

Econ 422
Summer 2010
PROBLEM SET 1
Due Monday 7/26/10

Note: The tools needed for the following problems are given in lectures and in the Hirshleifer and Varian reading. The main text does not cover them adequately (it's an MBA textbook and not an econ textbook).

1. Casper Milktoast has \$200,000 available to support consumption in periods 0 (now) and 1 (next year). He wants to consume exactly the same amount in each period. The interest rate is 8 percent. There is no risk. How much should he invest, and how much can he consume in each period?

2. An isolated society contains two groups of consumers (the same number of each kind). The consumers all have identical preferences with respect to present and future consumption, but they have different endowments. Group A has an endowment of (240, 160) for present and future. Group B has an endowment of (320, 440). Their preferences can be represented by the utility function

$$U(C_0, C_1) = (C_0 \cdot C_1)^{0.5}$$

These preferences have a marginal rate of substitution equal to

$$MRS = -\frac{C_1}{C_0}$$

- a. What is the budget constraint for members of the A group if the interest rate at which they can borrow or lend is r ?
- b. At an interest rate of r what is the value of an A group member's wealth?
- c. What conditions characterize the optimal consumption point for the consumers if there are exchange opportunities between the present and future at a real rate of interest r ?
- d. Show that the consumption function for present period for members of both groups is $C_0 = 0.5W$ where W is the consumer's wealth. Write an expression for the saving function for each group in a form that shows its dependence on the rate of interest.
- e. What condition must hold if r is to be an equilibrium interest rate for this society, assuming that they can exchange only among themselves. What is the equilibrium real interest rate?

3. You will find the example spreadsheet econ422Utility.xls on the class webpage helpful for this problem. Suppose an individual has the following utility function

$$U(C_0, C_1) = C_0^\alpha C_1^{1-\alpha}, \quad 0 < \alpha < 1$$

Let the endowments be Y_0 and Y_1 and let r denote the interest rate.

a. Use calculus to show that

$$MRS = -\frac{U_0}{U_1} = -\frac{\alpha}{1-\alpha} \frac{C_1}{C_0}$$

$$C_0^* = \alpha W_0, \quad C_1^* = (1-\alpha)(1+r)W_0$$

b. For $\alpha = 0.25, 0.5, 0.75$, plot utility, as a function of C_0 and C_1 , for a bivariate grid of values with C_0 and C_1 between 0.25 and 4 with increments of 0.25 (see the Utility tab in econ422Utility.xls for an example)

c. For $\alpha = 0.25, 0.5, 0.75$, solve for C_1 such that $U(C_0, C_1) = 1$, and plot the resulting indifference curve (see the IC tab in econ422Utility.xls for an example)

c. Let the initial endowment, Y_0 , be 10,000, and let the interest rate be 10%. Plot the budget constraint. For $\alpha = 0.25, 0.5, 0.75$, compute the optimal levels of C_0 and C_1 . How do these values vary with the value of α ? On the same graph with the budget constraint, show the indifference curve where the $MRS = -(1+r)$ (see the Budget Constraint tab in econ422xls for an example)

e. Using your Excel spreadsheet, for $\alpha = 0.25, 0.5, 0.75$, compute the new optimal levels of C_0 and C_1 when the interest rate is 5% and when it is 15%. Compare these values with the optimal levels when the interest rate is 10%.