Capital Market Equilibrium and the Capital Asset Pricing Model

Econ 422
Investment, Capital & Finance
Spring 2010
June 1, 2010

The Risk of Individual Assets

- Investors require compensation for bearing risk.
- We have seen that the standard deviation of the rate of return is an appropriate measure of risk for one’s portfolio.
- Standard deviation is not the best measure of risk for individual assets when investors hold diversified portfolios.
The Risk of Individual Assets (continued)

• For people holding a diversified portfolio it is the contribution of the individual asset to the portfolio’s standard deviation that matters.

• [If your portfolio involved only one asset, e.g. young Bill Gates, the portfolio standard deviation would be the standard deviation of the single asset.]

The contribution of an individual asset to the portfolio’s standard deviation: Beta

• Beta measures the sensitivity of an asset’s rate of return to variation in the market portfolio’s return.

• Beta for asset i can be computed as

\[ \beta_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)} = \frac{\sigma_{im}}{\sigma_m^2} \]
Beta as a Measure of Portfolio Risk: Class Example

• Suppose you hold an equally weighted portfolio with 99 assets
• What happens to the portfolio variance if a new asset, say IBM, is added to the portfolio?

Measuring Betas
The Market Model

• Beta can be interpreted as the slope coefficient in a regression of the return on the ith security, $r_i$, on the return for the market portfolio, $r_m$.
• The interpretation rests on the market model:

$$r_{it} = \alpha_i + \beta_i r_{mt} + \epsilon_{it}$$

• $r_m$ represents “market risk” and $\epsilon_i$ represents “firm specific” risk independent of the market. That is, $\text{cov}(r_{mt}, \epsilon_{it}) = 0$.

$$\hat{\beta}_i = \frac{\text{cov}(r_i, r_m)}{\text{var}(r_m)}$$
Beta as a Measure of Risk for Individual Assets

The intercept is given by \( \alpha \).

The slope is given by \( \beta = \frac{\text{cov}(r_i, r_m)}{\text{V}(r_m)} \).

Market return \( r_m \) and Asset i's return \( r_i \).

The Capital Asset Pricing Model (CAPM)

- CAPM describes the relationship between an asset's beta risk and its expected return as follows:

\[
E(r_i) = r_f + \beta_i \left[ E(r_m) - r_f \right]
\]

= riskfree rate + beta x market risk premium.
The Security Market Line (SML) Describes the CAPM Relationship

\[ E(r_i) = r_f + \beta_i [E(r_m) - r_f] \]

Slope is the market risk premium = \( E(r_m) - r_f \)

The Capital Asset Pricing Model Example

- Annual T-bill yield = 1%
- Beta for Microsoft = 0.93 (from Yahoo! See link for key statistics)
- Historical market risk premium = 7.1%

Then

\[ E[r_{msft}] = r_f + \beta_{msft} (E[R_{mt}] - r_f) \]

\[ = 1\% + 0.93 \times (7.1\%) = 7.603\% \]

Note: the return predicted from the CAPM is sometimes called the “risk-adjusted” return
CAPM and Efficient Portfolios

Example: Starbucks Stock

- \( E[R_{sbux}] = 0.10 \), \( SD(R_{sbux}) = 0.20 \)
- \( E[R_m] = 0.15 \), \( SD(R_m) = 0.10 \), \( r_f = 0.03 \)

- Find the beta for SBUX
- Find the efficient portfolio of T-bills and the market with the same expected return as SBUX

The Market Model and the Measures of Risk

- The total risk of an asset, held alone, i.e. not as part of a diversified portfolio, would be measured by its variance, \( V(r_i) \).
- According to the market model
  \[
  r_{it} = \alpha_i + \beta_i r_{mt} + \epsilon_{it} \quad \text{and} \quad V(r_{it}) = \beta_i^2 V(r_{mt}) + V(\epsilon_{it})
  \]
- Total risk = systematic market risk + unique risk
- The unique risk can be eliminated through diversification.
Portfolio Beta

The beta of a portfolio is a weighted average of the individual asset betas

\[ \beta_p = x_1 \beta_1 + x_2 \beta_2 + \cdots + x_n \beta_n \]

\( x_i \) = portfolio share for asset i

\( \beta_i \) = beta for asset i

Example

- Microsoft has a beta of 1.2
- Starbucks has a beta of 0.8
- Portfolio weights are \( x_{\text{msft}} = 0.25, \ x_{\text{sbux}} = 0.75 \)
- Portfolio beta = \( 0.25(1.2) + 0.75(0.8) = 0.9 \)
Applying the CAPM to Valuation

Recall the one-period holding period rate of return:

\[ r = \frac{P_1 - P_0 + D_1}{P_0} \]

At time \( t = 0 \), \( P_0 \) is known, but \( P_1 \) and \( D_1 \) are not known; they are random variables.

Take expectations:

\[ E(r) = \frac{E(P_1) - P_0 + E(D_1)}{P_0} \]

Solve for \( P_0 \):

\[ P_0 = \frac{E(P_1) + E(D_1)}{1 + E(r)} = \frac{E(P_1) + E(D_1)}{1 + r_f + \beta E(r_M) - r_f} \]

using the CAPM relation: \( E(r) = r_f + \beta[E(r_M) - r_f] \)

Applying the CAPM to Valuation (continued)

\[ P_0 = \frac{E(P_1) + E(D_1)}{1 + r_f + \beta[E(r_M) - r_f]} \]

- To value a future risky cash flow, discount the expected value of the cash flow to present value using the risk-adjusted expected return based on the CAPM.
Example: Stock valuation using CAPM

- $E[D_1] = 5$, $g = 0.10$, $r_f = 0.03$
- $\beta = 1.5$, $E[r_m] - r_f = 0.075$

$$E[r] = r_f + \beta(E[r_m] - r_f) = 0.03 + 1.5(0.075) = 0.1425$$

Constant growth: $P_0 = \frac{E[D_1]}{(E[r] - g)} = \frac{5}{0.1425 - 0.1} = \frac{5}{0.0425} = 117.64$