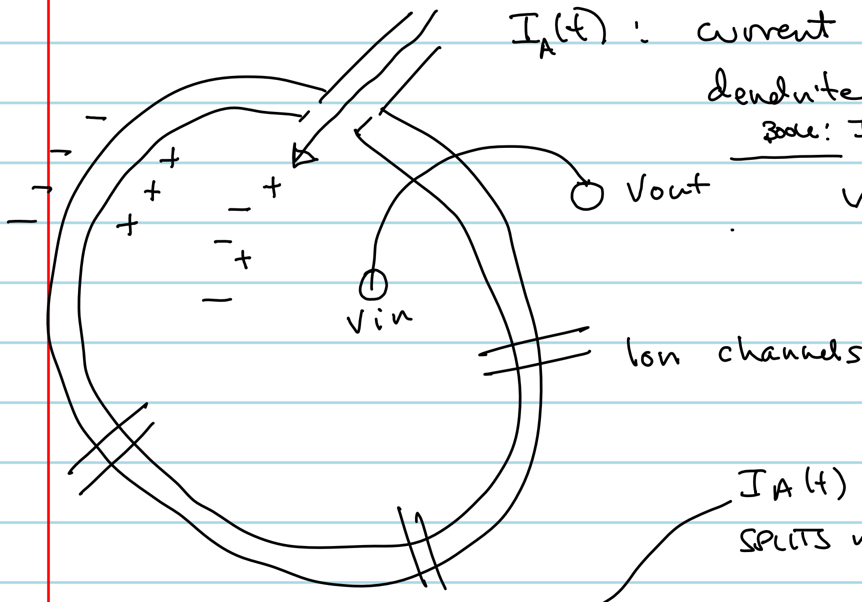
Equiv. circuit

model of single neuron dynamics

(1)



INPUT

$I_A(t)$: current (net + charge), from dendrite (OR experimenter) e.g.

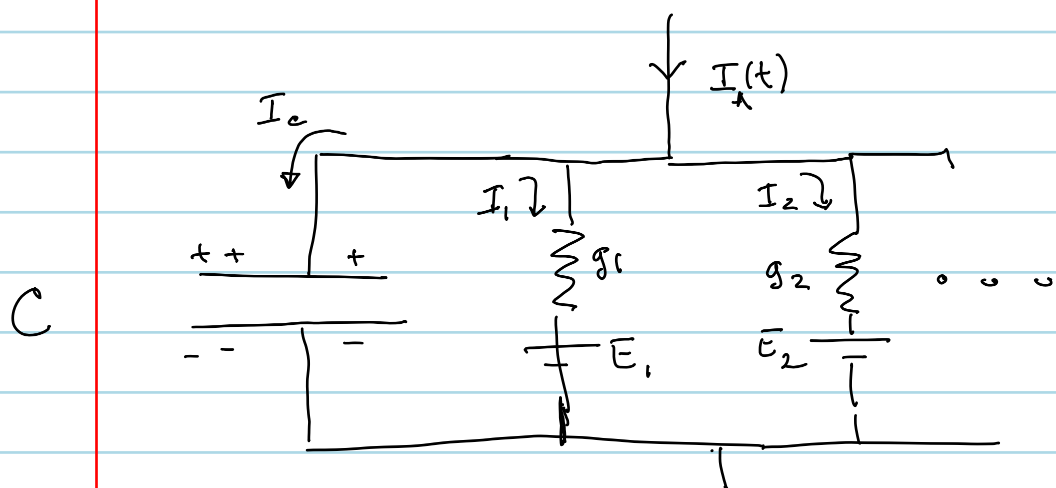
Code: $I_{stim}(t)$

$V = V_{in} - V_{out}$, voltage

$I_A(t)$ must go somewhere!

SPLITS into $\underbrace{I_c}_{\text{capacitive current}} + \underbrace{I_1 + I_2 + \cdots}_{\text{ion channel currents}}$

(2)



IN

OUT

CAPACITOR: $Q = CV$

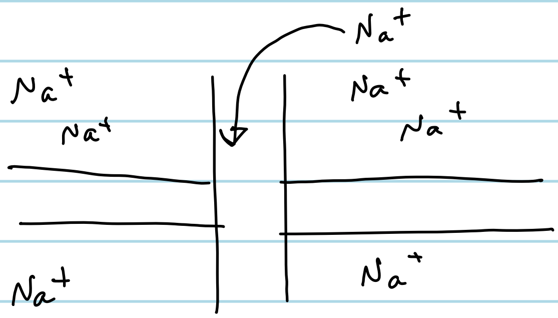
$$\frac{dQ}{dt} = I_C$$

Q: Charge on "top" wall / plate
C: capacitance
Current: Change in charge / unit time

$$\Rightarrow I_C = \frac{d}{dt}(CV) = C\frac{dV}{dt}$$

OHM'S LAW

(2) Ion channel: $I_1 = g_1(V - E_1)$.



- passes specific type or types of ions only, eg Na^+
- conductance $g = \frac{1}{R}$

"how open it is"

- Two "forces" drive current:
  (1) Diffusion: is there more Na^+ inside or outside
  (2) Voltage V, which pushes on charges

E_1 is "reversal" potential where these balance.

- Repeat for other channels. Pass different ions $\longrightarrow$ different E_k

- $I_A(t) = I_c(t) + \sum_k I_k(t)$

$$I_A(t) = C\frac{dV}{dt} + \sum_k g_k(V - E_k) \quad \ldots \text{so} \ldots$$

(1) $$C\frac{dV}{dt} = I_A(t) + \sum_k g_k(E_k - V)$$

- Passive Membrane / "RC Circuit" form of (1).

Combine all currents k that have g_k conductances that are approximately constant over time →

"net" current

$$C \frac{dV}{dt} = I_A(t) + g_L \; (E_L - V) \quad \ldots$$

AND ignore other "spike generating," etc currents.

(Formally, redefine $V \to V - E_L$)

- For convenience, set $E_L = 0$.
  → RC circuit eq^{ns} below

$$C \; \frac{dV}{dt} = I_A(t) - gV = I_A(t) - \frac{1}{R} V$$

$$\frac{dV}{dt} = \frac{-1}{RC} V + \frac{I_A(t)}{C}$$

- Differential equations + linear systems.

$$\frac{dy}{dt} = f(y, t)$$ [...rule / rate of change ...]

$$\frac{y(t+\Delta t) - y(t)}{\Delta t} = f(y, t)$$

$$y(t+\Delta t) = y(t) + f(y, t) \cdot \Delta t \quad ; \; y(0) = y_0.$$

Euler method! Works for ANY $f(\,)$.
$\Rightarrow$ (often only option...)

- Example:

$$\frac{dV}{dt} = V \quad ; \quad V(0) = V_0 = 1.$$

Solution: $V(t) = V_0 e^t$ (check).

Euler-illustrateV.m

(use $\Delta t = 0.1$, 0.001 + zoom)

Accuracy?

Each timestep, make local error $= k \cdot \Delta t^2$

→ Challenge: Use T-series to show this...

Global error: $\underbrace{\frac{T_{max}}{\Delta t}}_{\#\text{ timesteps}} \cdot k \Delta t^2 = k_2 \cdot \Delta t$

"1st order accurate method"

(R-K methods - like ode45 - get $k \Delta t^4$ - explain why important)