Econ 422
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Fall 2005
Final Exam

This is a closed book exam. However, you are allowed one page of notes (double-sided). Answer all questions. For the numerical problems, if you make a computational error you may still receive full credit if you provide the correct formula for the problem. Total points = 100. There is one extra credit problem worth 10 points. You have 2 hours and 10 minutes to complete the exam. Good luck.

I. Random Variables (20 points total, 5 points each)

1. Consider the following probability distribution for next year's price and dividend payment on Amazon stock

State of the world	Recession	Normal	Boom
Probability	0.2	0.6	0.2
P_1	\$70	\$80	\$90
D_1`	0	0	5

a. Compute the expected value of next year's price and next year's dividend. That is, compute E[P_1] and E[D_1].

$$E[P_1] = 70(0.20) + 80(0.60) + 90(0.2) = 80$$
$$E[D_1] = 0(0.20) + 0(0.60) + 5(0.2) = 1$$

b. Suppose the current price of Amazon stock is \$75. Compute the expected annual total return on an investment in Amazon stock. That is, compute E[R_1].

$$E[R] = \frac{E[P_1] + E[D_1] - P_0}{P_0} = \frac{80 + 1 - 75}{75} = 0.08$$

c. Compute the covariance between P_1 and D_1. That is, compute COV(P_1, D_1).

$$COV(P_1, D_1) = E[P_1 D_1] - E[P_1]E[D_1]$$
$$E[P_1 D_1] = (70)(0)(0.2) + (80)(0)(0.6) + 90(5)(.2) = 90$$
$$E[P_1 D_1] - E[P_1]E[D_1] = 90 - 80(1) = 10$$

2. In class, we looked at the historical distributions of annual returns on a number of different types of investments. In many instances, these historical distributions resembled normal distributions. The table below replicates some of these results:

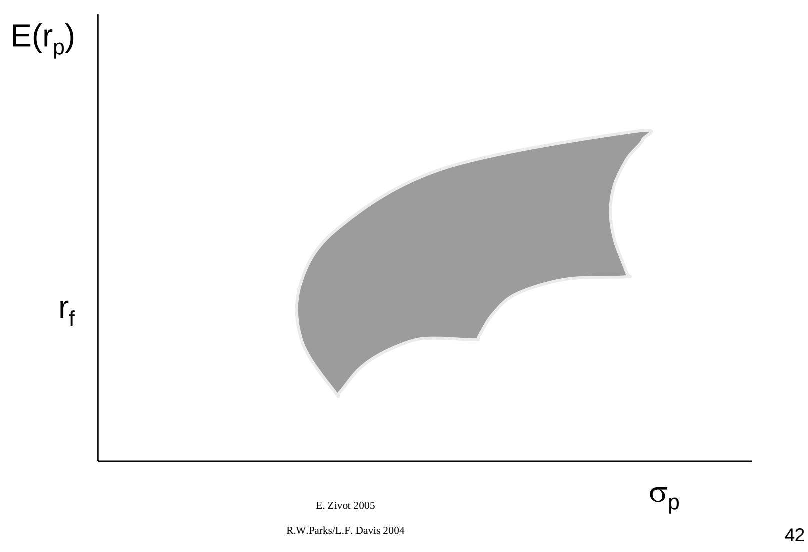
Asset	Sample Mean	Sample Standard Deviation
Small Capitalization Stocks	0.17	0.33
Large Capitalization Stocks	0.13	0.20

a. Using the information in the table above, sketch the normal distributions for annual returns on small stocks and large stocks implied by the mean and standard deviation values. On the graphs of the normal distributions, indicate the location of the mean and the location of the mean ± standard deviation.

II. Portfolio Theory (25 points total, 5 points each)

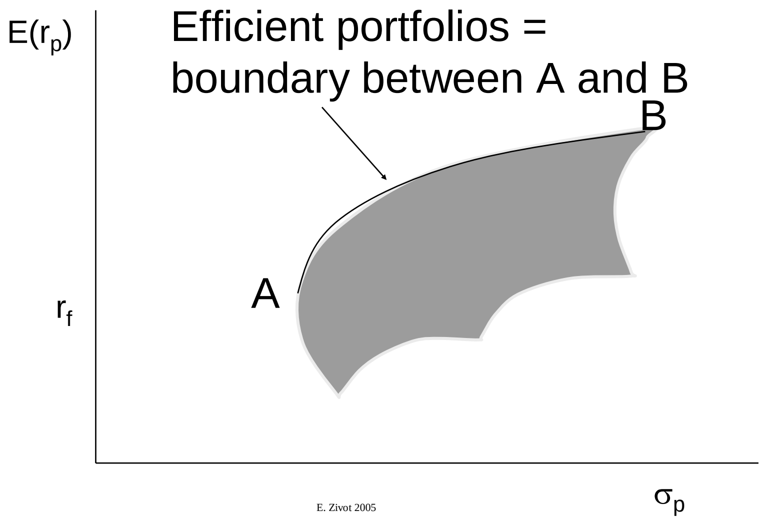
The graph below shows the expected return-standard deviation tradeoffs among portfolios of risky assets. The risk free T-bill has return r_f. Transfer the graph below to your blue book and use it to answer the following questions.

Set of Capital Market Risky Portfolios



a. Indicate the location of the efficient portfolios that are combinations of the risky assets only (excluding the risk free T-bill)

Set of Capital Market Risky Portfolios



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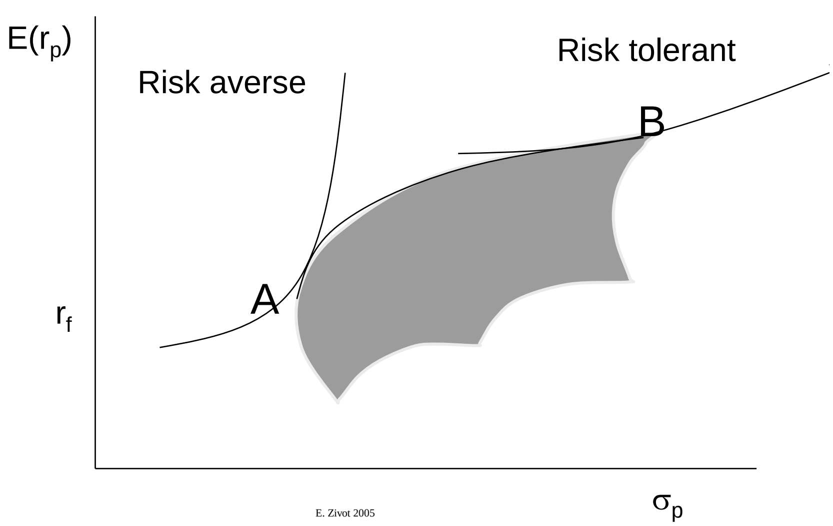
b. On the set of risky asset efficient portfolios, indicate a representative portfolio that would be chosen by a very risk averse investor and a representative portfolio that would be chosen by a risk tolerant investor.

Set of Capital Market Risky Portfolios



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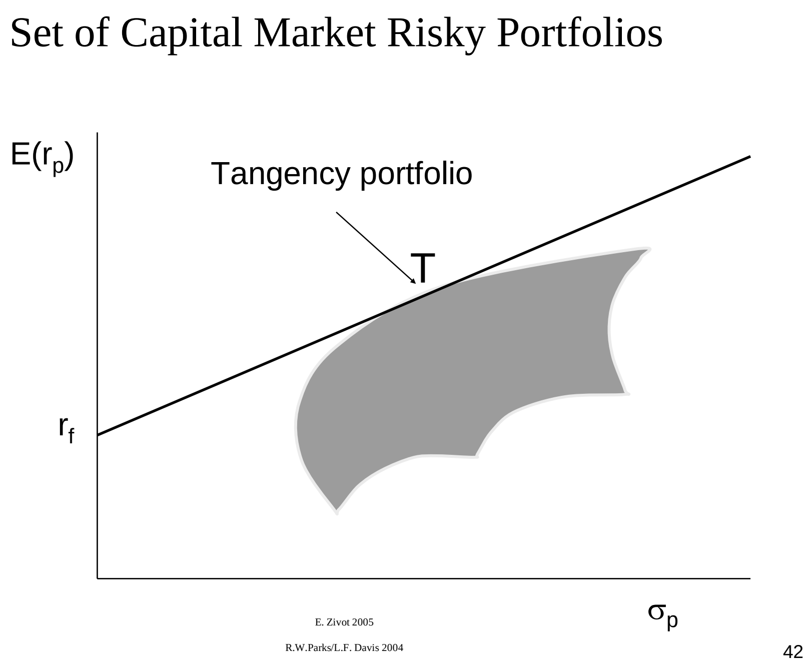
c. Sketch the set of efficient portfolios that are combinations of the risky assets and the risk free T-bill.



d. Using your answer to part (c), briefly explain the mutual fund separation theorem.

The mutual fund separation theorem says that the set of efficient portfolios of risky assets and T-bills can be described by simple combinations of two portfolios: (1) T-bills; (2) Tangency portfolio (portfolio of risky assets with the highest Sharpe's slope).

e. Transfer the graph below to your blue book. In the graph below, the annual T-bill rate is 3%, the annual expected return on the S&P 500 index is 8%, and the standard deviation of the annual return on the S&P 500 index is 10%. Determine the portfolio of T-bills and the S&P 500 that achieves an expected return of 10%, and show this portfolio on the graph. What is the standard deviation of the return on this portfolio?

Since efficient portfolios are simple combinations of T-bills and the S&P 500, we know that

$$E[R_p] = r_f + x_{sp500}(E[R_{sp500}] - r_f)$$
$$\Rightarrow x_{sp500} = \frac{E[R_p] - r_f}{E[R_{sp500}] - r_f}$$

Since our target portfolio expected return is 10% we solve

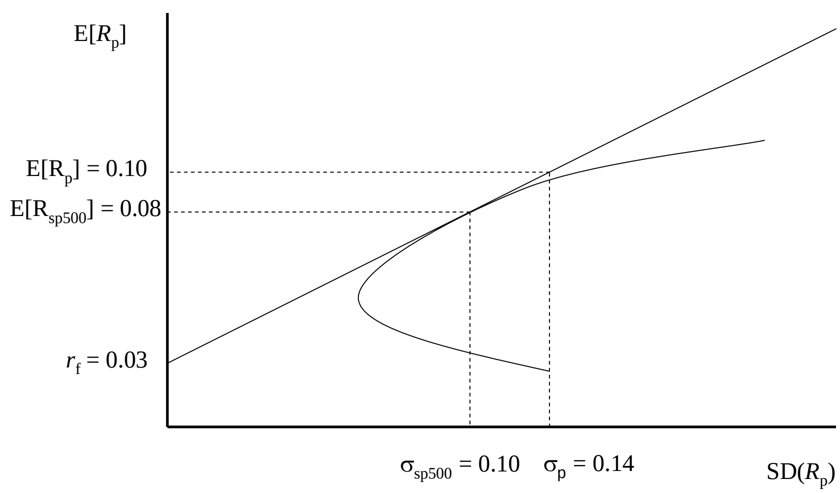
$$x_{sp500} = \frac{0.10 - 0.03}{0.08 - 0.03} = \frac{0.07}{0.05} = 1.4$$

The expected return on this portfolio is

$$E[R_p] = 0.03 + 1.4(0.08 - 0.03) = 0.10$$

The standard deviation of the return on this portfolio is

$$SD(R_p) = x_{sp500} SD(R_{sp500}) = 1.4(0.10) = 0.14$$



III. CAPM 20 points, 5 points each)

1. As a financial analyst in the capital budgeting division of Starbucks Corporation, you must make a recommendation to your boss regarding a proposed project to open a new store. As part of your analysis you come up with the following table showing the expected nominal cash flows associated with the project:

Year	0	1	2	3
Cash flow	-1,000,000	300,000	400,000	500,000

a. Assume that Starbucks Corporation is completely equity financed. If the annual T-bill rate is 3%, the market risk premium is 7%, and Starbucks' equity beta is 1.5, then do you recommend that Starbucks adopt the project?

First we must find the company cost of capital. Since Starbucks is all equity finance we can do this using the CAPM based on Starbucks equity beta:

$$E[R] = 0.03 + 1.5(0.07) = 0.135$$

Then we discount the project cash flows using the company cost of capital:

$$NPV = -1,000,000 + \frac{300,000}{1.135} + \frac{400,000}{(1.135)^2} + \frac{500,000}{(1.135)^3} = -83,212.5$$

Since the NPV is negative we recommend that Starbucks not do the project.

b. Now assume that Starbucks Corporation is partially debt financed and that its debt-to-equity ratio is 0.2. If the annual T-bill rate is 3%, Starbucks' equity beta is 1.5, and Starbucks' debt beta is 0.3, then do you recommend that Starbucks adopt the project?

To find the company cost of capital when the firm is partially debt finance we must determine the asset beta:

$$\beta_A = \frac{D}{D+E}\beta_D + \frac{E}{D+E}\beta_E$$

Using D/E = 0.2 we then solve

$$D = 0.2E \Rightarrow \frac{D}{D+E} = \frac{0.2E}{0.2E+E} = \frac{0.2}{1.2} = 0.167$$
$$\frac{E}{D+E} = 1 - \frac{D}{D+E} = 1 - 0.167 = 0.833$$

The asset beta is then

$$\beta_A = 0.167(0.3) + (0.833)(1.5) = 1.3$$

Using the CAPM the company cost of capital is

$$E[R] = 0.03 + 1.3(0.07) = 0.121$$

Discounting the project cash flows with the new company cost of capital gives

$$NPV = -1,000,000 + \frac{300,000}{1.121} + \frac{400,000}{(1.121)^2} + \frac{500,000}{(1.121)^3} = -59,134.4$$

Since the NPV is negative we recommend that Starbucks not do the project.

2. Suppose your broker gives you the following advice about investing in Zymogenetics stock: “You should not hold Zymogenetics stock in your portfolio because its annual return standard deviation is much higher than the annual return standard deviation on your existing portfolio”. Do you agree with your broker? Why or why not?

You should not necessarily agree with your broker. What matters is this contribution of Zymogenetics to the standard deviation of your portfolio return. This is determined by the beta of Zymogenetics with respect to your portfolio. If this beta is greater than one, then adding Zymogenetics to your portfolio will increase the variability of your portfolio and you should not add it. However, if the beta is less than one, then adding Zymogenetics to your portfolio will reduce the variability of your portfolio (a beneficial diversification effect) and you should consider adding it to your portfolio.

3. Suppose you own a portfolio composed of three Northwest stocks (Amazon, Boeing, Costco). The table below gives the betas (with respect to the S&P 500) of these stocks as well as their proportions in your portfolio:

Stock	Beta	Portfolio Weight
Amazon	1.5	0.3
Boeing	0.9	0.1
Costco	1.3	0.6

What is the beta of your portfolio?

The beta of your portfolio will be a weighted average of the betas of the individual stocks in the portfolio:

$$\beta_p = 1.5(.3) + 0.9(.1) + 1.3(.6) = 1.32$$

III. Market Efficiency (10 points total, 5 points each)

1. Consider the following types of information. If this information is immediately and fully reflected in security prices, what form of market efficiency is implied? (1 point for each part)

a. A company’s recent quarterly earnings announcement

semi-strong-form

b. Historical bond yields

weak-form

c. Deliberations of a company’s board of directors concerning a possible merger with another company *strong-form*

d. A brokerage firm's published research report on a particular company

semi-strong form

e. Movements in the Dow Jones Industrial Average as plotted in *The Wall Street Journal*.

weak-form

2. When investment managers present their historical performance records to potential and existing customers, market regulators require the managers to qualify those records with the comment that "past performance is no guarantee of future performance". In a perfectly efficient market, why would such a statement be particularly appropriate?

In a perfectly efficient market, all assets should be fairly priced. In other words, the expected return from investing in any asset should be based on the appropriate risk of the asset. High risk assets (as measured by beta, say) should generate higher average returns than lower risk assets. Notice that the statement is about expected returns. Even in a perfectly efficient market, sometime returns will be higher than expected and sometimes returns will be lower than expected. This is due to the random nature of returns. However, in an efficient market if returns turn out to be higher than expected in the past there is no reason to believe that they will continue to be higher than expected in the future.

IV. Options (25 points, 4 points each part)

1. Consider a European put option on a non-dividend paying stock. What five variables determine the value of the put option? For each variable, indicate how the value of the put option changes (+ or -) when the variable increases (holding all other variables fixed).

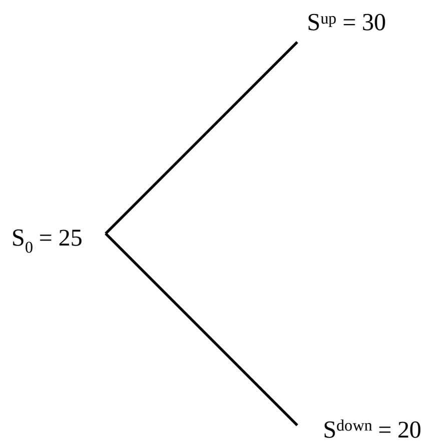
From put-call parity, the variables that determine a European put on a non-dividend paying stock are the same as those that determine a European call on a non-dividend paying stock. These are: S, X, T, r_f*,* σ*. To determine how increases in each of these variables influence the value of a put, use the put-call parity relationship*

$$C + PV(X) = S + P \Rightarrow P = C + PV(X) - S$$

$$= C(\overset{+}{S}, \overset{-}{X}, \overset{+}{T}, \overset{+}{r_f}, \overset{+}{\sigma}) + PV(X) - S$$

Increases in S reduce put value (because the payoff is X – S); increases in X increase put value (because the payoff is X – S); increases in T increase put value; It is not clear how increases in r_f *will affect the put value because increases in* r_f *increase the call value but reduce PV(X); Increases in* σ *will increase put value.*

2. Consider using the simple binomial model to value one year European call and put options on Starbucks stock with an exercise price of $25. The tree diagram below shows the hypothesized evolution of Starbucks stock over the next year:



Assume that the annual T-bill rate is 3%.

a. At the expiration date of the options, what are the values of the call and the put if the stock price goes up and what are the values if the stock price goes down?

At expiration of the option (1 year) the values of the call and put will be

up state: $C^{up} = \max(0, 30-25) = 5; P^{up} = \max(0, 25-30) = 0$
down state: $C^{down} = \max(0, 20-25) = 0; P^{down} = \max(0, 25-20) = 5$

b. What is the current value of the call option?

To determine the value of the call, we can use the replicating portfolio method. We want to find a portfolio of the stock and a risk free bond (with face value $1) that exactly replicates the payoffs on the call option at maturity:

up state: $x_s(30) + x_b = 5$
down state: $x_s(20) + x_b = 0$
$\Rightarrow x_b = -20x_s$
$\Rightarrow 30x_s - 20x_s = 5 \Rightarrow x_s = \frac{5}{30-20} = 0.5$
$\Rightarrow x_b = -20(0.5) = -10$

To avoid arbitrage opportunities, it must be the case that the current value of the replicating portfolio equals the current value of the call option. The current value of the replicating portfolio is

$$C = 0.5(25) - \frac{10}{1.03} = 2.791$$

Alternatively, the call can be valued using risk neutral probabilities. In a risk neutral world, the expected return on all assets should be the risk free rate. Using this rule with the stock allows us to determine the risk neutral probabilities:

$$E[R] = p(0.2) + (1-p)(-0.2) = 0.03 = r_f$$
$$\Rightarrow p = \frac{0.23}{0.40} = 0.575,\ 1-p = 0.425$$

The value of the call is the expected payout discounted at the risk free rate:

$$C = \frac{C^{up}p + C^{down}(1-p)}{1+r_f} = \frac{5(0.575) + 0(0.425)}{1.03} = 2.791$$

c. Given the current value of the call option, determine the current value of the put option.

To determine the value of the put, use the put-call parity relationship:

$$C + PV(X) = S + P \Rightarrow P = C + PV(X) - S$$
$$\Rightarrow P = 2.791 + \frac{25}{1.03} - 25 = 2.063$$

d. What happens to the current values of the call and put options if the T-bill rate rises to 4% per year?

Here we can simply repeat the above calculations using the higher T-bill rate. Notice that the increase in the T-bill rate does not change the composition of the replicating *portfolio. The only change comes when we determine the current value of the replicating portfolio:*

$$C = 0.5(25) - \frac{10}{1.04} = 2.885$$

Here we see that the call increases in value. By put-call parity

$$P = 2.885 + \frac{25}{1.04} - 25 = 1.830$$

and we see that the put value declines.

VI. Extra Credit (10 points, 5 points each)

Consider an individual whose utility of wealth function is $U(W)=W^{1/3}$. If the person chooses occupation A, his or her wealth is given by the following wealth distribution:

Wealth:	1,000,000	2,000,000
Probability:	0.8	0.2

If the person chooses occupation B, his or her wealth is given by the following wealth distribution:

Wealth:	1,100,000	1,300,000
Probability:	0.5	0.5

a. Which occupation will the person choose and why? Or will the person be indifferent to the alternatives? Explain.

The individual will choose the occupation that maximizes the expected utility of wealth. The expected utilities for the two occupations are

$$E[U(W_A)] = 0.8(1,000,000)^{1/3} + 0.2(2,000,000)^{1/3} = 105.198$$
$$E[U(W_B)] = 0.5(1,100,000)^{1/3} + 0.2(1,300,000)^{1/3} = 106.184$$

Since occupation B gives the higher expected utility, choose B.

b. Compute the certainty equivalent wealth and risk premium for occupation A.

The certainty equivalent wealth is the amount of wealth with certainty that gives the same expected utility as the gamble: $U(W^c) = E[U(W_a)]$. *Therefore, we solve*

$$(W^c)^{1/3} = 105.198 \Rightarrow W^c = (105.198)^3 = \$1,164,200$$