightarrow T = \frac{\ln(2)}{\ln(1+r)} = \frac{0.6931}{\ln(1+r)} \approx \frac{0.7}{r}$$

The approximation above uses the fact that $\ln(1+r) \approx r$ *if r is close to zero. If we express the interest rate in percent (e.g., multiply r by 100), then this result is known as the "rule of 70". That is,*

$$T \approx \frac{70}{r \times 100}$$

Using the rule of 70, it is easy to determine the number of years for an investment to double. Just divide 70 by the annual growth rate as a percent. Using the rule of 70, if r=2%, 5% and 10% then the corresponding values of T are

$$T \approx \frac{70}{2} = 35$$
$$T \approx \frac{70}{5} = 14$$
$$T \approx \frac{70}{10} = 7$$

3. At the University of Washington, employees have the ability to save for retirement in a tax deferred 403B program administered by TIAA-CREF. This program is a defined benefit program in that employees put aside a percentage of their salary in specific mutual fund administered by TIAA-CREF, and these funds grow tax free until the employee retires. Upon retirement the employee has accumulated savings from the program and is given the option of purchasing a various annuities from TIAA-CREF to replace the lost monthly salary. For this question, suppose a retired professor has accumulated savings of $1 million upon retirement.

a. Suppose TIAA-CREF offers a 20 year annuity making annual payments starting next year. If the discount rate is 3 percent per year, what is the annual annuity payment?

Here we use the finite annuity formula:

$$PV = C \times PVA(r,T)$$

$$PVA(r,T) = \frac{1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

and solve for the annuity payment C:

$$\$1{,}000{,}000 = C \times PVA(0.03,20) \Rightarrow C = \frac{\$1{,}000{,}000}{14.87747} = \$67{,}215.71$$

b. Suppose TIAA-CREF offers a 20 year annuity making annual payments starting next year, and that the payments increase with the inflation rate. If the inflation rate is 2 percent per year and the discount rate is 3 percent per year, what is the first annuity payment?

Now we use the finite growing annuity formula:

$$PV = C \times PVGA(r,g,T)$$

$$PVGA(r,g,T) = \frac{1}{r-g}\left(1 - \frac{(1+g)^T}{(1+r)^T}\right)$$

and solve for the first annuity payment C:

$$\$1{,}000{,}000 = C \times PVGA(0.03,0.02,20) \Rightarrow C = \frac{\$1{,}000{,}000}{17.7267} = \$56{,}412.09$$

c. Suppose TIAA-CREF offers a 20 year annuity making *monthly* payments starting next month. If the discount rate is 3 percent per year, what is the *monthly* annuity payment assuming monthly compounding?

This is the same as part a, except now we must use monthly compounding:

$$PV = C \times PVA(r/12, T \times 12)$$

$$PVA(r/12, T \times 12) = \frac{1}{r/12}\left(1 - \frac{1}{(1+r/12)^{T\times 12}}\right)$$

Solving for the monthly payment C gives

$$\$1{,}000{,}000 = C \times PVA(0.03/12, 20 \times 12) \Rightarrow C = \frac{\$1{,}000{,}000}{180.3109} = \$5{,}545.98$$

III. Bond Pricing and the Term Structure of Interest Rates (25 points, 5 points each)

The following is a list of prices for zero coupon bonds (STRIPS) of various maturities (taken from finance.yahoo.com):

Maturity (years)	Price of zero coupon bond
1	96.56
2	92.48
3	88.42
4	84.65

The bond prices are quoted as a percentage of par (face) value, and the par value is $1,000. Please answer the following questions:

a. Calculate the spot rates associated with each bond, and plot the term structure of interest rates.

The spot rates are computed using

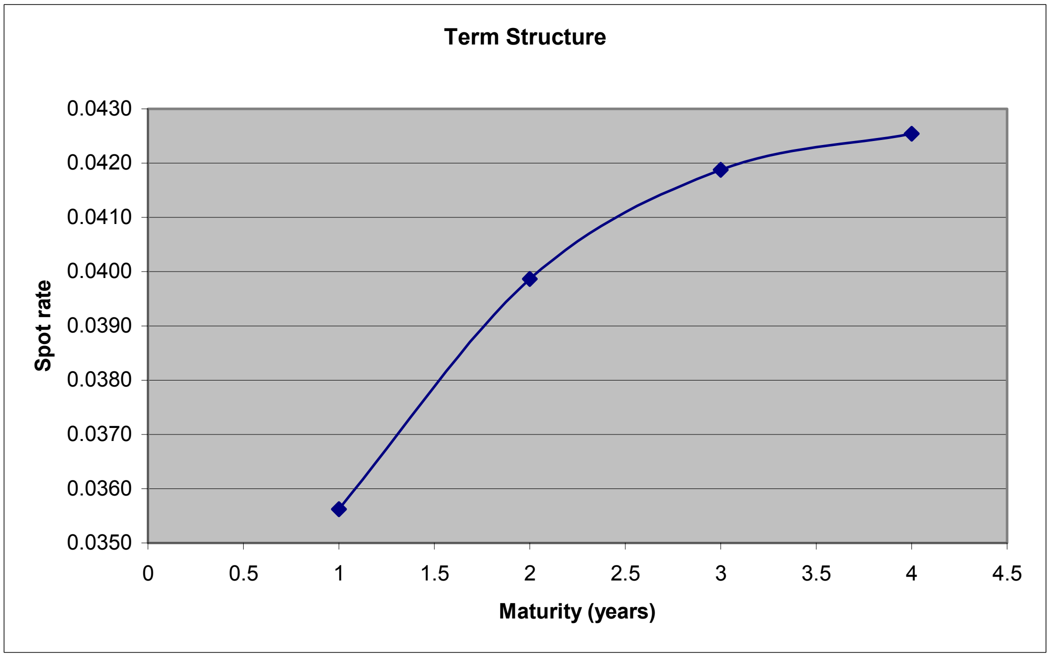
$$r_{0,1} = \left(\frac{1000}{965.6}\right) - 1 = 0.0356$$

$$r_{0,2} = \left(\frac{1000}{924.8}\right)^{1/2} - 1 = 0.0399$$

$$r_{0,3} = \left(\frac{1000}{884.2}\right)^{1/3} - 1 = 0.0419$$

$$r_{0,4} = \left(\frac{1000}{846.5}\right)^{1/4} - 1 = 0.0425$$

The term structure is plotted below.



b. Calculate the implied 1-year forward rates, $f_{t-1,t}$, for t=2, 3, 4

The implied forward rates are computed using

$$f_{1,2} = \frac{(1+r_{0,2})^2}{1+r_{0,1}} - 1 = 0.0441$$

$$f_{2,3} = \frac{(1+r_{0,3})^3}{\left(1+r_{0,2}\right)^2} - 1 = 0.0459$$

$$f_{3,4} = \frac{(1+r_{0,4})^4}{\left(1+r_{0,1}\right)^3} - 1 = 0.0445$$

c. If the expectations hypothesis of the term structure holds, what does the information in the yield curve say about the course of future interest rates?

The expectations hypothesis states that the implied forward rates are the best forecasts of the future spot rates. Using the implied forward rates from part b, we see that the expectations hypotheses predicts slightly rising one year spot rates for two years and then a slight decrease.

d. What is the price of a 2 year coupon bond making annual coupon payments with a annual coupon rate of 3% and a face value of $1,000?

$$PV = C_0 + \frac{C_1}{1+r_{0,1}} + \frac{C_2}{\left(1+r_{0,2}\right)^2}$$

$$= \frac{\$30}{1.0356} + \frac{\$1,030}{1.0399^2} = \$981.51$$

Extra credit (5 points). Compute the value of the yield-to-maturity on the 2 year coupon bond from part d above.

To find the yield-to-maturity on the bond, solve the following equation:

$$\$981.51 - \frac{\$30}{1+r} - \frac{\$1030}{(1+r)^2} = 0$$

This is a quadratic equation. The positive root is given by $r = 0.0398$.

IV. Valuing Stocks (25 points, 5 points each)

1. Microsoft stock is expected to pay an annual dividend of $0.32 at the end of the year, and then the dividend is expected to grow at 5% per year forever. The required rate of return (market capitalization rate) on Microsoft stock is 10% per year.

a. What is the current price of Microsoft stock?

Here we use the growing perpetuity formula:

$$P = \frac{D}{r-g} = \frac{\$0.32}{0.10-0.05} = \$6.40$$

b. Suppose the expected growth rate of the dividend changes from 5% to 6%. What is the expected percentage change in the stock price after this increase in the dividend growth rate?

This problem can be solved in two ways. The exact answer follows from valuing the stock at the new growth rate and then computing the percentage change in price

$$P^{new} = \frac{DIV}{r-g} = \frac{\$0.32}{0.10-0.06} = \frac{\$0.32}{0.04} = \$8$$

$$\%\Delta P = (P^{new} - P)/P = (8-6.4)/6.4 = 0.25 = 25\%$$

An approximate answer uses the derivative of P with respect to g:

$$\frac{dP}{dr} = -P(r-g)^{-1} \Rightarrow \frac{dP}{P} = -(r-g)^{-1}dr$$

$$= -(0.1-0.05)^{-1}(0.06-0.05) = -20(0.01) = 0.2 = 20\%$$

Notice that the approximate answer is a little off.

2. Google stock is currently selling for $381.89, its earning per share (EPS) is $4.51 and its price-earnings ratio (P/E) is 84.60. Assume that Google has an annual market capitalization rate of 20%.

a. What is the present value of growth opportunities (PVGO) for Google? What percentage of the current stock price does PVGO represent?

Here we use the formula

$$P = \frac{EPS}{r} + PVGO \Rightarrow PVGO = P - \frac{EPS}{r} = \$381.89 - \frac{\$4.51}{0.20} = \$359.34$$

3. You are a financial analyst at a large investment firm and your job is to forecast the price of Boondogle stock. Currently Boondogle does not pay any dividend, but you forecast that Boondogle will pay a dividend of $2 starting 3 years from today and that the dividend will grow at 5% per year forever afterwards. Your boss tells you to assume a market capitalization rate of 8% per year for Boondogle stock. Given this information, what is your forecast of the current price of Boondogle stock?

At time 2, the stock may be viewed as a growing perpetuity. The price at time 2 is therefore

$$P_2 = \frac{D}{r-g} = \frac{\$2}{0.08-0.05} = \$66.67$$

However, we want the price today. That price is

$$P_0 = \frac{P_2}{(1+r)^2} = \frac{\$66.67}{(1.08)^2} = \$57.16$$

V. Capital budgeting (10 points)

Starbucks has to choose between two industrial strength espresso machines which do the same job but have different lives: the Italian machine provides service for two years, while the French machine provides service for three years. The two machines have the following costs, expressed in real terms: (Neither has a scrap value at the end of its life.)

Year	Italian Machine	French Machine
0	$100,000	$125,000
1	$6,000	$4,000
2	$6,000	$4,000
3		$4,000

Assuming that the opportunity cost of capital is 5 percent in real terms and ignoring taxes, which machine would you choose? Explain and show your work.

We discussed two solutions to this type of problem. The first solution is based on computing the equivalent annual cost (EAC) for each project. This is based on first computing the PV of the costs for the two projects, and then computing the fixed annual payment (EAC) that has the same PV of cost. For PV of the costs for the two projects are

$$PVC_{Italian} = \$100,000 + \frac{\$6,000}{1.05} + \frac{\$6,000}{(1.05)^2} = \$111,156.46$$

$$PVC_{French} = \$125,000 + \frac{\$4,000}{1.05} + \frac{\$4,000}{(1.05)^2} + \frac{\$4,000}{(1.05)^3} = \$135,892.99$$

The EAC are computed using the finite annuity formula

$$EAC = \frac{PVC}{PVA(r,T)},\ PVA(r,T) = \frac{1}{r}\left(1 - \frac{1}{(1+r)^T}\right)$$

For the Italian machine, PVA(r, T) is

$$PVA(0.05,2) = \frac{1}{0.05}\left(1 - \frac{1}{(1.05)^2}\right) = \$1.86$$

For the French machine, PVA(r, T) is

$$PVA(0.05,4) = \frac{1}{0.05}\left(1 - \frac{1}{(1.05)^3}\right) = \$2.72$$

Therefore, the EAC values for the Italian and French machines are

$$EAC_{Italian} = \frac{\$111,156.46}{\$1.86} = \$59,780.49$$
$$EAC_{French} = \frac{\$135,892.99}{\$2.72} = \$49,901.07$$

Since the EAC for the French machine is less, choose it over the Italian machine..

The second method is based on replicating the projects enough times so that they cover the same number of periods and then computing the PV of each project. Here, the Italian machine must be replaced 2 times (in years 3, 6) and the French machine must be replaced once (in year 3). Under this approach the PV of costs are computed using

$$PV_{Italian} = \$111,156.46 + \frac{\$111,156.46}{(1.05)^2} + \frac{\$111,156.46}{(1.05)^4} = \$303,427.35$$
$$PV_{French} = \$135,892.99 + \frac{\$135,892.99}{(1.05)^3} = \$253,282.47$$

As with the equivalent annual cost approach, the French machine is cheaper.