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Choice Under Uncertainty

Econ 422: Investment, Capital & Finance
 University of Washington
 Summer 2006
 August 15, 2006

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Decision Making Under Uncertainty

Course Chronology:

1. Intertemporal Choice: Exchange & Production
2. Introduction of Financial Markets—Lending & Borrowing
3. Interest Rate Determination
4. Present Value Calculations
5. Valuation of Financial Instruments: Stocks & Bonds
6. NPV Rule for Investment Decision Making
7. Random Variable/Probability Theory
8. *Intertemporal Choice with Uncertainty*

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How Do Maximizing Agents Make Choices in the Presence of Uncertainty?

Following Random Variable review you might propose the following:
 - Choices made to maximize expected values (expected returns, etc.)

Is this consistent with observed behavior?

Lottery Example
 Expected value is low, but individuals pay more than expected return to win?

Washington State LOTTO
Odds of winning: 1 in 6,991,908
Jackpot: March 2004 \$1.6 MM
Expected Return: Jackpot * Prob (winning Jackpot) = \$0.22
Cost of ticket: \$1 for 2 plays or \$0.50/play
Observed behavior: Paying more than expected return

*Explanation(s): Investors are irrational??
 Behavioral Economics—Individuals overestimate probability of success or winning = “overconfident”;*

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Rational Selection Among Lotteries

- Consider 4 choices: **W X Y Z**
- Your preference rankings are as follows:
 $W > X > Y > Z$
- Now suppose there is an additional choice U, where U represents a ‘gamble’ or ‘lottery ticket’—you receive W with probability p and you receive X with probability (1-p)

Do you prefer Z or U, where $E[U] = pW + (1-p) X$?

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UW Rational Selection Among Lotteries Cont.

$W > X > Y > Z$

- Again consider there is an additional choice V , where V represents a ‘gamble’ or ‘lottery ticket’—you receive Y with probability q and you receive Z with probability $(1-q)$

Do you prefer Z or V , where $E[V] = qY + (1-q)Z$?

How does your answer change for changes in q ?

- “Money in Hand” or “Sure Thing Principle”

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UW Rational Selection Among Lotteries Cont.

$W > X > Y > Z$

- Create a new opportunity T that provides W with probability r and Y with probability $1-r$ such that $E[T] = rW + (1-r)Y$
- Do you prefer T or X ?
- What does your choice depend on?

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UW Decision Making Under Uncertainty

- Early contributors to decision making under uncertainty—gamblers—believed that comparing expected values of outcomes (alone) would work as a decision rule
- It may come as no surprise that early contributors to finance theory were ‘gamblers.’ Do financial markets today reflect this heritage?
- The St. Petersburg Paradox suggests that this idea does not in general hold with consistent rational behavior

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UW The St. Petersburg Paradox

The game: Flip a fair coin until the first head appears

The payoff: If the first head appears on the k^{th} flip, you get $\$2^k$

- How much would you be willing to pay for a chance to play this game?

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The St. Petersburg Paradox

The game: Flip a fair coin until the first head appears

The payoff: If the first head appears on the k^{th} flip, you get $\$2^k$

Using an expected value rule, you should be willing to pay at least the expected value of the payoff from playing the game.

- What is the expected payoff?

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Expected Payoff for the St. Petersburg Game

Recall the expected payoff will be the probability weighted sum of the possible outcomes.

Note: The tosses are independent, a tail on the previous toss does not influence the outcome of the subsequent toss. Head has a $1/2$ or 50% chance of occurring on any single toss.

Outcomes = "Head appears in toss #":	1	2	3 ...	k
Probability head occurs on given toss:	$1/2$	$1/4$	$1/8$	$1/2^k$
Payoff = 2^k :	2	4	8	2^k

$$\text{Expected Payoff} = 1/2 * 2 + 1/4 * 4 + 1/8 * 8 + \dots + 1/2^k * 2^k + \dots = 1 + 1 + 1 + \dots = \infty$$

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Conclusion: The Expected Utility Hypothesis

- In situations involving uncertainty (risk), individuals act as if they choose on the basis of **expected utility** – the utility of expected wealth, consumption, etc. -- rather than expected value.
- In our discussions we can think of individuals choosing between different probability distributions of wealth

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Example: The Expected Utility Hypothesis

- Let W_a be W_a for certain, i.e., $p_a = 1$
- Let W_b provide W_1 with probability p_1 or W_2 with probability p_2 :

$$E(W_b) = p_1 W_1 + p_2 W_2, \text{ where } p_1 + p_2 = 1$$

- Assume that the utility function over wealth, $U(W)$ is monotonically increasing, more wealth is preferred to less wealth.

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Example: The Expected Utility Hypothesis

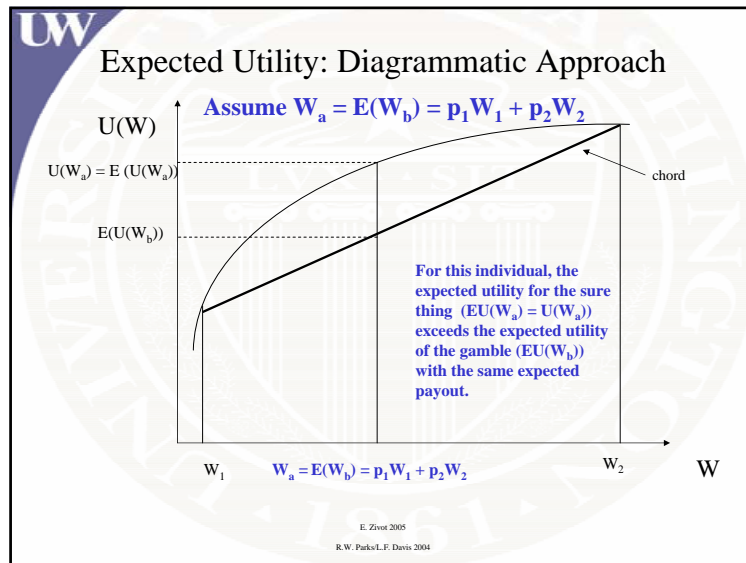
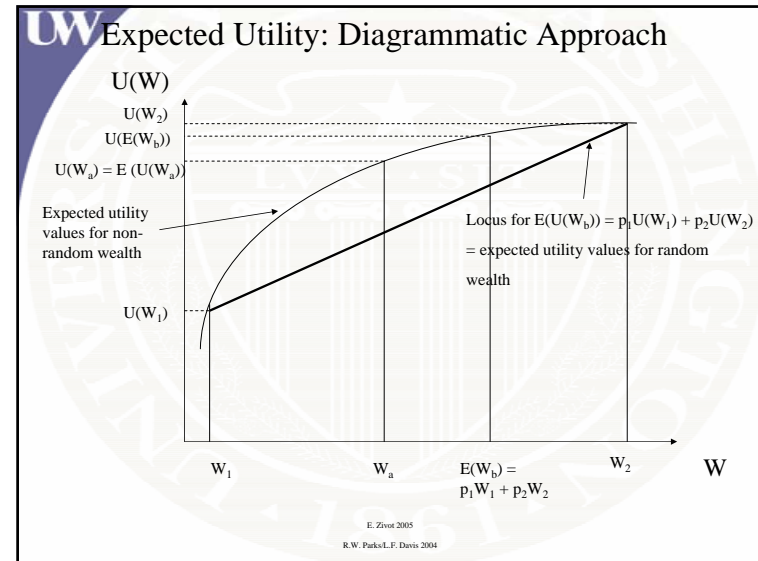
- The expected utility for the possible two wealth situations are as follows:

$$E(U(W_a)) = U(W_a) \text{ (for certain wealth)}$$

$$E(U(W_b)) = p_1 * U(W_1) + p_2 * U(W_2) \text{ (for random wealth)}$$

Expected Utility Theory states that individual will choose between these two wealth opportunities (W_a and W_b) based on expected utility.

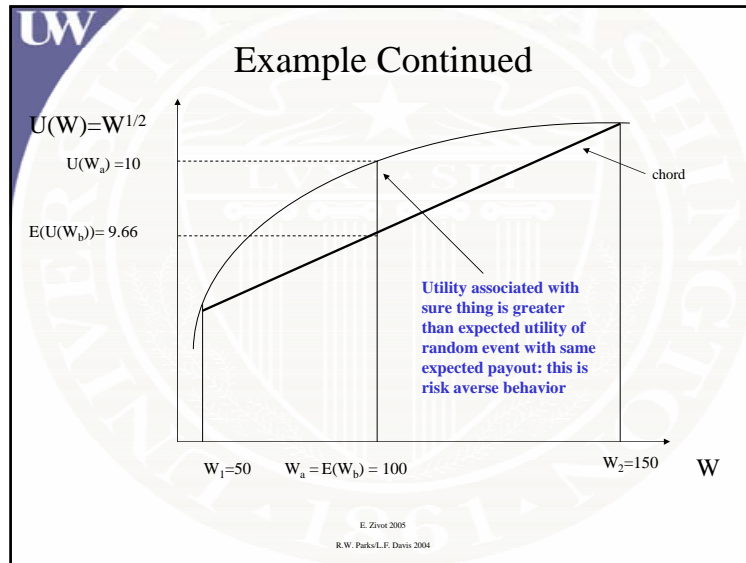
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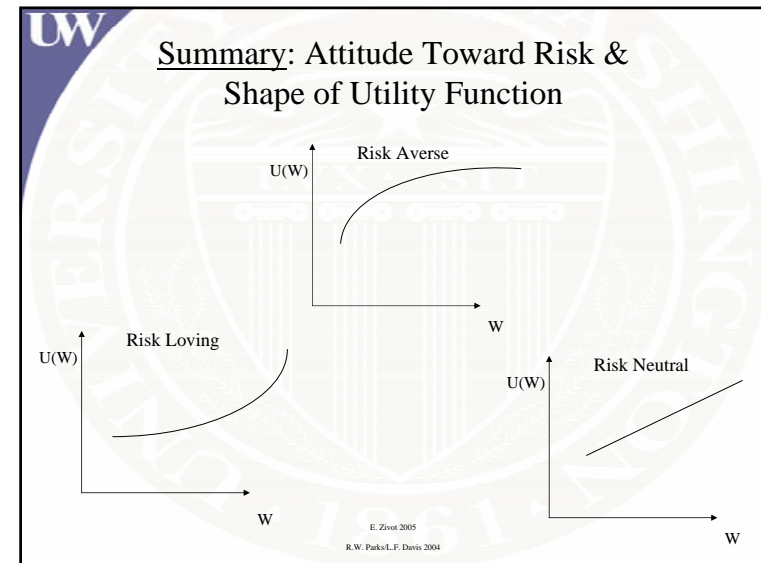
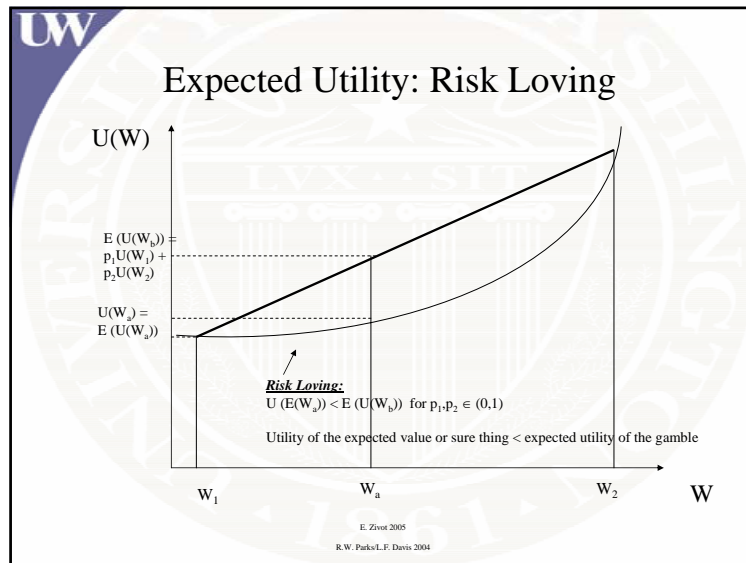
Example

- $U(W) = W^{1/2}$
- $W_a = 100$ with probability 1
- $E[U(W_a)] = U(W_a) = 100^{1/2} = 10$
= expected utility for sure thing
- $W_b = 50$ with $p_1 = 0.5$
= 150 with $p_2 = 0.5$
- $E[W_b] = 50(0.5) + 150(0.5) = 100 = W_a$
- $E(U(W_b)) = 50^{1/2}(0.5) + 150^{1/2}(0.5)$
= $7.07(0.5) + 12.25(0.5) = 9.66$
= expected utility for gamble

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- ### Definition of Risk Aversion
- A risk averse individual will refuse to accept a fair gamble versus the sure thing.
 - The utility function for a risk averse individual must be concave (from below) such that the chord lies below the utility function.
 - The chord or linear locus represents a risk neutral investor that is indifferent between the fair gamble and the sure thing. For the risk neutral investor the expected utility of the gamble equals the expected value of the gamble.
 - A risk loving individual is represented by a utility function which is convex (from below) such that the chord lies above the utility function. A risk loving individual will prefer the fair gamble over the sure thing.
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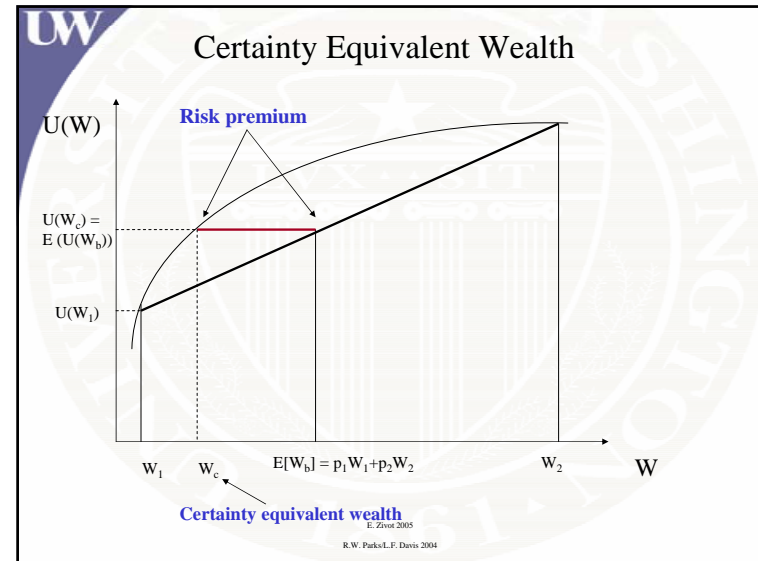
Certainty Equivalent

- A risk averse person prefers a sure thing to a fair gamble
- Is there a smaller amount of certain wealth, W_c , that would be viewed as equivalent to the gamble?
- Define the Certainty Equivalent as follows:

$$U(W_c) = E[U(W_b)] = p_1U(W_1) + p_2U(W_2)$$
- Risk Aversion implies $W_c < p_1W_1 + p_2W_2 = E[W_b]$
- Risk Premium: $E[W_b] - W_c$

$$= p_1W_1 + p_2W_2 - W_c > 0$$

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Example Continued

- Find the the certainty equivalent wealth associated with $E[W_b] = 100$:

$$U(W_c) = W_c^{1/2} = E[U(W_b)] = p_1U(W_1) + p_2U(W_2) = 9.66$$

$$\Rightarrow W_c = (9.66)^2 = 93.32$$

- Risk premium: $E[W_b] - W_c = 100 - 93.32 = 6.68$

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