Capital Market Equilibrium and the Capital Asset Pricing Model

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The Risk of Individual Assets

- Investors require compensation for bearing risk.
- We have seen that the standard deviation of the rate of return is an appropriate measure of risk for one's portfolio.
- Standard deviation is <u>not</u> the best measure of risk for individual assets when investors hold diversified portfolios.

The Risk of Individual Assets (continued)

- For people holding a diversified portfolio it is the contribution of the individual asset to the portfolio's standard deviation that matters.
- [If your portfolio involved only one asset, e.g. young Bill Gates, the portfolio standard deviation would be the standard deviation of the single asset.]

The contribution of an individual asset to the portfolio's standard deviation: Beta

- Beta measures the sensitivity of an asset's rate of return to variation in the market portfolio's return.
- Beta for asset i can be computed as

$$\beta_i = \frac{\operatorname{cov}(r_i, r_m)}{V(r_m)} = \frac{\sigma_{im}}{\sigma_m^2}$$

Beta as a Measure of Portfolio Risk: Class Example

- Suppose you hold an equally weighted portfolio with 99 assets
- What happens to the portfolio variance if a new asset, say IBM, is added to the portfolio?

Measuring Betas
The Market Model

- Beta can be interpreted as the slope coefficient in a regression of the return on the ith security, r_i, on the return for the market portfolio, r_m.
- The interpretation rests on the market model:

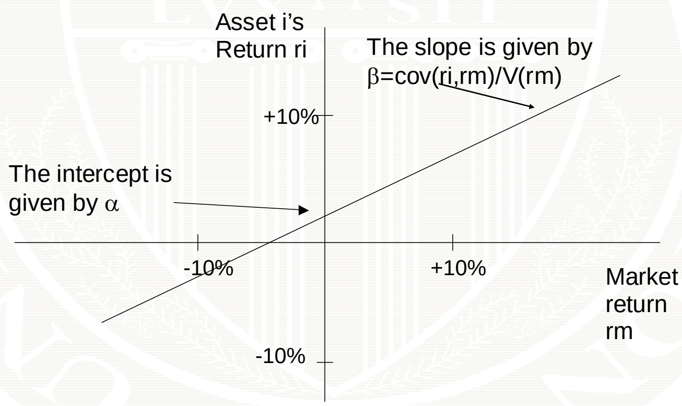
$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it}$$

Yahoo! performs linear regression using 5 yrs of monthly returns & R_m = S&P 500

- r_m represents "market risk" and ε_i represents "firm specific" risk independent of the market. That is, $cov(r_{mt}, \varepsilon_{it}) = 0$.

$\hat{\beta}_i$ = least squares regression estimate $= \dfrac{\widehat{cov}(R_i, R_m)}{\widehat{var}(R_m)}$

Beta as a Measure of Risk for Individual Assets



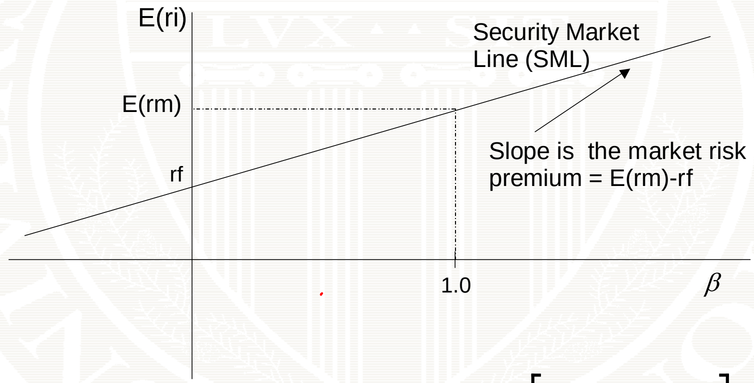
The Capital Asset Pricing Model (CAPM)

- CAPM describes the relationship between an asset's beta risk and its expected return as follows:

$$E(r_i) = r_f + \beta_i \left[E(r_m) - r_f \right]$$

= riskfree rate + beta x market risk premium.

The Security Market Line (SML) Describes the CAPM Relationship



$$E(r_i) = r_f + \beta_i\left[E(r_m) - r_f\right]$$

The Capital Asset Pricing Model Example

- Annual T-bill yield = 1%
- Beta for Microsoft = 0.93 (from Yahoo! See link for *key statistics*)
- Historical market risk premium =7.1%

Then

$$E[r_{msft}] = r_f + \beta_{msft}(E[R_{mt}] - r_f)$$
$$= 1\% + 0.93*(7.1\%) = 7.603\%$$

Note: the return predicted from the CAPM is sometimes called the “risk-adjusted” return

CAPM and Efficient Portfolios

Example: Starbucks Stock

$E[R_{sbux}] = 0.10$, $SD(R_{sbux}) = 0.20$

$E[R_m] = 0.15$, $SD(R_m) = 0.10$, $r_f = 0.03$

- Find the beta for SBUX
- Find the efficient portfolio of T-bills and the market with the same expected return as SBUX

The Market Model and the Measures of Risk

- The total risk of an asset, held alone, i.e. not as part of a diversified portfolio, would be measured by its variance, $V(r_i)$.
- According to the market model

$$r_{it} = \alpha_i + \beta_i r_{mt} + \varepsilon_{it} \quad \text{and}$$

$$V(r_{it}) = \beta_i^2 V(r_{mt}) + V(\varepsilon_{it})$$

- Total risk = systematic market risk + unique risk
- The unique risk can be eliminated through diversification.

Portfolio Beta

The beta of a portfolio is a weighted average of the individual asset betas

$$\beta_p = x_1\beta_1 + x_2\beta_2 + \cdots + x_n\beta_n$$

$x_i =$ portfolio share for asset i

$\beta_i =$ beta for asset i

Example

- Microsoft has a beta of 1.2
- Starbucks has a beta of 0.8
- Portfolio weights are $x_{msft} = 0.25$, $x_{sbux} = 0.75$
- Portfolio beta = 0.25(1.2) + 0.75(0.8) = 0.9

Applying the CAPM to Valuation

Recall the one-period holding period rate of return:

$$r = \frac{P_1 - P_0 + D_1}{P_0}$$

At time $t = 0$, P_0 is known, but P_1 and D_1 are not known; they are random variables.

Take expectations:

$$E(r) = \frac{E(P_1) - P_0 + E(D_1)}{P_0}$$

Solve for P_0:

$$P_0 = \frac{E(P_1) + E(D_1)}{1 + E(r)} = \frac{E(P_1) + E(D_1)}{1 + r_f + \beta[E(r_M) - r_f]}$$

using the CAPM relation: $E(r) = r_f + \beta[E(r_M) - r_f]$

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Applying the CAPM to Valuation (continued)

$$P_0 = \frac{E(P_1) + E(D_1)}{1 + r_f + \beta[E(r_M) - r_f]}$$

- To value a future risky cash flow, discount the expected value of the cash flow to present value using the risk-adjusted expected return based on the CAPM.

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Example: Stock valuation using CAPM

- $E[D_1] = 5$, $g = 0.10$, $r_f = 0.03$
- $\beta = 1.5$, $E[r_m] - r_f = 0.075$

$$E[r] = r_f + \beta(E[r_M] - r_f) = 0.03 + 1.5(0.075) = 0.1425$$

$$\text{Constant growth: } P_0 = \frac{E[D_1]}{(E[r]-g)} = \frac{5}{0.1425-0.1} = \frac{5}{0.0425} = 117.64$$