Hedging Financial Risk

Econ 422

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Hedging and Insuring

- Both hedging and insuring are methods to manage or reduce financial risk.

- Insuring involves the payment of a premium (a small certain loss) for the reduction or elimination of the possibility of a larger loss.

- Hedging involves a transaction that reduces the risk of financial loss by giving up the possibility of a gain.

- Hedging often involves the use of derivative securities.

General Principles of Hedging

- Assume that you own risky asset A and want to reduce your risk exposure.

- Find an asset B whose price is (highly) correlated with that of A, i.e., there is a linear relationship between A's price and B's.

- Estimate the parameters of this relationship by running a regression of A's price on B's price:

    $p_A = \alpha + \delta p_B + \varepsilon, \quad \delta = cov(p_A, p_B)/var(p_B)$

- Delta measures the sensitivity of expected changes in A's price to expected changes in B's price:

    $E[p_A] = \underline{\alpha} + \underline{\delta E[p_B]}$

- Delta is referred to as the hedge ratio.

General Principles of Hedging, Cont.

- If the prices of A and B are <u>perfectly correlated</u>, with a hedge ratio of δ, then $\varepsilon = 0$ and you could construct a perfect hedge by selling (short) δ units of B. Thus, your portfolio would have a long position of one unit of A and a short position of δ units of B.

- If the price of A rises by $1, the price of B (in this case) rises by ($1/$\delta$).

- The value of your portfolio changes by:
    $1 - \delta (\$1/\delta) = 0$

- You have eliminated the risk

General Principles of Hedging, Cont.

- If the prices of A and B are <u>not perfectly correlated</u>, then $\varepsilon \neq 0$ and you could still construct a hedge by selling (short) δ units of B, but the hedge would not be perfect.

- Your portfolio would have a long position of one unit of A and a short position of δ units of B.

- If the price of A rises by \$1, the price of B (in this case) rises by ($1/\delta$) on average, but not always.

- The value of your portfolio changes on average by:
  - \$1 - δ ($1/ \delta$) = 0 but not always

- You have eliminated the risk on average, but not always.

Hedging Using Returns

- In practice, hedging using regression is usually done with returns instead of prices, for certain statistical reasons:

$$r_A = \alpha + \delta r_B + \text{ error}$$

The interpretation of δ remains the same: it is an estimate of the hedge ratio

General Principles of Hedging (Example)

- XYZ Corp holds \$12.5 million of IBM stock. It wants to reduce the risk associated with this asset, using a market index as a hedge instrument.
- We know based on the CAPM, that IBM's return is correlated with the market return and the relationship is

$$r_{IBM} = \alpha + \beta r_M + \varepsilon = 0.78 + 0.72 r_M + \hat{\varepsilon}, \ R^2 = 0.4$$

$$\hat{\delta} = \hat{\beta}_{IBM} = 0.72$$

To construct a hedge, sell 0.72 * \$12.5 million = \$9 million of the market index.

- Q: Is the hedge perfect?

Risk of Hedge Portfolio

$$x_{IBM} = 1 = \text{share of wealth invested in IBM}$$
$$x_{MKT} = -9/12.5 = -0.72 = \text{ share of wealth sold short}$$
$$r_p = x_{IBM} r_{IBM} + x_{MKT} r_{MKT} = \text{hedge portfolio}$$
$$\beta_{IBM} = 0.72 = \text{hedge ratio}$$
$$\beta_{MKT} = 1$$
$$x_{IBM} \beta_{IBM} + x_{MKT} \beta_{MKT} = \text{Portfolio beta}$$
$$= 1(0.72) - 0.72(1) = 0$$

Hedge portfolio has a zero beta: it is *market neutral*

Market risk has been elminiated but specific risk remains!

Hedging with Options

Recall, from the Black-Scholes option pricing formula

$C = \delta \times S - \text{ bank loan}$

$\delta = N(d_1) = \text{ option hedge ratio}$

$\Rightarrow S = \dfrac{1}{\delta} C + \text{ bank loan}$

$\Rightarrow \dfrac{\partial S}{\partial C} = \dfrac{1}{\delta} = \# \text{ of options required to hedge stock}$

Forward Contract

- An agreement to buy or sell an asset at a certain future time for a certain price.

- Typically between two financial institutions or between a financial institution and a corporate client.

- Normally not traded on an exchange.

- One of the parties to a forward contract assumes a long position—by agreeing to purchase the asset at the specified price in the future; whereas, the other party assumes a short position—agrees to sell the asset on the same date for same price.
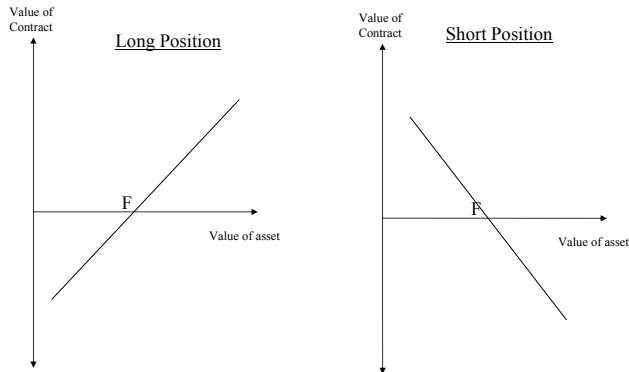
Forward Contract

- This contract creates for both buyer and seller both the _right_ and the _obligation_ to transact at the specified terms.

- Specified price = 'delivery price' determined such that at time of contracting, value of contract is zero.

- Example: Contract today to buy a house for $200,000 with closing in two months. Title to the house and the money are exchanged in two months at terms agreed upon today in the contract

- Example: Bank A contracts to buy from Bank B and Bank B contracts to sell to Bank A 10 million euros for $10.2 million, six months from now.

Forward Contracts

- Usually held to maturity and settled at maturity

- One party looses, one party gains

- The loser may have an incentive to default, so there is always a credit problem.

- Thus, forward contracts are usually written between large institutions with good credit and an ongoing relationship
    – Pre-paid variable forward contracts

Position Diagram for Forward Contract

Value of Contract — **Long Position**

Value of Contract — **Short Position**

F

F

Value of asset

Value of asset

Futures

- Futures are forward contracts managed by an exchange

- Oldest actively traded derivative instrument. Initially used with agricultural and commodities; e.g., pork bellies, cattle, sugar, wool, coffee, frozen orange juice, copper, gold, aluminum, gold, tin.

- Underlying financial assets include stock indices, currencies, Treasuries.

- Trade on the CBOT (Chicago Board of Trade) or CME (Chicago Mercantile Exchange). The buyer and seller don't meet or know each other. The exchange contracts separately with buyer and seller, then matches them.

- The exchange establishes margin accounts for each party to guarantee fulfillment of contract

- The futures market is extremely liquid

Futures

- Futures contract is an agreement between two parties to buy or sell an asset at a certain time (delivery) in the future for a certain price.

- Contracts are standardized, specifying exactly what is to be delivered, where and when.

- Seller has some choice concerning the delivery date (usually within the final month)

- The contract is "marked to market," i.e., there is settlement after every trading day. This arrangement reduces default risk.

Example: Marking to Market

- A 3 day futures contract (which is marked to market) and a 3 day forward contract (which is not) call for A to buy and B to sell 1000 ounces of silver three days from now $6.00 per ounce.

- The price of silver moves to:
  - $6.10 on day 1
  - $6.05 on day 2
  - $6.12 on day 3

- Regarding the forward contract, A gains $0.12 *1000 = $120 and B looses $120 since silver is trading at a higher price on settlement day.
  - *The forward contract can either be settled by B actually selling to A the 1000 ounces of silver for $6,000 (when it is worth $6,120) or by making a payment of $120 to A.*

Example: Marking to Market

- Regarding the <u>futures contract</u>, at the end of the trading on the first day, B would pay and A would receive ($6.10 - $6.00) = $0.10*1000 = $100 as the settlement for the first day's price rise of $0.10. Once this payment is made, the contract is rewritten with a price of $6.10.

- At the end of day 2, A would pay and B would receive ($6.10 - $6.05) = $0.05*1000 = $50 as settlement for the second day's price decline. The contract would be rewritten at $6.05.

- At the end of day 3, B would pay and A would receive a payment of ($6.12 - $6.05) = $0.07*1000 = $70.

- Note: The total net payment made by B is the same $120.

- If at the beginning of day 3, B had chosen to deliver the silver, B would sell 1000 ounces for $6050 (why?), but would have made net settlement payments of $50. A would have paid a net of $6,000 for the silver now worth $6,120.

- B looses $120 and A gains $120 of value

Marking to Market

- Marking to market largely eliminates the default risk by the process of daily settlement. The futures exchange (CBOT or CME) takes care of the marking to market accounting

- Although the net totals are the same as with the forward contract, the timing of the payments is different.

- The timing of payments can be very important.

Hedging with Futures

- Consider a wheat farmer who expects to harvest 100,000 bushels of wheat in one month. The price that will prevail in one month is uncertain, hence the farmer is exposed to considerable financial risk.

- The farmer could reduce or eliminate this risk by entering into a futures contract to sell wheat in one month at a price, say $3.00/bushel, agreed to today.

- Consider a miller who will need to buy 100,000 bushels of wheat in one month. The miller is exposed to financial risk because of the uncertainty about the price that will prevail in one month.

- The miller could eliminate this risk by entering into a futures contract to buy wheat in one month at the $3.00/bushel futures price.

Hedging with Futures

- Consider three possible prices for wheat one month from now: $2.50, $3.00 and $3.50.
- The farmer has hedged his 100,000 bushel crop with a futures contract at $3.00/bushel.
- The futures contract will be settled in cash rather than with delivery of wheat.
- The farmer sells his crop to the local grain dealer at the spot or market price in one month.
- Show what the farmer will receive for the crop, combining the revenue from the actual sale with the settlement proceeds for the futures contract.

Hedging with futures

- Draw a position diagram for the farmer showing the dependence of his/her wealth on the price of wheat if no hedging is used. Assume wheat is the farmer's only asset, (100,000 bushels of it.)
- Show the farmer's position after taking a short position in a futures contract. (100,000 bu)
- Do the same thing for the miller.

Are Futures Risky?

- If you take a speculative position in a futures contract, either long or short, that position is risky.

- If you already have a long or short position in the physical commodity, then taking the opposite position in the futures contract is risk reducing.

- The risk of the futures contract depends on the context in which it is used.

The Relationship Between Spot and Futures Prices

- The spot price of a commodity at any date is the cash price of the commodity at that time.

- The forward or futures price is the price that would be established for a forward or futures contract.

- Certain 'no arbitrage' conditions link spot and futures prices

Futures Prices for Financial Futures

- Suppose that 1.5 years from today you would like to own a financial asset, e.g. 1000 shares of an S&P index fund or a 10 yr. Treasury Bond.

- One way to achieve this goal is to buy the asset now and hold it for 1.5 years.

- Another way to achieve this goal would be to buy a forward or futures contract for the asset.

- Using the first approach approach you would pay the current spot price, P_s, for the asset. You would also get any cash flows (e.g. dividends, coupon payments) generated by the asset during the 1.5 years that you hold it.

Futures Prices for Financial Futures

- Using the second approach you would pay the agreed upon futures price F at the future date T = 1.5 years from today. You would not get the intervening cash flow from the asset.
- In both cases you would own the asset at time T=1.5.
- To prevent arbitrage, it must be the case that

$$\frac{F}{(1+r_f)^T} = P_S - PV(\text{Cash flow})$$

$$\Rightarrow F = (1+r_f)^T \left(P_S - PV(\text{Cash flow})\right)$$

Futures Pricing: Example

- Determine futures price of 10 yr T-bond with delivery date in 6 months (T = 0.5)
- Suppose that the spot price for a 10 yr. Treasury Bond is 100
- The current six month interest rate is 0.06 (annual rate)
- The PV of the anticipated coupon interest payment over the next six months is 5

Futures Pricing: Example

- Using the no-arbitrage condition, the price of the 6-month futures contract is

$$F = (1+r_f)^T(P_S - PV(\text{Cash flow}))$$
$$= (1.06)^{0.5}(100 - 5)$$
$$= 97.808$$

Futures Pricing: Equity Index

No arbitrage relationship:

$$\frac{F}{(1+r_f)^T} = P_S - PV(\text{Cash flow})$$

$$\Rightarrow F = (1+r_f)^T \left(P_S - PV(\text{Cash flow})\right)$$

Cash flow for equity index is dividend payment

Futures and spot prices
for commodities

- No-arbitrage condition for commodity futures.

$$\frac{F}{(1+r_f)^T} = P_S + \text{PV(Storage costs)} - \text{PV(Convenience yield)}$$

- Again compare the costs and benefits of buying a futures contract vs. buying the commodity and holding it.
- Convenience yield (non-negative) is the benefit associated with having possession of the physical commodity. It is likely to be inversely related to the size of existing inventories.