Quantitative reductions of HH - type models:

Timescale Separation and Generalized Integrate + Fire models.

CH. 5 of GERSTNER.

$$\begin{cases} \frac{dV}{dt} = f_v(V, \vec{x}) + I(t) \\ \frac{dx_j}{dt} = f_j(V, x_j) \end{cases}$$

$$= \left(x_j^{\infty}(V) - x_j\right) / \tau_j(V)$$

Say... $j = 1 \ldots N_{fast}$ of these conductance variables are "fast": $\tau_j(V) << 1$ for values of V near "resting" equilibrium... OR just for typical values of V before a spike.

ie, letting $\tilde{\tau}_j(V)$ be $O(1)$,

$$\frac{dx_j}{dt} = \frac{1}{\varepsilon_j} \left(x_j^{\infty}(V) - x_j\right) / \tilde{\tau}_j(V), \quad \text{ie}$$

$x_j(t) \longrightarrow x_j^{\infty}(V)$ with very rapid dynamics,

at least when V is subthreshold, ie before a spike.

So... original system

(don't need to write)

$$\frac{dV}{dt} = f(\underbrace{x_1, x_2, x_3}_{N_{fast}}, \dots, x_{N-1}, x_{N-2})$$

$$N_{FAST} \left\{ \begin{aligned} \frac{dx_1}{dt} &= f_1(V, x_1) \\ \frac{dx_2}{dt} &= f_2(V, x_2) \end{aligned} \right.$$

$$\vdots$$

$$\frac{dx_N}{dt} = f_N(V, x_N)$$

BECOMES...

$$\frac{dV}{dt} = f(\underbrace{x_1^\infty(V), \dots, x_{N_{FAST}}^\infty(V)}_{N_{FAST}}, x_{N_{FAST+1}}, x_{N_{FAST+2}}, \dots, x_N)$$

$$\frac{dx_{N_{FAST+1}}}{dt} = f_{N_{FAST+1}}(V, x_{N_{FAST+1}})$$

$$\frac{dx_N}{dt} = f_N(V, x_N)$$

Have eliminated fast variables. Formalize:

(geometric) singular perturbation theory.

Next ... say the remaining variables are <u>SLOW</u> :

eg. $$\frac{dx_N}{dt} = \varepsilon \frac{x_N^{\infty}(V) - x_N}{\tilde{\tau}_N(V)}$$ ... for "typical values" of V before spike.

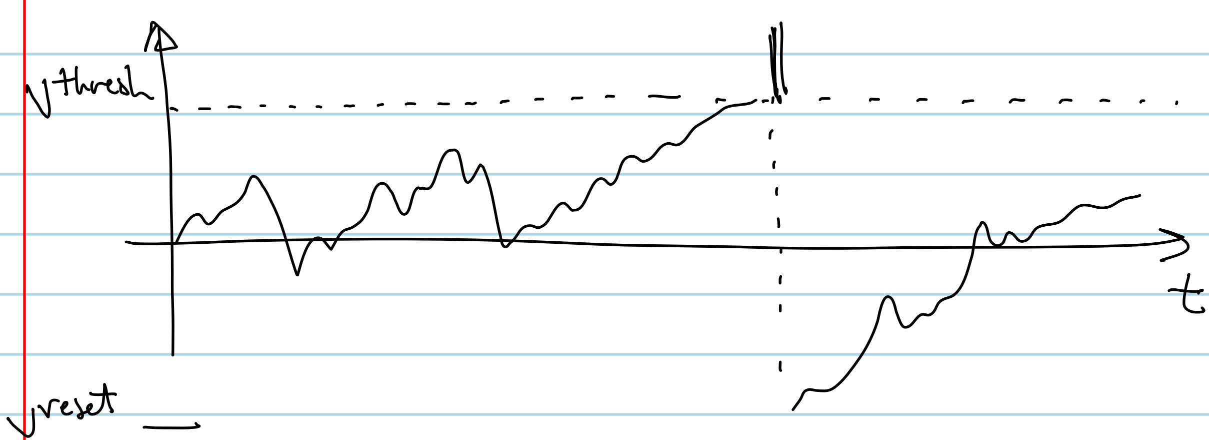
Then say ... when V subthreshold,

$x_N(t) \approx$ <u>const.</u> (Let that const. be $\bar{x}_N$).

AND ... left w/ <u>single</u> ODE:

$$\frac{dV}{dt} = f\Big(\underbrace{x_1^{\infty}(V),\ x_2^{\infty}(V), \ldots}_{\text{fast}},\ \underbrace{\bar{x}_{N-1},\ \bar{x}_N}_{\text{slow}}\Big) \qquad (1)$$

... for V below spiking threshold V thresh.



Then, "declare" spike ; and reset $V(t) \to V^{reset}$ . . .
then follows (i) again.

This is [As will see below, w/ realistic $x^{\infty}(V)$,] the NONLINEAR INTEGRATE + FIRE MODEL.

(5.15 in Gerstner)

E.g. . . . The original 1952 HH eq$^{\underline{n}}$s are of form . . .

$$\left\{\begin{array}{ll} C\dot{V} = \bar{g}_{Na}\, m^3 h\,(E_{Na}-V) + \bar{g}_k\, n^4\,(E_k - V) + g_L(E_L - V) + I(t) & \\ \dot{m} = f_m(V,m) & \longrightarrow \underline{\text{fast}} \\ \dot{n} = f_n(V,n) & \longrightarrow \text{slow} \\ \dot{h} = f_h(V,h) & \longrightarrow \text{slow . . .} \end{array}\right.$$

So get (Gerstner 5.15)

$$C\dot{V} = \overbrace{\bar{g}_{Na}\,(m^{\infty}(V))^3\,\bar{h}\,(E_{Na}-V) + \bar{g}_k\,\bar{n}^4\,(E_k - V) + g_L(E_L - V)}^{F(V)} + I(t)$$

Fourcaud-Trocme 2003:

$$F(V) \simeq \frac{-V - V_{rest}}{\tau} + \frac{\Delta}{\tau}\exp\left(\frac{V - V_t}{\Delta}\right)\text{, i.e.,}$$

is well-fit by linear + exp. terms. This defines the

Slide 5.3] (plot this.)

($\underline{\text{EXPONENTIAL}}$ INTEGRATE + FIRE MODEL.) $\underline{(\text{EIF})}$

· The EIF has also been successfully fit to data.

Badel et al, 2008. Idea:

$$c\,\frac{dV}{dt} = F(V(t)) + I(t)$$

Known current inject into neuron, measure resulting $V(t)$ (& hence $\frac{dV}{dt}$).

Let $F(V(t)) = \left\langle c\,\frac{dV}{dt} - I(t) \right\rangle$

for given $V(t)$

Avg. over all times when voltage = $V(t)$.

[Slide... Gerstner 5.5 ... AND 5.6, which gives addn! time-dep.
5.4

by re-fitting model in diff. time windows following spike.

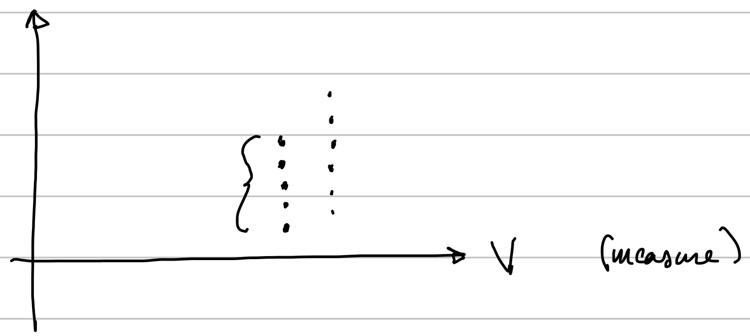
* "Best" value of C is one that minimizes the width of the V-conditioned distrib. above.

SIDE/EXTRA NOTES...

Thinking about ... choosing value of C:

$C\frac{dV}{dt} - I(t)$

(measure)



"base point"

Fix, V.

For any C, make scatter of $C\frac{dV}{dt} - I(t)$.

← For each, mean = F(V)

choose C so that this scatter minimized

(averaged over all values of V)

Trivial, $C=0$ works only if $I(t) \equiv$ const ...