Introduction to Options

Econ 422: Investment, Capital &
Finance
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Derivatives

- A derivative is a security whose payoff or value depends on (is derived from) the value of another security, the 'underlying' security.

- Derivatives are referred to as contingent claims, the claim is contingent on the underlying asset.

- Examples of derivatives:
  – Options
  – Forward Contracts
  – Futures
  – Swaps

Financial & Non-Financial Option Examples

Financial Examples:

- Traded Options (CBOE, NYSE)
- Convertible bond (embedded option within debt instrument)
- Prepayment option on mortgage
- Venture Capital follow-on investment option
- Employee stock options

Non-Financial Examples:

- Enrollment in this course— 'right to attend lectures but not obligated'

Financial 'Option' Definitions

- An option provides you the right, without obligation, to buy or sell an asset at a pre-specified price in the future.

- An option is a *derivative* security in that its value is contingent upon the underlying asset.

Financial 'Option' Definitions

- 'Plain vanilla' options:
  - **Call Option**: provides the holder/owner the right to *buy* an asset at a pre-specified price in the future.
  - **Put Option**: provides the holder/owner the right to *sell* an asset at a pre-specified price in the future.
  - 'specific price' = *Strike Price*
  - 'asset' referred to as 'underlying'
  - 'price' of option = 'value' or 'premium' of option
  - 'expiration' date = 'maturity'

Financial 'Option' Definitions

Two types of exercise rights:

- **European option**: rights can only be invoked or 'exercised' on a specific date = expiration date

- **American option**: rights can be invoked or 'exercised' any time on or before the expiration date

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MSFT June 10 Call Options

June 1 close price: 25.89

View By Expiration: **Jun 10** | Jul 10 | Oct 10 | Jan 11 | Jan 12

CALL OPTIONS   Expire at close Friday, June 18, 2010

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
22.00	MSFT100619C00022000	3.30	0.00	3.90	4.00	0	5
23.00	MSFT100619C00023000	3.30	↑0.30	2.96	3.05	101	410
24.00	MSFT100619C00024000	2.27	↑0.02	2.08	2.14	597	4,812
25.00	MSFT100619C00025000	1.33	↓0.01	1.28	1.33	1,979	13,780
26.00	MSFT100619C00026000	0.67	↑0.04	0.67	0.70	7,123	29,756
27.00	MSFT100619C00027000	0.32	↑0.05	0.28	0.32	4,719	16,640
28.00	MSFT100619C00028000	0.14	↑0.04	0.11	0.13	1,091	22,544
29.00	MSFT100619C00029000	0.06	↑0.01	0.05	0.06	498	11,291
30.00	MSFT100619C00030000	0.02	0.00	0.02	0.03	218	28,034
31.00	MSFT100619C00031000	0.02	0.00	0.01	0.02	127	20,413
32.00	MSFT100619C00032000	0.01	0.00	N/A	0.01	12	29,798
33.00	MSFT100619C00033000	0.01	0.00	N/A	0.01	32	14,360
34.00	MSFT100619C00034000	0.01	0.00	N/A	0.01	20	5,692
35.00	MSFT100619C00035000	0.01	0.00	N/A	0.01	0	1,570

MSFT June 10 Put Options

June 1 close price: 25.89

PUT OPTIONS   Expire at close Friday, June 18, 2010

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
20.00	MSFT100619P00020000	0.04	↑0.01	0.02	0.03	15	385
22.00	MSFT100619P00022000	0.05	↓0.03	0.07	0.08	5	1,126
23.00	MSFT100619P00023000	0.13	0.00	0.13	0.14	685	5,162
24.00	MSFT100619P00024000	0.24	↓0.01	0.23	0.25	1,174	14,507
25.00	MSFT100619P00025000	0.43	↑0.03	0.43	0.46	2,626	39,594
26.00	MSFT100619P00026000	0.82	0.00	0.79	0.83	2,282	29,558
27.00	MSFT100619P00027000	1.45	↑0.07	1.41	1.47	698	10,192
28.00	MSFT100619P00028000	2.20	↑0.05	2.22	2.28	404	13,191
29.00	MSFT100619P00029000	2.81	↓0.39	3.15	3.25	158	10,198
30.00	MSFT100619P00030000	3.90	↓0.11	4.10	4.20	98	10,025
31.00	MSFT100619P00031000	4.85	↓0.33	5.05	5.20	34	5,213
32.00	MSFT100619P00032000	5.39	0.00	6.05	6.20	0	2,957
33.00	MSFT100619P00033000	6.75	↓0.40	7.10	7.20	30	894
34.00	MSFT100619P00034000	7.75	↓0.40	8.10	8.20	38	33
35.00	MSFT100619P00035000	8.75	↓0.40	9.10	9.20	96	65

Time T Payoff for Call Option on MSFT

- Suppose you own a *European Call option* on MSFT
- X = strike price = 25
- T = maturity date (eg. June 18, 2010)
- S(T) = share price of MSFT stock at maturity
- Assume MSFT does not pay dividends
- C(T) = value of call option at maturity

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Time T Payoff for Call Option on MSFT

- S(T) < X => Call option is worthless (out of the money) => C(T) = 0
- S(T) > X => Call option is exercised and has value S(T) – X (in the money) => C(T) = S(T) – X
- General formula for payoff at maturity:
$$C(T) = \max\{0, S(T) - X\}$$

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Time T Payoff for Call Option

Consider you own a *European Call option* on the stock of MSFT.
MSFT does not issue dividends.
S = share price of MSFT stock
X = strike price of call option

Let C(T) = value of the call option *at expiration, time T*

If S < X $\Rightarrow$ option is worthless ("out-of-the-money")
         $\Rightarrow$ C(T) = 0

If S > X $\Rightarrow$ option has value ("in-the-money")
         $\Rightarrow$ C(T) = S - X

Graphically: C(T) = max{0,S-X}

C(T)
Value of call option at expiration

S - X

Question: Why would you want to own/hold a call option?

X   **S(T)**

Value of underlying at expiration

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Time T Payoff for Put Option on MSFT

- Suppose you own a *European Put option* on MSFT
- X = strike price = 25
- T = maturity date (eg. June 18, 2010)
- S(T) = share price of MSFT stock at maturity
- Assume MSFT does not pay dividends
- P(T) = value of put option at maturity

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Time T Payoff for Put Option on MSFT

- $S(T) < X \Rightarrow$ Put option is exercised (in the money) and has value $P(T) = X - S(T)$
- $S(T) > X \Rightarrow$ Put option is worthless (out of the money) and has value $P(T) = 0$
- General formula for payoff at maturity:
$$P(T) = \max\{X - S(T), 0\}$$

Time T Payoff for Put Option

Consider you own a *European Put option* on the stock of MSFt company.
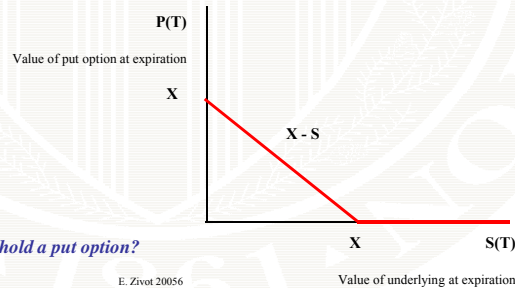S = share price of MSFT stock
X = strike price of put option

Let P(T) = value of the put option *at expiration, time T*

If S < X $\Rightarrow$ option has value ("in-the-money")
           $\Rightarrow$ P(T) = X-S
           (By exercising the Put option ,you sell stock at S and receive X in return.)

If S > X $\Rightarrow$ option is worthless ("out-of-the-money")
           $\Rightarrow$ P(T)= 0

Graphically: P(T) = max{0,X-S}

P(T)

Value of put option at expiration

X

X - S

X   **S(T)**

Question: Why would you want to own/hold a put option?

Value of underlying at expiration

Payoff vs. Profit

- Payoff diagram does not account for initial cost of call or put option.
  - e.g., cost today, $C(t)$, of Jun 10 MSFT call with strike price 25 is $0.46 plus commission costs (cc)
- Profit diagram subtracts cost of option contract from option payoff
  - Call profit: $\max(0, S(T)-X) - C(t) - cc$
  - Put profit: $\max(X-S(T), 0) - P(t) - cc$

Writing Options

- Previously we consider the payoffs associated with owning (long position) call or put options
- What if instead of holding/owning the call and/or put options, you wrote/sold them (short position)?
- What are the payoffs and profits associated with writing options?

Short the Options

- **Short the Call**—By shorting a call, you sell the right, but not the obligation to someone else to purchase from you an asset at a given price at some point in the future.

Denote -C(T) as being short the value of the call option at expiration, time T

Note:

If S < X ⟹option is worthless ("out-of-the-money"), the owner will not purchase your asset from you, i.e., does not exercise the option
⟹-C(T) = 0

If S > X ⟹option has value ("in-the-money"), the owner will exercise option and purchase your asset
⟹ -C(T) = X - S
(You receive X from the holder of the option and in return give up your stock)

Graphically: -C(T) = max{0,-(S-X)} or max{0,X-S}

- **Short the Put**—By shorting a put, you sell the right, but not the obligation to someone else to sell to you an asset at a given price at some point in the future.

Denote -P(T) as being short the value of the put option at expiration, time T

Note:

If S < X ⟹option has value ("in-the-money"), the holder of the option will want you to purchase their asset at a price X greater than the asset price
⟹ -P(T) = S - X
(You pay X to receive the asset valued at S.)

If S > X ⟹ option is worthless ("out-of-the-money"), option is not exercised such that you are not required to purchase the asset
⟹- P (T) = 0

Graphically: - P(T) = max{0,-(X-S)} or max{0, S-X}

C(T)

X   S(T)

X - S

P(T)

X   S(T)

S - X

-X

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Portfolio Insurance: Protective Put

- Purchase one share of stock with current price S_0
- Purchase one out-of-the money put option on same stock with exercise price $X < S_0$ with cost P_0
- Draw the payoff at maturity diagram (protective put)
- Draw the profit at maturity diagram

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Portfolio Insurance: Bond + Call

- Purchase one risk-free zero coupon bond with a face value of X
- Purchase one call option on stock with exercise price X
- Draw the payoff at maturity diagram

Bull Spread Strategy

- Buy 1 call option on MSFT stock with exercise price X_1. Cost is C_1
- Write 1 call option on MSFT stock with exercise price $X_2 > X_1$. Receive C_2
- Draw the payoff at maturity diagram of the option strategy (called Bull Spread)
- Draw the profit at maturity diagram of the option strategy
- Why would you consider such a strategy?

Straddle Strategy 1

- Buy 1 call option on MSFT stock with exercise price X. Cost of call is C_0
- Buy 1 put option on MSFT stock with exercise price X. Cost of put is P_0
- Draw the payoff at maturity diagram of the option strategy (called Straddle)
- Draw the profit at maturity diagram of the option strategy
- Why would you consider such a strategy?

Straddle Strategy 2

- Buy 1 share MSFT stock at price S_0
- Purchase 2 put options on MSFT stock with exercise price X. Cost of each put is P_0
- Draw the payoff at maturity diagram of the option strategy (called Straddle)
- Draw the profit at maturity diagram of the option strategy

Put-Call Parity

- Suppose you own a put option with strike price X and a share of the underlying stock?
- What is the value of your holdings at the maturity date of the option as a function of the underlying stock price?

Long the Put and Stock

- Long the Put:

  $P(T) = \max\{0, X\text{-}S\}$

- Long the Stock:

  $S(T) = S$

- Long the Put and the Stock:

  $P(T) + S = \max\{0, X\text{-}S\} + S$

  $P(T) + S = $ S for S > X

  X for S ≤ X

P(T)
X
X - S
X   S(T)

S
X
X   S(T)

(X – S) + S = X
S
X
X   S(T)
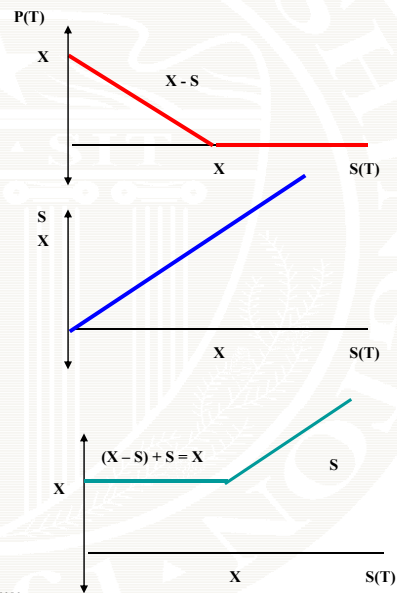
Payoff Diagrams

- Suppose you own a call option with strike price X and a riskless bond (T-Bill) with face value X that matures at the same time as the call option?

- What is the value of your holdings at the maturity date of the option as a function of the underlying stock price?

Long the Call and Risk-less Asset
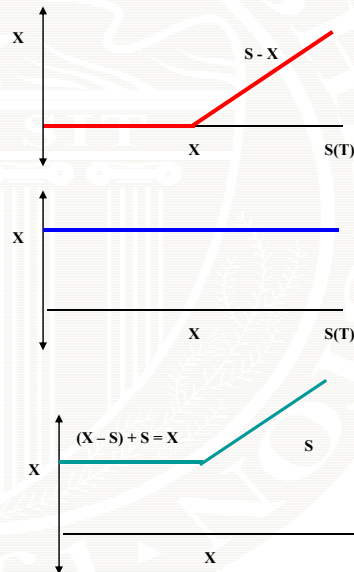
- Long the Call

  $C(T) = \max\{0, S\text{-}X\}$

- Long the Risk-less Asset

  $X(T) = X$

- Long the Call and Risk-less Asset

  $C(T) + X(T) = \max \{0, S\text{-}X\} + X$

  $C(T) + X(T) = X$ for $S \leq X$

  $\quad\quad S$ for $S > X$

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Put-Call Parity

•From our time T position diagrams:

$$P(T) + S(T) = C(T) + X(T)$$

•Put-Call Parity: Absence of arbitrage opportunities implies that the current value of above portfolios are equal

$$\text{Put} + \text{Stock} = \text{Call} + PV(X) = \text{Call} + X/(1+r_f)^{(T-t)}$$

Holds for all t, X is strike for both Put and Call, T is expiration for both contracts and PV(X) = X at expiration. Note, T – t is the time to maturity in years.

Example: Put-Call Parity

- SBUX stock is currently trading at $33 and does not pay a dividend
- 1 yr put option on SBUX with strike price X=$35 costs $2.10
- 1 yr T-Bill rate is $r_f = 0.10$
- Use put-call parity to determine the price of a 1 yr call option on SBUX with strike price X=$35

Example: Put-Call Parity

- Put-call parity:

  Put + Stock = Call + $X/(1+r_f)^{(T-t)}$

- $\$2.10 + \$33 = Call + 35/(1.1)^1$

  => Call = $\$2.10 + 33 - 35/1.1 = 3.28$
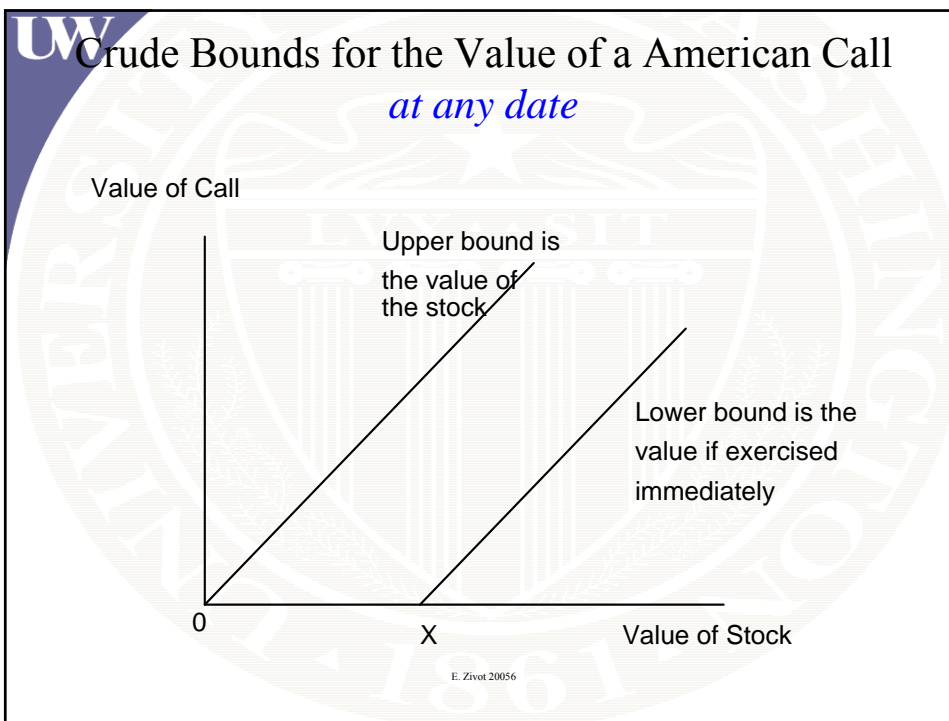
The Value of a Call Option at any date Depends on:

- The price of the underlying share S
- The exercise or strike price X
- The riskfree interest rate r_f
- The variability of the stock rate of return σ
- The amount of time to expiration (in years) T

[If the stock pays dividends, the option also depends on the anticipated dividends, D.]

$$C = f(S, X, r_f, \sigma, T, [D])$$

Crude Bounds for the Value of a American Call
at any date

Value of Call

Upper bound is
the value of
the stock

Lower bound is the
value if exercised
immediately

0

X

Value of Stock

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The value of an option is increased by increased variability

- Stock has a current price S=100
- Next period the price will be either: 80   or   120

(Assume that the two states are equally likely, so that the expected future stock price is 100.)

- Call option with X=100 and T=1

  The call will be worth either   0   or   20

- Now suppose that the variance (or standard deviations) of the future price distribution is increased, so that prices will be either 60 or 140. The option will pay off either 0 or 40. It must clearly be worth more.

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Value of Call when Stock Price is Large

Put-Call Parity:

$$\text{Put} + \text{Stock} = \text{Call} + PV(X) = \text{Call} + X/(1+r)^{(T-t)}$$

$$\Rightarrow \text{Call} = \text{Stock} + \text{Put} - X/(1+r)^{(T-t)}$$

When stock price is large, put value ≈ 0 and

$$\text{Call} \approx \text{Stock} - X/(1+r)^{(T-t)}$$

American Options with More Time to Maturity are Worth More

- Intuition: A longer maturity gives the option holder more rights (e.g., more time in which the option may be potentially exercised. Having more rights cannot lower the value of the Call

The Effect on the Value of a Call of Changing One of the Underlying Variables

$$C = f(S^+, X^-, r_f^{\ +}, \sigma^+, T^+, [D^-])$$

- An increase in S increases the price of a call
- An increase in X <u>lowers</u> the price of a call
- An increase in r_f increases the price of a call
- An increase in σ increases the price of a call
- An increase in T increases the price of a call
- [An increase in expected dividends lowers the price of a call.

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American vs. European Call Options

- Suppose that the underlying shares are expected to pay no dividends during the term of the option.
- Would you ever exercise the American option before expiration?
  - No, because value of call increases with maturity
  - Always better to sell the call than to exercise
- What does this imply about the relationship between the value of an American call and a European call on non-dividend paying shares?
  - They are the same!

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An Option View of the Value of Common Stock (Equity)

Consider an all-equity company (ABC) with balance sheet (at market values):

Assets 30	Debt 0
	Equity 30
30	30

The payoff from a call option on ABC's stock with X=50:

Asset Value at time T	Value of Call with X=50
40	0
50	0
60	10
70	20

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An Option View of the Value of Common Stock (Equity)

Consider leveraged company (XYZ) with balance sheet (at market values):

Assets 30	Debt 25
	Equity 5
30	30

(The debt requires a payment of 50 at time T.)

The payoff from XYZ's stock is:

Asset Value at time T	Value of the equity
40	0
50	0
60	10
70	20

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A Call Option View of Equity

- The example above shows:
- The equity of a leveraged firm has the same payoff as a call option
    - the exercise price equals the required payment of principal <u>and</u> interest on the debt
    - the option and the debt have the same maturity date.

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A Call Option View of the Risky Debt

- The bond holders own the firm's assets, but they have sold a call to the shareholders.
- If at expiration the firm's assets are worth more than the promised payments on the bonds, the shareholders exercise their call option by paying off the debt and buying back the firm.
- If the firm's assets are worth less than this exercise price, then the call option is not exercised. The firm defaults on its debt payments and the bondholders retain the firm. (which is worth less than they were promised.

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Put-Call Parity Gives An Alternative Option View

- Recall the put-call parity relationship:

  $$C = P + S - PV(X)$$

- Alternatively, the shareholders own the firm, but they owe the present value of X, the promised principal and interest on the debt for certain, <u>but</u> they also own a put option giving them the right to sell the firm's assets to the bondholders for X, which would exactly offset their debt obligation.

- [Note that in this case the assets of the firm are the underlying asset for the option.]

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Option Valuation Approaches

- Utilize notion that a portfolio comprised of the underlying stock and the present value of a loan which replicates the payoffs of the option will have the same value *(Replicating Portfolio approach)*.

1. Binomial Method (Discrete Time)

2. Risk Neutral Valuation

3. Black-Scholes Formula (Continuous Time)

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Valuation Approaches

Binomial Method

Stock price of ABC today: S = 100

Annual risk free rate is r_f = 0.05

6-month risk free rate is 0.05/2 = 0.025
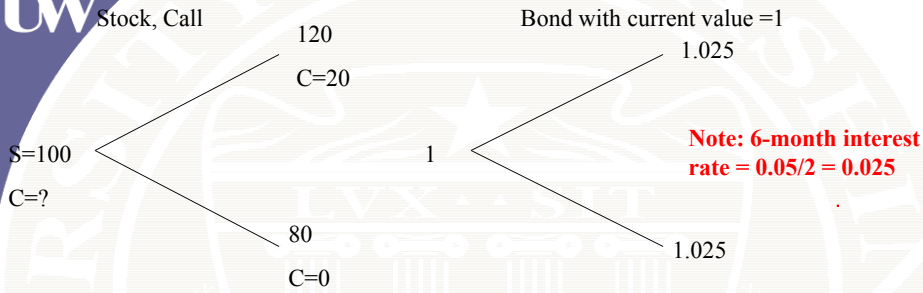
Stock price in six months is either S = 80 or
S = 120

Value of 6 month Call option on stock ABC
at maturity, with strike price X = 100 :

    C = 0 if S = 80
    C = 20 if S = 120

What is the value of the call today??

S = 100
C = ??

S = 120
C = 20

S = 80
C = 0

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Stock, Call

120
C=20

S=100
C=?

80
C=0

Bond with current value =1

1.025

1

1.025

**Note: 6-month interest
rate = 0.05/2 = 0.025**

.

Replicating portfolio of Stock and Bond in 6 months: $\Delta \times S + B \times (1.025)$

Goal: Find Δ and B such that payoff of replicating portfolio in 6 months is the same as the Call option

Up state: $\Delta \times 120 + B \times 1.025 = 20$

Down state: $\Delta \times 80 + B \times 1.025 = 0$

Solving for Δ and B gives: $\Delta = 0.5$, B = -39.024

Current value: $0.5S - 39.024B$

    $= 0.5(100) - 39.024(1) = 10.98$

= Current Call value by no arbitrage arguments

Replicating portfolio: Long 0.5 shares of stock; Borrow 39.024 bonds with current value =1

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Option Delta or Hedge Ratio

- The number of shares in the leveraged portfolio needed to replicate the payoff on the call is known as the option delta, Δ.

- In the example since the call was replicated by a leveraged position with 0.5 shares of stock, the option delta $= 1/2$

- As a result

$$\text{Value of Call} = \text{Delta} \times S - \text{Bank Loan}$$

Note: $\Delta C \approx \text{Delta} \times \Delta S => \text{Delta} = \Delta C / \Delta S$

Use Put-Call Parity to Value Put

• Recall, Put-Call Parity relationship

$$P + S = C + PV(X)$$

In the previous example

$\quad$ C = 11, S = 100, X = 100, r_f = 2.5% per 6 mo.

Therefore, the value of a put with exercise price X = 100 maturing in 6 months is

$$P = C - S + PV(X) = 11 - 100 + 100/1.025 = 8.56$$

Risk Neutral Valuation

1. Assume investor is risk-neutral. Then

2. Expected return from asset gamble must be equal to the risk-less opportunity (no risk premium); i.e.,
$$E[r] = r_f$$

3. Current value of risky asset must be equal to the PV of expected future cash flows discounted at risk-free rate; i.e,
$$PV = E[\text{cash flow}]/(1 + r_f)$$

4. Expected value computations are based on *risk-neutral probabilities*

Computing Call Value Using Risk Neutral Probabilities

Step 1: Determine risk neutral probabilities

In risk neutral world
$$E(\text{return on ABC stock}) = r_f$$
where
$$E(\text{return on ABC stock}) = p*\text{Return up} + (1-p)*\text{Return down}$$

Now

Return up = $(S^{up} - S)/S = (120 - 100)/100 = 0.2$

Return down = $(S^{down} - S)/S = (80 - 100)/100 = -0.2$

Computing Call Value Using Risk Neutral Probabilities

Therefore, risk neutral probabilities solve

E(return on ABC stock) = p*(.20) + (1-p)*(-.20) = .025

=> p = .5625 = *risk neutral probability of up state*

1-p = .4373 = *risk neutral probability of down state*

Note: Using the risk neutral probabilities we have

$$S = PV = E[\text{cash flow}]/(1 + r_f)$$

i.e.,

100 = S = [120(.5625) + 80(.4375)]/1.025

Computing Call Value Using Risk Neutral Probabilities

Step 2: Determine Call value using risk neutral probabilities and the relationship

$$C = PV = E[\text{cash flow}]/(1 + r_f)$$

The expected value of the call at expiration is

$$E[C] = .5625*\$20 + (.4375)*0 \approx \$11.25$$

The PV of E[C] using the 6-month risk free rate is

$$PV(E[C]) = 11.25/1.025 = \$10.98 = C$$

We have two ways to value a Call Option

1. Construct a leveraged portfolio using borrowing together with purchase of shares in an amount (the option delta) that makes the payoff the same as that of the option.

2. Pretend that investors are risk-neutral and require a risk free return. Calculate the implied probability of a stock price increase. Calculate the expected payoff at expiration on the call option and discount the result to present value using the risk-free rate.

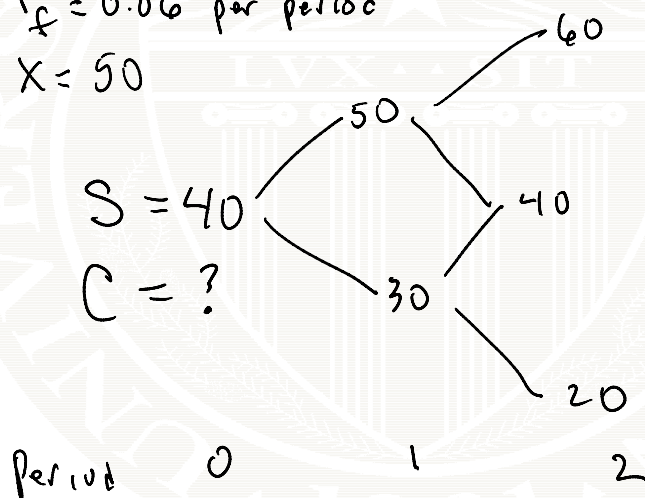# Multiperiod Binomial Model
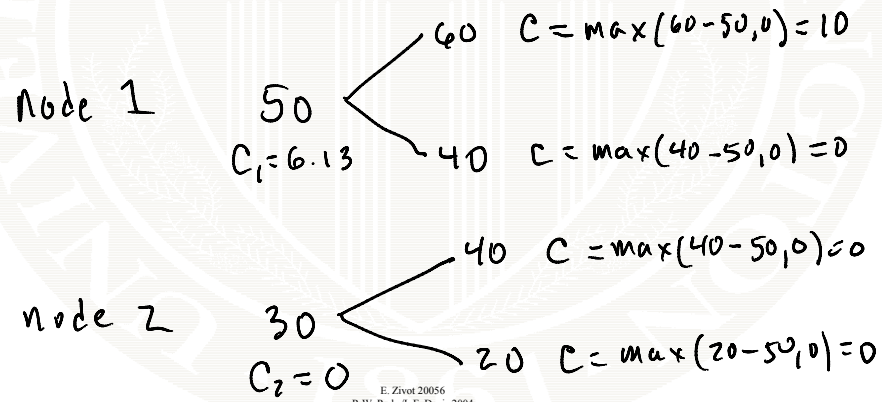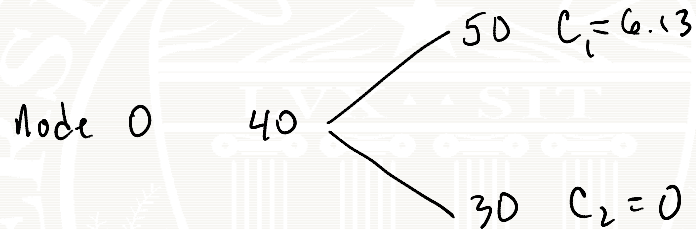

$r_f = 0.06$ per period

$X = 50$

$S = 40$

$C = ?$

Period   0   1   2

60

50

40

30

20

Multiperiod Binominal Model

Trick: Use single period binomial Model
on each "node" of binomial tree

Node 1   50 $\begin{array}{l} \nearrow 60 \quad C = \max(60-50,0) = 10 \\ \searrow 40 \quad C = \max(40-50,0) = 0 \end{array}$

$C_1 = 6.13$

node 2   30 $\begin{array}{l} \nearrow 40 \quad C = \max(40-50,0) = 0 \\ \searrow 20 \quad C = \max(20-50,0) = 0 \end{array}$

$C_2 = 0$

Multiperiod Binomial Model

Node 0   40 $\begin{array}{l} \nearrow 50 \quad C_1 = 6.13 \\ \searrow 30 \quad C_2 = 0 \end{array}$

Using replicating portfolio $\Delta = 0.3065, B = -8.67$

$$C_0 = 40(0.3065) - 8.67$$
$$= 3.59$$

End of Period Share Prices
Take on Many Possible Values

- If the period is broken into many small subintervals, we can use the binomial (up or down) approach within each subinterval. This process allows us to replicate a complicated price distribution for the end of the period.
- The Black-Scholes option pricing model assumes that stock prices follow a random walk in continuous time.
- The Black-Scholes result can also be derived using the binomial approach where we consider the limit as the number of subintervals goes to infinity.

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The Black-Scholes
Option Pricing Formula

- Provides the price of a European call option on a non-dividend paying stock
- Formula involves the familiar 5 variables:
  - S = stock price
  - X = exercise price
  - r_{fc} = annual riskfree interest rate, continuously compounded
  - σ = annual standard deviation of the stock's rate of return
  - T = the amount of time (in years) to expiration

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Black-Scholes Formula

$$C = S \cdot N(d_1) - PV(X) \cdot N(d_2) = S \cdot N(d_1) - Xe^{-r_{fc}T} \cdot N(d_2)$$

$$d_1 = \frac{\ln\left[\frac{S}{PV(X)}\right]}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2} = \frac{\ln\left[\frac{S}{X}\right] + r_{fc}T}{\sigma\sqrt{T}} + \frac{\sigma\sqrt{T}}{2}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$N(d) = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} = \Pr(X \le d), \ X \sim N(0,1)$$

[$N(d)$ is the cumulative Normal distribution function.
It gives the area under the Normal density function
to the left of d.]

Intuition Behind Black-Scholes Formula

Value of Call option = [delta × share price] – [bank loan]

$$[N(d_1) \times S] \qquad - \qquad [N(d_2) \times PV(X)]$$

Applying Black-Scholes

- S = 55
- X = 55
- σ = 0.4069 (SD of annual stock return distribution)
- T = 0.5 (years to maturity)
- r_f = 0.04 (annual rate)
- r_{fc} = ln(1 + r_f) = 0.039221 (cc annual rate)

Applying Black-Scholes

$$d_1 = \frac{\ln(55/55) + 0.039221 \times 0.5}{0.4069\sqrt{.5}} + \frac{0.4069\sqrt{.5}}{2}$$

$$= 0.17794$$

$$d_2 = 0.17794 - 0.4069\sqrt{.5} = -0.109718$$

$$N(0.17794) = 0.570615$$

$$N(-0.109718) = 0.456291$$

$$C = 55(0.570615) - 55e^{-0.039221 \times 0.5}(0.456291)$$

$$= 6.775155$$

Applying Black-Scholes to Value Put

- Use Put-Call Parity to value put option

  $P + S = C + PV(X) \Rightarrow P = C + PV(X) - S$

- Using data from previous example

  $P = 6.775 + 55*\exp(-0.0392*0.5) - 55 = 5.707$

The Effect on the Value of a Call of Changing One of the Underlying Variables

$$C = f(S^+, X^-, r_f^{\,+}, \sigma^+, T^+, [D^-])$$

- An increase in S increases the price of a call
- An increase in X <u>lowers</u> the price of a call
- An increase in r_f increases the price of a call
- An increase in σ increases the price of a call
- An increase in T increases the price of a call
- [An increase in expected dividends lowers the price of a call.

See spreadsheet econ422Options.xls for examples

Employee Compensation Options
Contract Features, example

- 1,000 options.
- $X = S_0$ i.e. stock price at grant date
- Long term, e.g. 7-10 years.
- Vesting schedule: 1/8 vest after one year, 1/8 every six months thereafter. (Options fully vested after 4.5 years.)
- Options exercisable only after vested.
- Vested options must be exercised within 90 days of termination of employment. Options nontransferable.
- Unvested options lost if employment ends.

Employee Compensation Options
Advantages and disadvantages

- Advantages and disadvantages for the company:
  – Incentive for employee to work for company goals
  – Vesting and other features of the grant tie employees to firm.
  – Reduces cash flow needs, a form of financing.
  – Shifts risk to employees.
  – May turn out to be costly.

Employee Compensation Options
Advantages and disadvantages

- Advantages and disadvantages for the employee:
  - Employee incentives aligned with company goals.
  - May provide rich payoff.
  - Risk born by employee, not diversified.
  - Reduces mobility.

Valuing Employee Compensation Options

- e.g. MSFT Grant of 1000 options today, vesting as above.
- From Yahoo!, $X = S_0 = \$27$
- $T = 7$ years.
- From Yahoo!, $r_f = 7$ yr spot rate = 4.1%
- $r_{fc} = \ln(1 + r_f) = 4.01\%$
- Assume no dividends paid.
- Estimate of σ using 5 years of historical returns = 35% per yr
- BS call price = \$7.47 per share (see Excel spreadsheet)
- Current stock option value: 1000*\$7.47 = \$7,470

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Example:
Evaluating Job Offers

	Establishment Industries	Digital Organics
Number of Options	100,000	100,000
Exercise Price	$25	$25
Maturity	5 years	5 years
Current stock price	$22	$22
Stock price volatility (annual standard deviation)	24%	36%
BS Value of Stock Options assuming $r_f = 4\%$		

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Other Interesting Financial Options

- **Index Options**
  - Examples:
    - S&P 500 Index (European) option
    - S&P 100 Index (American) option
    - S&P 100 Index (American) option
  - Settlement in cash not stock
  - LEAPS (Long Term Equity Anticipation Securities) traded
  - Used by Portfolio managers for limiting downside risk (Portfolio Insurance)
    - Beta = 1 example
- **Currency Options**
  - Contract size large: 62,500 Swiss Francs, 50,000 Australian Dollars
- **Exotics**
  - "knock outs" or "knock ins" (barrier options)
    - "down and out" or "down and in"
    - Option goes away or appears if certain price hit
  - "as you like it"
    - After a specified period of time, investor chooses if option is put or call